

Emission-Line Spectroscopy and Analysis

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November 2017

Various topics to be addressed:

- Processes that govern ISM/nebular excitation and ionization
- Diagnostic information obtained from different kinds of transitions
- Why most ionized (nebular) regions have kinetic $T_e \sim 10^4$ K
- The Balmer decrement (H recombination spectrum)
- Why forbidden lines are prominent in ionized gas regions
- e-databases useful for spectroscopy

Reference Books:

- Osterbrock, D. E. & Ferland, G. J. 2006, *Astrophysics of Gaseous Nebulae and Active Galactic Nuclei* 2nd edn (Sausalito: Univ. Sci. Books)
- Pradhan, A.K. & Nahar, S.N. 2011, *Atomic Astrophysics & Spectroscopy* (Cambridge: Cambridge Univ. Press)
- Bashkin, S. & Stoner, J. 1975, *Atomic Energy Levels & Grotrian Diagrams* (North-Holland: Amsterdam)
- v2.05 Atomic Line List website: <http://www.pa.uky.edu/~peter/newpage/>
- NIST Atomic Spectra Database website: <https://www.nist.gov/pml/atomic-spectra-database>

stanley bashkin and john o. stoner, jr

atomic energy levels & grotrian diagrams 1

hydrogen I – phosphorus XV



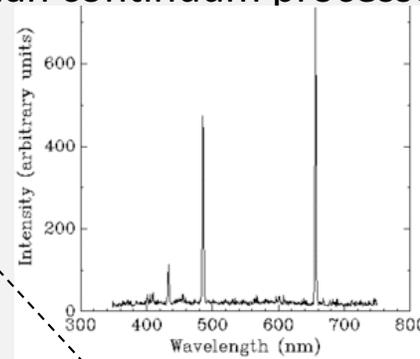
north-holland/american elsevier

Lecture 1

- Emission spectra
- Spectroscopic notation
- Atomic processes
- Interaction rates
- Cross sections

Emission Spectrum: produced in **Statistical Equilibrium**:

- Mean free path of photons $>$ size of object ('*opt thin*')
- Due to bound-bound (line) transitions have larger cross sections than continuum processes for emission



Emission Spectrum

- Originate in low density objects
- (Photon) mean free path $= 1/(n\sigma)$

$$(n \propto R^{-3} \text{ for constant mass}) \propto R^3$$

Therm Equilib: applies whenever physical processes that determine thermodynamic properties of a gas (P, n, T) occur over distance scales less than distances over which n & T change. (Mean free path small \rightarrow isolation)

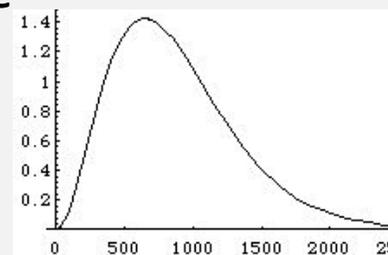
Continuous Spectrum: produced whenever

- Mean free path of photons \ll size of object ('*opt thick*')
- In high density, **Thermodynamic Equilibrium** applies:

Isolation \leftrightarrow 'Detailed Balance' \leftrightarrow reaction rates = their inverse

- > intensity, $I_\nu = \text{Planck fn } B_\nu(T)$
- > velocity distribution, $f(v) = \text{Max-Boltzmann fn}$
- > ionization & excitation = Saha and Boltzmann eqns

Continuous Spectrum





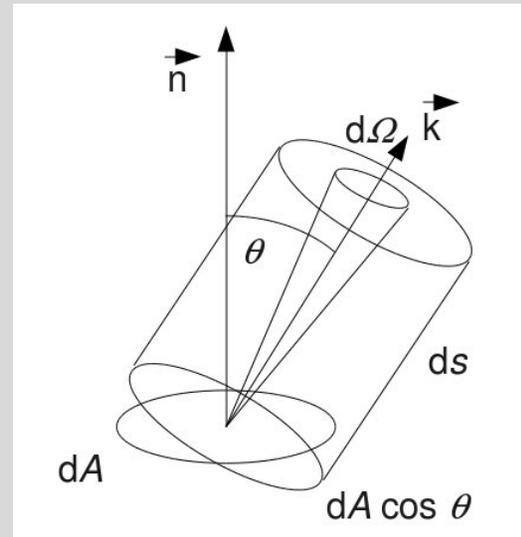
Emission Spectrum

Define:

