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Relativistic spin effects in the Earth-Moon system

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Relativistic precession and nutation effects applied to the Earth and Moon coordinate systems are discussed in detail. Apart from the effects common to the two reference systems (including the well-known geodetic precession), the lunar reference frame undergoes an additional precession of 28.9 millarc sec/century. This value is theoretically within the range of the lunar laser ranging (LLR) technique in the forthcoming years, but impossible to decorrelate from other secular effects of selenophysical origin. Because of a nearly perfect cancellation of the de Sitter and Lense-Thirring phenomena, additional nutations of the lunar referential are below the sensitivity of the LLR. An analogous cancellation occurs also for the secular part of the Lense-Thirring precession of the Earth-Moon referentials. A terminological ambiguity of the geodetic precession constant notion, related to the inclusion of some of the Lense-Thirring terms, is pointed out.

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I. INTRODUCTION

Recently, Damour and co-workers [1–3] have developed a general scheme for the modeling of post-Newtonian dynamics (both translational and rotational) of a system of weakly self-gravitating extended bodies. A detailed theory of the coordinate systems is a natural part of the approach.

In this paper we address the problem of the correspondence between local systems attached to a given moving body and the global system used to describe the motion of all bodies. Although principles of general relativity admit arbitrary coordinate systems, it appears that judicious choice makes the mathematical problem much simpler. Such a kind of coordinate system is commonly called *dynamically nonrotating* because of the absence of the relativistic Coriolis effects [3] (sometimes also "quasi-inertial frame" [4–6]). Another commonly recommended choice for the rotational state of the local coordinate systems, called *kinematically nonrotating*, does not relate to the dynamical principles but to the simplicity of the coordinate transformation between the local and global systems. In the latter approach, recommended by the IAU Working Group on Reference systems [7,8], the direction of the local frame axes coincides with that of the global one. In this case one has to apply the relativistic Coriolis force "$\propto \mathbf{\Omega}_\tau \times \mathbf{S}$" in local body systems. In both approaches the relativistic spin effects remain. One naturally faces the question of implementation and observability of these phenomena in case of actual dynamical systems.

The Earth-Moon system seems to be promising for detection of the relativistic effects thanks to the availability of very high accuracy positional data obtained by the lunar laser ranging (LLR) technique. The lunar motion is thus the most precisely known among the natural bodies because of the current LLR technology precision of about 2–3 cm for the distance. In the coming decade, with the use of multicolor lasers, an improvement is expected up to the ultimate limit of about 2–3 mm for normal points of the lunar motion [Ch. Veillet (personal communication)]. Corresponding technical improvements are being implemented on the French lunar laser station at the Observatoire de la Côte d'Azur. Translated into amplitudes of the lunar librations the current best performances are on the level of 1 milliarc sec with an ultimate precision limit of about 0.3 milliarc sec.

Thanks to these high quality data the Earth-Moon dynamics has already been used for important relativistic tests. First, one has to mention the significant contribution to the equivalence principle testing via "the Nordtvedt effect" [9–12]. Second, the principal term of the de Sitter precession of the Earth-Moon center-of-mass frame has been detected with a precision of about 1% [10–14]. Finally, a theoretical modeling of a new class of relativistic lunar librations has been recently presented [15]. On the other hand, one must be aware that from the relativistic point of view the Earth-Moon system is "polluted" by a lot of phenomenologically modeled geophysical and selenophysical phenomena, namely those related to the inelastic deformation of the two bodies.

In this paper, we focus on detailed elaboration of the relativistic rotational effects of the reference frames in the Earth-Moon system. A subclass of these effects, common to the two referentials, may be attributed to the rotation of the Earth-Moon center-of-mass system (including the widely known de Sitter term having the magnitude of about 1.9194 arc sec/century). However, because of mutual Earth-Moon dynamics both referentials are also submitted to individual precession and nutations. Their principal terms are evaluated. It is shown that the addi-

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tional precession of the lunar reference frame is within the capabilities of the LLR technique. However, because of difficulties in decorrelating it from the other (phenomenologically modeled) effects, it is hardly seen in the data. A similar situation occurs for the newly described relativistic nutations of the lunar reference system because of fine cancellation of the de Sitter and Lense-Thirring phenomena. Corresponding terms of the Earth reference system coincide with those computed recently by Brumberg and co-workers [16,17]. Also the ambiguity in the precise definition of the geodetic precession constant is briefly discussed.