Intensity, I_{ν} = radiant energy crossing surface per unit (area•time•solid angle•freq) [erg/(cm² sec ster Hz)]

Flux, F_{ν} = radiant energy crossing surface *in all directions* per unit (area•time•freq) [erg/(cm² sec Hz)]

$$= \int I_{\nu} \cos \Theta d\Omega$$



Equation of Radiative Transfer:

$$dI_{\nu}/ds = j_{\nu} - \kappa_{\nu} I_{\nu} \longrightarrow I_{\nu} = \int j_{\nu} ds \quad \text{if no absorption}$$

(Volume) Emission coefficient, j_{ν} = energy emitted per unit (volume•time•solid angle) [erg/(cm³ sec ster)]
for transition $i \rightarrow j$

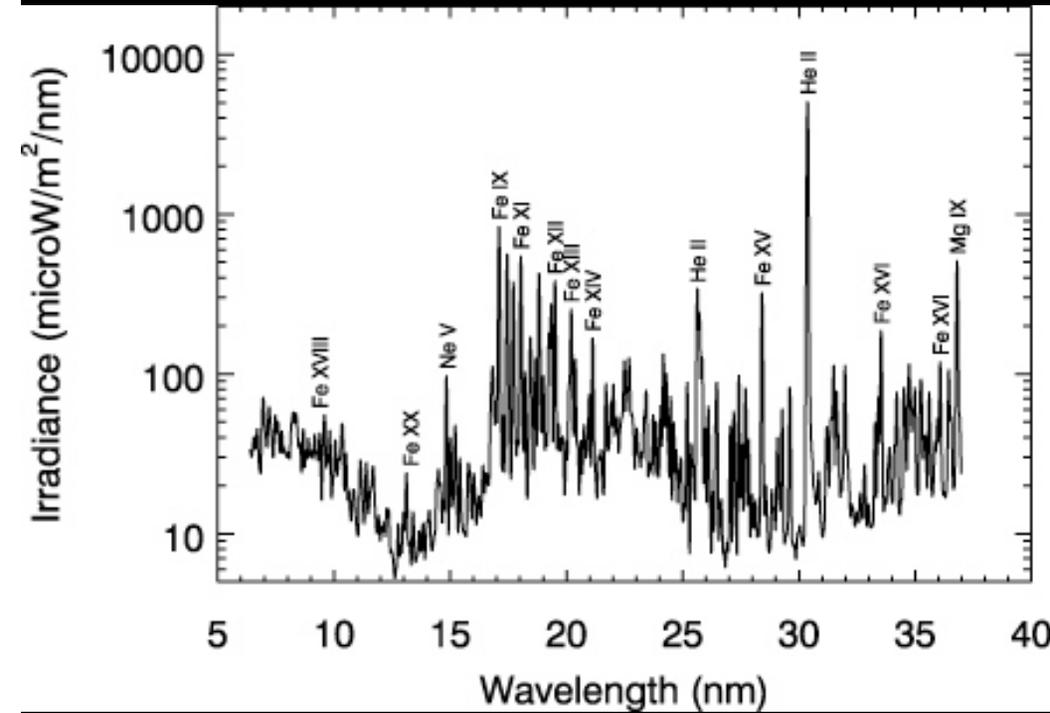
$$= n_i A_{ij} h\nu_{ij}/4\pi \quad \text{for emission *line* transitions } i \rightarrow j$$

(Volume) Absorption coefficient, κ_{ν} = $n a_{\nu} = 1/(\text{mean free photon path})$ [cm⁻¹]