Finally, it is to be noticed that the phenomena discussed in this paper are not to be mistaken for the relativistic perigee precession of the lunar orbit because of the Earth mass gravity monopole [18,19].

II. RELATIVISTIC SPIN EFFECTS FOR REFERENCE SYSTEMS

A. Background

Formally, the relativistic Coriolis force term in the equations of rotational motion in a given local coordinate system of body A reads[1]

$$\left(\frac{dS_a^A}{dT_A}\right)_R^{\text{Coriolis}} = \frac{1}{2c^2}\epsilon_{abc}S_b^A H_c^A , \tag{2.1}$$

where S_a^A is the first post-Newtonian (1PN) spin vector of body A, T_A is the local coordinate time, ϵ_{abc} is the usual Levi-Civita fully antisymmetric symbol, and H_a^A is the central gravitomagnetic field in the coordinate system of the Ath body. Detailed analysis shows [3] that the latter quantity is composed of two parts, one depending on the dynamical state of the external bodies and another depending only on the dynamical state of the reference frame of the Ath body (i.e., on the manner how it is related to the global coordinate system). If V_A and A_A denote, respectively, the velocity and acceleration of the origin of the coordinate system of the Ath body, R^i_{Aa} the orthogonal rotation matrix involved in the transformation between the local system of body A and the global coordinate systems, $\bar{w}^A$ and $\bar{\mathbf{w}}^A$ the scalar and vector post-Newtonian potentials of the external gravitational field measured in the local system of body A, one has

$$\begin{aligned} H_a\left(T_A\right) = & -4\epsilon_{ijk}R^i_{Aa}\left[\partial_j\bar{w}^A_k + v^j_A\partial_k\bar{w}^A\right]_{\mathcal{L}_A} \\ & + \epsilon_{abc}\left[V^A_b A^A_c + c^2\frac{dR^i_{Ab}}{dT_A}R^i_{Ac}\right]_{\mathcal{L}_A} . \end{aligned} \tag{2.2}$$

An approximation of the potentials sufficient for our purposes is given by

$$\bar{w}^A = G\sum_{B\neq A}\left[\frac{M_B}{r_B}\right]_{\mathcal{L}_A} + O(2) , \tag{2.3a}$$

$$\bar{w}^A_i = G\sum_{B\neq A}\left[\frac{M_B}{r_B}v^i_B - s^{ji}_B\partial_j\left(\frac{1}{r_B}\right)\right]_{\mathcal{L}_A} + O(2) , \tag{2.3b}$$

where M_B's are the mass monopoles of the individual bodies, $r^i_B = x^i - z^i_B$ with z^i_B being the coordinate, in the global reference system, of the frame with origin at B and $s^{ij}_B = \epsilon_{ijk}s^k_B = \epsilon_{ijk}R^k_{Aa}S^a_B$. Symbol $\mathcal{L}_A$ means that the bracket term has to be evaluated at the origin of the coordinate system of body A and $O(2) = O(c^{-2})$ points neglected post-Newtonian terms. Notice the absence of spin contribution to the scalar potential $\bar{w}^A$. As for the rotation matrix R^i_{Aa}, either it is submitted to the trivial condition $R^i_{Aa} \equiv \delta_{ia}$ to express the absence of kinematical rotation or one adjusts its value in order to cancel the gravitomagnetic field $H^A_a(T_A)$ at any instant, which is equivalent to the absence of dynamical rotation. Although this choice has been adopted in this paper, our results can be easily transposed in the other way, with the appearance of the Coriolis term in Eq. (2.1).

Introducing the angular velocity vector Ω^A_i characterizing rotation of the dynamically nonrotating frame of the body A with respect to the global coordinate system by

$$\Omega^A_i = -\frac{1}{2}\epsilon_{ijk}\frac{dR^j_{Aa}}{dT_A}R^k_{Aa} , \tag{2.4}$$

we obtain

$$\vec{\Omega}^A = -\frac{1}{2c^2}\vec{\mathbf{v}}_A\times\vec{\mathbf{a}}_A + \frac{2}{c^2}\vec{\mathbf{v}}_A\times\nabla\bar{w}^A + \frac{2}{c^2}\nabla\times\vec{\bar{\mathbf{w}}}^A , \tag{2.5}$$

which has to be compared with corresponding terms in [18,20,21]. Notice, that in the parametrized version of the post-Newtonian theory factors 2 in the last two terms of Eq. (2.5) are to be replaced by $(1+\gamma)$, where γ is the traditional Eddingtonian post-Newtonian parameter. If only gravitational forces are applied one advantageously combines the first two terms into a single one with factor $(\gamma+\frac{1}{2})$. In agreement with the common terminology [18] we shall refer to them as the geodetic terms. The last term in Eq. (2.5) then expresses gravitomagnetic or Lense-Thirring phenomena.

In the following, we shall investigate the influence of all terms in Eq. (2.5) on the dynamics of the coordinate frames in the Earth-Moon(-Sun) system. We adopt the following notation: index C is reserved for the Sun, index B for the Moon (with mass M_B) and index A for the Earth (with mass M_A). Since Eq. (2.5) is limited to the terms in $1/c^2$, we can use for the vectors of position, velocity, and acceleration Newtonian approximation. The center-of-mass of the Earth-Moon system is assumed to move on an elliptic orbit with semimajor axis a_1, eccentricity e_1, and mean motion n_1 around the Sun, which is supposed to be located at the center-of-mass of the global coordinate system. We thus neglect the solar motion with respect to the global system, an

[1] As discussed in detail in Ref. [3], it is this term which can take the kinematical interpretations of the body A reference frame relativistic rotation.

approximation justified by the inverse mass ratio of the Sun and the Earth-Moon system of about 328 900. The mutual Earth-Moon dynamics is approximated by the elliptical motion with semimajor axis a_2, eccentricity e_2, inclination I with respect to the ecliptic plane and mean motion n_2. The following abbreviations will be used: $M_{AB} \equiv M_A + M_B$, $\iota = \sin I$. If $\mathbf{R}$ denotes the position of Earth-Moon center-of-mass in the global system and $\mathbf{r}(= \mathbf{r}_{BA}) = \mathbf{z}_B - \mathbf{z}_A$ the Earth-Moon vector, positions of the Earth and Moon in the global coordinate system are

$$\mathbf{z}_A = \mathbf{R} - \frac{M_B}{M_{AB}}\mathbf{r} , \tag{2.6a}$$

$$\mathbf{z}_B = \mathbf{R} + \frac{M_A}{M_{AB}}\mathbf{r} , \tag{2.6b}$$

and similarly for the velocities. An analogous relation can also be established for the accelerations provided the solar tidal terms are neglected in the Earth-Moon center-of-mass system. Within the accuracy aimed at in this paper, these terms can be safely omitted.

Our representation of the Earth-Moon(-Sun) dynamics is thus much simpler than in the case considered by Brumberg and co-workers [16,17] who implemented for similar calculations the analytical planetary theory VSOP87 [22]. However, simplified formulas given in the following sections yield a better view of the structure of the particular terms and allow us to localize and discuss clearly the discrepancy between geodetic precession constant given by Brumberg *et al.* [16,17] and by those of the IERS Standard [8] (also, e.g., in [23]).

As the investigated system is close to the planar three-body problem, remind that $\iota \approx 0.087$, the main relativistic precession and nutations contribute to the rotation around the ecliptic normal. It is thus natural to adopt an ecliptic global coordinate system with the z axis normal to the ecliptic plane. The following two sections are devoted to the analysis of the geodetic and Lense-Thirring terms related to the rotations around the ecliptic normal for both the terrestrial and lunar reference systems. In order to simplify notations, we omit the index "z" in the computed quantities. Final subsection D is devoted to the computation of new nutations of the lunar coordinate system around the in-ecliptic plane axes. It will be demonstrated that their amplitudes are surprisingly greater than those for the corresponding nutations about the third axis as a result of (i) a fine-tuning and cancellation of the geodetic and Lense-Thirring terms contributing to the out-of-ecliptic "z" component, and (ii) an associated long period, given by the lunar node rate, with the in-ecliptic components.