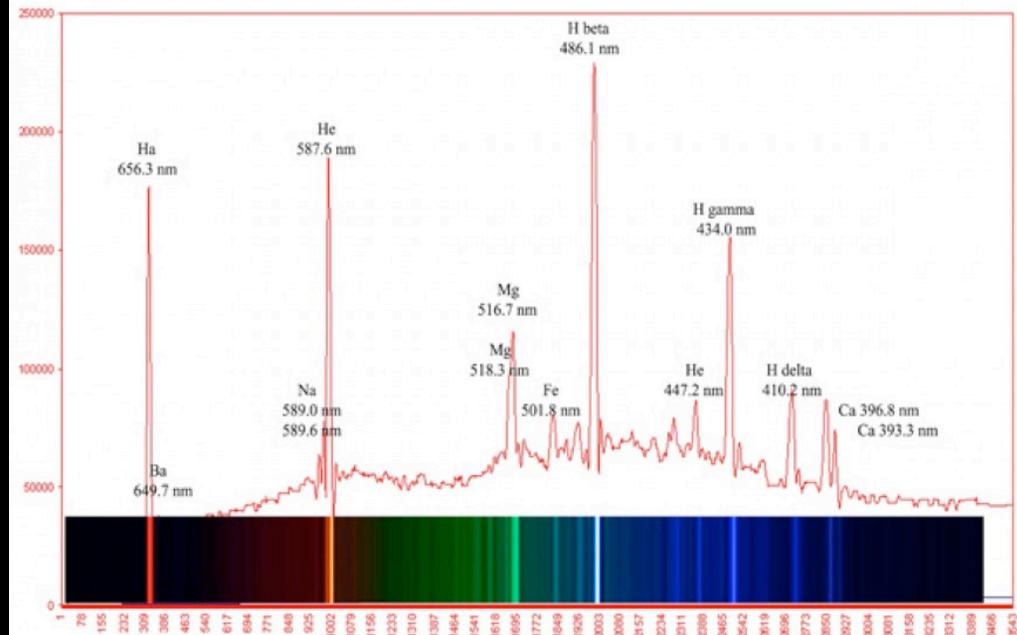
where n = density of absorbers [cm⁻³]

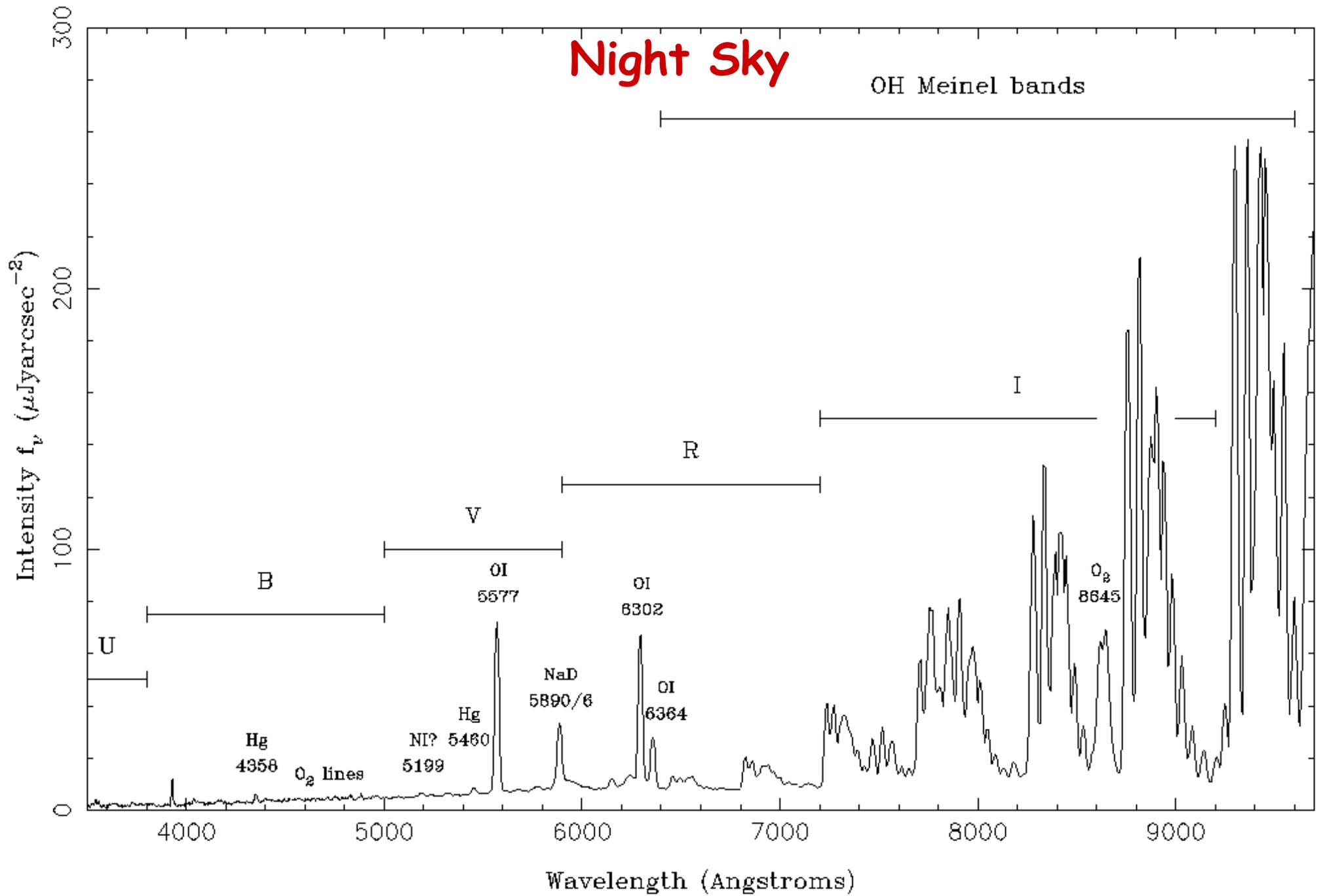
a_{ν} = absorption cross section [cm²]