B. Application to the Earth-Moon(-Sun) system — geodetic terms

First, we focus on the secular geodetic phenomena resulting from the first two terms in Eq. (2.5). Inserting global frame velocity and accelerations of the investigated bodies A and B we obtain the secular rates

$$\Omega_A = \left(\gamma + \frac{1}{2}\right)\left[\left(\frac{n_1 a_1}{c}\right)^2 \frac{n_1}{1-e_1^2} + \left(\frac{M_B}{M_{AB}}\right)^2 \left(\frac{n_2 a_2}{c}\right)^2 n_2 \frac{\sqrt{1-\iota^2}}{1-e_2^2}\right] , \tag{2.7}$$

and

$$\Omega_B = \left(\gamma + \frac{1}{2}\right)\left[\left(\frac{n_1 a_1}{c}\right)^2 \frac{n_1}{1-e_1^2} + \left(\frac{M_A}{M_{AB}}\right)^2 \left(\frac{n_2 a_2}{c}\right)^2 n_2 \frac{\sqrt{1-\iota^2}}{1-e_2^2}\right] , \tag{2.8}$$

characterizing the relativistic precession of the corresponding referentials. After an additional integration of Eqs. (2.7) and (2.8), one sets the angles of precession of the two referentials [to be denoted θ_A and θ_B]

$$\theta_A = \left(\gamma + \frac{1}{2}\right)\left[\left(\frac{n_1 a_1}{c}\right)^2 \frac{l'}{1-e_1^2} + \left(\frac{M_B}{M_{AB}}\right)^2 \left(\frac{n_2 a_2}{c}\right)^2 l \frac{\sqrt{1-\iota^2}}{1-e_2^2}\right] , \tag{2.9}$$

and

$$\theta_B = \left(\gamma + \frac{1}{2}\right)\left[\left(\frac{n_1 a_1}{c}\right)^2 \frac{l'}{1-e_1^2} + \left(\frac{M_A}{M_{AB}}\right)^2 \left(\frac{n_2 a_2}{c}\right)^2 l \frac{\sqrt{1-\iota^2}}{1-e_2^2}\right] . \tag{2.10}$$

Here, l and l' are the mean anomalies of the Moon and the Sun. The first parts of the previous formulas, which are common to the two coordinate systems, are the usual de Sitter precession which have been known since 1916 [24]. Its rate is of about 1.9194 arc sec/century [23,25] and it is due to the solar gravity field. The second terms in Eqs. (2.9) and (2.10) are specific to each reference frame and follow from the mutual Earth-Moon gravitational interaction. Their difference reads

$$\delta\Omega_{AB} = \left(\gamma + \frac{1}{2}\right)\left(\frac{n_2 a_2}{c}\right)^2 n_2 \frac{\sqrt{1-\iota^2}}{1-e_2^2}\frac{M_A - M_B}{M_{AB}} . \tag{2.11}$$

If the Earth-Moon system parameters are substituted one obtains $\delta\Omega_{AB} \approx 28.9$ milliarc sec/century. Naturally, most of this is because of a secular advance of the lunar reference frame. Because of the small value of $(M_B/M_{AB})^2$ the Earth reference frame is subjected to an additional relativistic precession of about 0.004 milarc sec/century.

Notice that previous formulas for the secular rates of the two local coordinate systems relating to the orbiting bodies A and B are given in closed form in small parameters (ι, e_1, and e_2).

Periodic terms called nutations (following the terminology of Fukushima [25]), are superimposed to the geodetic precession. Analogously to the case of the secular effects mentioned above, the main relativistic nutation terms have their origin in the solar gravity (and are thus of "the tidal origin"). Annual and semiannual terms derived by Fukushima [25] have amplitudes of 0.153 milliarc sec and 0.002 milliarc sec (see also [15,26]). They are again common both to the Earth and Moon reference frames. In the following, we shall derive another class of relativistic nutations arising from the mutual Earth-Moon dynamics. As a consequence, their period equals one synodic month. Following the classical theories of the lunar motion, we shall denote the mean longitudinal elongation of the Moon and the Sun by the Delaunay variable D and the ratio of the mean motion of the Earth-Moon center-of-mass around the Sun and the lunar synodic mean motion by m [thus $m = n_1/(n_2 - n_1)$], and we have $m \approx 0.081$.