Solar Corona & Chromosphere

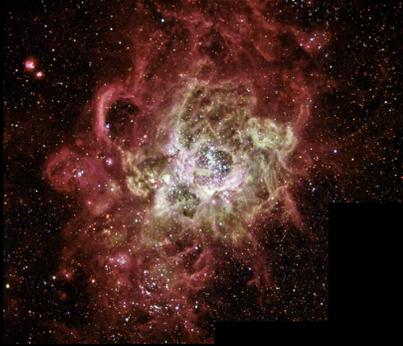


The Solar Chromosphere Spectrum (Flash Spectrum)





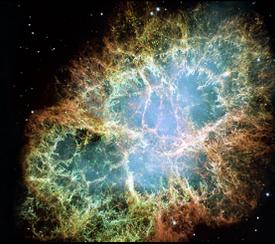
Ionized ISM Constituents



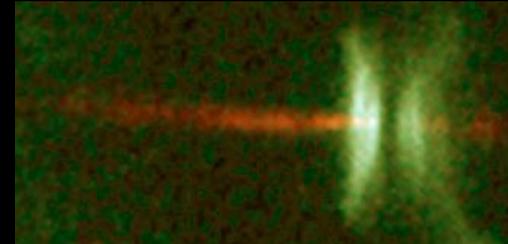
H II Regions



Planetary Nebulae



Nova & Supernova Remnants



Herbig-Haro Object Jet

Size

1-100 pc

0.2 pc

1 - 50 pc

1,000 au

Mass

$10^{4-6} M_{\odot}$

$0.3 M_{\odot}$

$1-10 M_{\odot}$

$10^{-6} M$

Lifetime

10^{7-8} yr

2×10^4 yr

10^4 yr

10^6 yr

Abundances

Solar

CNO~2 Solar

>10 Solar

~Solar

Expansion velocities

5-10 km/s

20-30 km/s

10^{2-3} km/s

100 km/s

Energy source

O Association
hot stars

RG→WD hot central star

Explosion of SN
+ pulsar

Stellar grav
Potential; mag field

Atomic Energy Levels

- Energy levels are determined by the **Schroedinger Eqn**

$$H\psi = E\psi$$

where H is the Hamiltonian of a charged particle with spin in an electric field:

$$H = p^2/2m - Ze^2/4\pi\epsilon_0 r + (\hbar/2m) \underline{s} \cdot \underline{l} + (\hbar/2m) \underline{s} \cdot \underline{s}$$

In quantum mechanics the momentum operator p can be replaced by $\hbar/i \partial/\partial x$.

For a spherically symmetric system $H\psi = E\psi \longrightarrow -\hbar^2/2m \nabla^2\psi - Ze^2/4\pi\epsilon_0 r = E\psi$

In spherical polar coordinates:

$$-\hbar^2/2mr^2 \left[\partial/\partial r (r^2 \partial\psi/\partial r) + 1/\sin\theta \partial/\partial\theta (\sin\theta \partial\psi/\partial\theta) + 1/\sin^2\theta (\partial^2\psi/\partial\Phi^2) \right] - Ze^2\psi/4\pi\epsilon_0 r = E\psi$$

The solution of the Schroedinger eqn for the single e^- atom can be written generally in the form

$$\psi_{nlm} = A_{nlm} L_{nl}(r) P_{lm}(\cos\theta) e^{im\Phi}$$

with eigenstate energy: E_{nlms}


Laguerre & Legendre polynomials

Energy Levels & Spectroscopic Notation

- The energy levels of many-electron atoms/ions are specified by the quantum numbers of the electrons:

$n_i \rightarrow$ orbit of each electron i

$\ell_i \rightarrow$ orbital angular momentum of each electron, with $\ell_i < n_i$ ($\ell_i=0$ [s], $\ell_i=1$ [p])

$L \rightarrow$ orbital angular momentum of the state = $\sum \ell_i$ (vector) ($L=1$ [S], $L=2$ [P])

$S \rightarrow$ spin angular momentum of the state = $\sum s_i$ (vector) (where $s_i = \frac{1}{2}$)

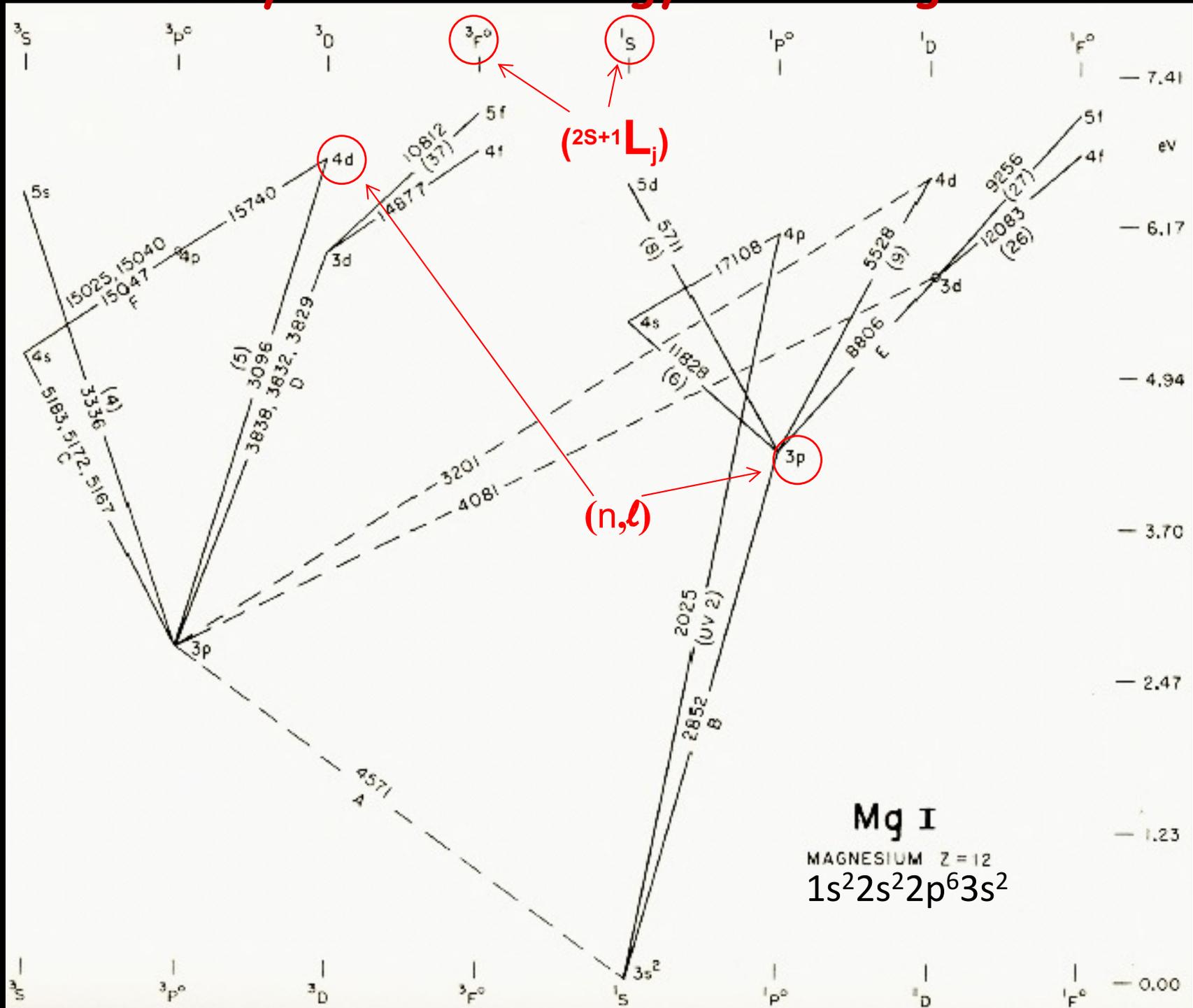
$J \rightarrow$ total angular momentum of the state = $\underline{L} + \underline{S}$ (vector)