Inserting the global frame velocity and accelerations of the involved bodies into the first two terms of Eq. (2.5) and retaining only terms of the zeroth order in the power series in eccentricity e_2 and inclination ι we obtain (apart from the previously mentioned Fukushima terms)

$$\delta\Omega_A = -\left(\gamma + \frac{1}{2}\right) \frac{M_B}{M_{AB}} \frac{a_2}{a_1} \left(\frac{n_1 a_1}{c}\right)^2 n_2 \left(1 + \frac{n_2}{n_1}\right) \cos D + O(e_2, \iota^2) \,, \tag{2.12a}$$

$$\delta\Omega_B = \left(\gamma + \frac{1}{2}\right) \frac{M_A}{M_{AB}} \frac{a_2}{a_1} \left(\frac{n_1 a_1}{c}\right)^2 n_2 \left(1 + \frac{n_2}{n_1}\right) \cos D + O(e_2, \iota^2) \,, \tag{2.12b}$$

and the corresponding solution for the local system nutations,

$$\delta\theta_A = -\left(\gamma + \frac{1}{2}\right) \frac{M_B}{M_{AB}} \frac{a_2}{a_1} \left(\frac{n_1 a_1}{c}\right)^2 \frac{(1+m)(1+2m)}{m} \sin D + O(e_2, \iota^2) \,, \tag{2.13a}$$

$$\delta\theta_B = \left(\gamma + \frac{1}{2}\right) \frac{M_A}{M_{AB}} \frac{a_2}{a_1} \left(\frac{n_1 a_1}{c}\right)^2 \frac{(1+m)(1+2m)}{m} \sin D + O(e_2, \iota^2) \,. \tag{2.13b}$$

In the case of the lunar referential these synodic terms have an amplitude comparable to the first Fukushima nutation term (approximately 0.1 milliarc sec). We shall, however, see in the next section, that a great part of the effect is canceled by the corresponding synodic term because of the Lense-Thirring effect.

C. Application to the Earth-Moon(-Sun) system — Lense-Thirring terms

Next, we evaluate the principal effects due to the last, Lense-Thirring, term of Eq. (2.5). By restricting to the secular effects and substituting the vector potential (2.3b) into formula (2.5) we obtain two kinds of effects. The first is because of the solar angular momentum and reads

$$\Omega_A = -\frac{\gamma+1}{c^2} \frac{G S_C}{a_1^3 (1-e_1^2)^{3/2}} \,, \tag{2.14}$$

while the second, linked to the total angular momentum of the Earth-Moon system, reads

$$\Omega_A = (\gamma+1) \frac{M_A M_B}{M_{AB}^2} \left(\frac{n_2 a_2}{c}\right)^2 n_2 \frac{\sqrt{1-\iota^2}}{1-e_2^2} = \frac{\gamma+1}{c^2} \frac{G L_{AB}^z}{a_2^3 (1-e_2^2)^{3/2}} \tag{2.15}$$

[G is the gravitational constant]. The same contributions appear in the Lense-Thirring precession of the body B frame. Interestingly, both secular Lense-Thirring phenomena nearly cancel out for the Earth-Moon system. The Earth-Moon angular momentum term (2.15) yields a value of about 0.47 milliarc sec/century and the solar induced term (2.14) amounts to -0.28 milliarc sec/century. One must notice, however, that the latter depends on the total solar angular momentum S_C, which is not well known [27].

In the most complete investigation of the relativistic spin effects of the Earth reference frame by Brumberg and co-workers [16,17], the authors seem to include arbitrarily the second Lense-Thirring precession (2.15) in the definition of the geodetic precession constant (in the broader sense following their terminology). As a result, their corrected value of the geodetic precession constant is 1.9199 arc sec/century instead of the value 1.9194 arc sec/century deduced from the first two terms of Eq. (2.5). In our opinion it might not be a favorable route, as the first Lense-Thirring term (2.14), investigated previously, is of the same order of magnitude as the second one. Moreover, not only has it an opposite sign but its value is uncertain, at least by a factor of 2, because of the uncertainty of the total solar angular momentum [27]. It may thus happen by chance that the Brumberg *et al.*'s additional term in the geodetic precession constant will be canceled by an opposite contribution because of the solar rotation Lense-Thirring term. However, it should be acknowledged that the discussed Lense-Thirring originated terms are by a factor of 30 smaller than the current precision level of the dynamical determination of the geodetic precession [9–12]. The dominant cause of this uncertainty is because of the poorly known second zonal harmonic J_2 of the Moon [11,12]. Of course, an improvement in the determination of this parameter by future lunar missions combined with a larger time span of the LLR data will make the determination of the geodetic precession more accurate. However, with current state of the art, it is unlikely that the dynamics of the Earth-Moon system could contribute significantly to a precise determination of the Lense-Thirring effect, and other opportunities are discussed, for instance, in Ref. [18].

As for the periodic terms in the Lense-Thirring (LT) effect (LT nutations), they are given by[2]

$$\delta\Omega_A = (\gamma+1)\,\frac{M_B}{M_{AB}}\frac{a_2}{a_1}\frac{n_2}{n_1}\left(\frac{n_1 a_1}{c}\right)^2 n_2 \cos D + O(e_2,\iota^2)\ , \tag{2.16a}$$

$$\delta\Omega_B = -\,(\gamma+1)\,\frac{M_A}{M_{AB}}\frac{a_2}{a_1}\frac{n_2}{n_1}\left(\frac{n_1 a_1}{c}\right)^2 n_2 \cos D + O(e_2,\iota^2)\ , \tag{2.16b}$$

which give after integration the nutation angles, as

$$\delta\theta_A = (\gamma+1)\,\frac{M_B}{M_{AB}}\frac{a_2}{a_1}\left(\frac{n_1 a_1}{c}\right)^2 \frac{(1+m)^2}{m}\sin D + O(e_2,\iota^2)\ , \tag{2.17a}$$

$$\delta\theta_B = -\,(\gamma+1)\,\frac{M_A}{M_{AB}}\frac{a_2}{a_1}\left(\frac{n_1 a_1}{c}\right)^2 \frac{(1+m)^2}{m}\sin D + O(e_2,\iota^2)\ . \tag{2.17b}$$

Collecting the synodic nutations given previously, both of the geodetic and Lense-Thirring origin, we observe that a significant part of them cancels out. The final relativistic synodic signal reads

$$\delta\theta_A = \frac{1}{2}\frac{M_B}{M_{AB}}\frac{a_2}{a_1}\left(\frac{n_1 a_1}{c}\right)^2 \frac{1+m}{m}\,(1-2\gamma m)\sin D + O(e_2,\iota^2)\ , \tag{2.18a}$$

$$\delta\theta_B = -\frac{1}{2}\frac{M_A}{M_{AB}}\frac{a_2}{a_1}\left(\frac{n_1 a_1}{c}\right)^2 \frac{1+m}{m}\,(1-2\gamma m)\sin D + O(e_2,\iota^2)\ . \tag{2.18b}$$

Substituting the relevant numerical values into formulas (2.18) we obtain an amplitude of 0.03 milliarc sec for the lunar coordinate system, and 4×10^{-4} milliarc sec for the terrestrial coordinate system. The latter value related to the Earth referential coincides with the result of Brumberg and co-workers (Ref. [16], Eq. (4.3)).

Tables I(a) and I(b) show briefly various phenomena studied throughout the text. Phenomena common to the two referentials are distinguished from the individual terms.

D. Application to the Earth-Moon(-Sun) system — ecliptic terms

Finally, we discuss briefly relativistic nutations around the in-ecliptic axes $(\mathbf{e}_x, \mathbf{e}_y)$. The two main motivations in performing this work are as follows: (i) a fine cancellation of the lunar nutations discussed previously, (ii) possible appearance of small divisors related to the small rate angles (e.g., longitude of the lunar orbit node Ω). Careful inspection of the Brumberg *et al.* [16] results strongly supports this possibility.

Substituting necessary components of the global system velocities and accelerations into Eq. (2.5) we find dominant nutation terms in the form

$$\delta\theta^x_B = -\left(\gamma+\frac{1}{2}\right)\left(\frac{M_A}{M_{AB}}\right)^2\left(\frac{n_2 a_2}{c}\right)^2 \iota\,\frac{n_2}{\dot\Omega}\cos\Omega\ , \tag{2.19a}$$

$$\delta\theta^y_B = -\left(\gamma+\frac{1}{2}\right)\left(\frac{M_A}{M_{AB}}\right)^2\left(\frac{n_2 a_2}{c}\right)^2 \iota\,\frac{n_2}{\dot\Omega}\sin\Omega\ . \tag{2.19b}$$

The amplitude of both terms in Eqs. (2.19) is approximately 0.09 milliarc sec, greater than that of the relativistic nutations around the axis normal to the ecliptic plane. Considering a rotating axis $\mathbf{n}(\Omega) = \cos\Omega\,\mathbf{e}_x + \sin\Omega\,\mathbf{e}_y$ in the ecliptic plane, one can also represent nutations

TABLE I. Schematic description of the relativistic spin effects related to the definition of the local (dynamically nonrotating) lunar (LS) and terrestrial (TS) coordinate systems. Secular effects, denoted as "precession," are given in part (a), periodic effects, denoted as "nutations," are given in part (b) (P in the fourth column indicates the period). Common terms which can be equally assigned to the Earth-Moon center-of-mass coordinate system (including the principal de Sitter term) appear between the columns of the two reference systems. Common Lense-Thirring precession is split into two pieces: (i) LT–1 originating in the coupling between the solar gravity field and the angular momentum of the Earth-Moon system (the term added to the newly proposed value of "the geodetic precession constant"), (ii) LT–2 originating in the coupling between the solar angular momentum and the Earth-Moon translational motion. The individual geodetic and Lense-Thirring nutations of both reference systems are of opposite sign and of comparable magnitude.

(a) Precession (milliarc sec/century)

Source terms	TS	Common	LS
Principal geodetic		1919.36	
Individual geodetic	0.004		28.86
LT − 1		0.47	
LT − 2		-0.28 (error ≈ 100%)	

(b) Nutations (milliarc sec)

Source terms	TS	LS	P
Fukushima − 1	0.153 13		1 year
Fukushima − 2	0.001 92		0.5 year
Individual geodetic	-0.001 47	0.119 64	29.5 days
Individual LT	0.001 82	-0.148 32	29.5 days

[2]We again omit all eccentricity and inclination ι-dependent terms.

(2.19) as a precession around $\mathbf{n}(\Omega)$ with the rate of 2.58 milliarcsec/century. Finally, one must bear in mind that excitations associated with the long-period planetary perturbations are likely to exist (see [16]).

Similar search for the dominant terms in the case of the terrestrial reference frame allows to recover 18.6 year terms in [16]. Interestingly, these Earth system nutations are of Lense-Thirring origin. Inversely, corresponding lunar nutation terms given in formulas (2.19) originate in the geodetic part of Eq. (2.5).

III. CONCLUSION

The relativistic precession and nutation of the lunar and terrestrial coordinate systems has been examined. The main terms connected to the Earth-Moon center-of-mass referential have been identified together with individual terms of the two local coordinate systems related to the mutual Earth-Moon dynamics. In particular, those related to the relativistic effects of the terrestrial reference system coincide with the principal terms given by Brumberg *et al.* [16].

New terms include mainly additional relativistic precession of the lunar reference system with the rate of 28.9 milliarc sec/century. This effect is in principle "measurable" using the best LLR data, but because of the secular character that can be hardly separated from the tidal secular effects (e.g., [28]). But it might be wise to subtract this value from the observed lunar data to achieve a better fitting of the tidal terms to the observed ranges. The corresponding secular Lense-Thirring term of the common Earth-Moon center-of-mass system is very small, largely because of the cancellation of the term linked, respectively, to the solar and the Earth-Moon angular momenta. Even with an improved LLR technology, its measurement will be hardly feasible. Important experimental confirmation of the Lense-Thirring effect thus requires the use of other techniques (e.g., satellite born). The amplitudes of the new relativistic lunar nutations are unfortunately very small, in part as a result again of a nearly complete cancellation of the geodetic and Lense-Thirring terms.

Corresponding results for the Earth reference frame are also small if compared with the capabilities of the current astrometric techniques reaching maximally the level of a few 0.01 milliarc sec [29,30]. However, future astrometric projects [31,32] at the microarc sec level will need to allow for the Earth frame precession and nutations given in the previous sections and in Ref. [16].

Finally, it is useful to draw attention to the question of terminology when speaking of the geodetic precession. Its "broader sense" pioneered by Brumberg and co-workers [16] (later also included in [33,34]) includes in fact a Lense-Thirring term resulting from a coupling between the solar gravity field and the angular momentum of the Earth-Moon system. Another Lense-Thirring term because of the coupling between the solar angular momentum and the translational motion of the Earth-Moon center of mass, ignored in the analysis of Brumberg *et al.*, however, is of the same order of magnitude but of opposite sign. Moreover, the latter term has an uncertainty of nearly 100% because of the badly determined solar angular momentum. Terminological unification among the astronomical community is desirable in the future, as the IERS Standard recommended value still follows geodetic precession in the "narrow sense" based only on the first two terms of the fundamental formula (2.5).

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