- Specification of these quantities represents a distinct state with a unique energy. For "L-S coupling", the energy levels are most sensitive to the quantum numbers $n_i - \ell_i - L - S - J$ in that order.
- The spectroscopic notation of different energy states uses the convention:

$$(n_i, \ell_i) \ ^{2S+1}L_J \quad \text{for example: } [1s^2 2s^2 2p^6] 3s^2 \ ^1S_0$$

For example, the configurations $1s^2 \ ^1S_0$ & $1s2p \ ^3P_2$ represent two different energy states of helium, with the second state (3P_2) having the higher energy because one electron is in a higher orbit.

Many-Electron Energy Level Diagram



Ionized ISM Physical Processes

	Process	Description	Comments
R A D I A T I V E	Photoionization	$A^i + h\nu \rightarrow A^{i+1} + e^-$	Energy threshold: $h\nu \geq \chi_I$. Main source of ISM ionization.
	Recombination	$A^{i+1} + e^- \rightarrow A^{i*} + h\nu$	Source of bound-free continuum radiation. Inverse of photoionization
	Spontaneous Radiative Decay	$A^{i*} \rightarrow A^i + h\nu_1$	No energy threshold. Most common form of de-excitation. Primary source of emission lines
	Photoexcitation/ Fluorescence Excitation	$A^i + h\nu \rightarrow A^{i*}$	$h\nu = \Delta E$. Populates a few specific levels. Usually followed by spontaneous radiative decay
C O L L I S I O N A L	Collisional: Ionization Excitation De-excitation	$A^i + e^- \rightarrow A^{i+1} + e^- + e^-$ (ionization) $\rightarrow A^{i*} + e^-$ (excitation) $A^{i*} + e^- \rightarrow A^i + e^-$ (de-excitation)	Threshold: $\frac{1}{2} m_e v^2 \geq \chi_I$ (i.e., $kT_e \geq \chi_I$) $\frac{1}{2} m_e v^2 \geq \chi_{exc}$ (e⁻ coolant) No energy threshold
	Dielectronic Recombination	$A^{i+1} + e^- \rightarrow A^{i**}$	No radiation emitted. Large cross section, subject to condition: $\frac{1}{2} m_e v^2 = \chi_{exc}^2 - \chi_{exc}^1$
	Autoionization	$A^{i**} \rightarrow A^{i+1} + e^-$	Often follows dielectronic recomb if there is no stabilizing emission: $A^{i**} \rightarrow A^{i*} + h\nu$
	Charge Exchange	$A^i + H^0 \leftrightarrow A^{i-1} + H^+ + \Delta E$	Large resonance ($\Delta E \sim 0$) cross section. Controls $O^0 \leftrightarrow O^+$ ionization

A^{i*} = excited state of ion i; A^{i**} = doubly excited state of ion i

General 'Interaction Rate' Equation

(from Transport Theory)

* Define $\sigma = \pi r_0^2$ as the cross section for interactions between particles of types A & B, so that if an A & B particle come within distance r_0 the interaction occurs (statistically). For most interactions $\sigma(v) \propto v^{-n}$ ($n > 0$) because higher velocity particles have greater self energy and spend less time influenced by the interaction field.

Let n_A, n_B = number density of particles A & B (cm^{-3})

$f(v) dv$ = particle speed distribution function (usually Maxwell-Boltzmann) $\propto v^2 \exp(-mv^2/2kT)$

$\sigma(v)$ = interaction cross section (cm^2) (typically $\sigma \sim 10^{-16} \text{ cm}^2$ for particle interactions)

v_0 = threshold velocity for interaction to occur (from energetics)

Then, the no. of interactions per unit volume and time is easily shown to be

$$\text{Number of A-B interactions}/(\text{cm}^3 \text{ sec}) = n_A n_B \langle \sigma(v) v \rangle \quad \text{cm}^{-3} \text{ s}^{-1}$$

$$\text{where } \langle \sigma(v) v \rangle \equiv q(T) \equiv \int_{v_0}^{\infty} \sigma(v) v f(v) dv$$

'collision coefficient'

$$\propto T^{-1/2} \exp(-mv_0^2/2kT) \quad [\text{for } \sigma(v) \propto v^{-2}]$$

* For radiation the photon density n_ν is related to the mean intensity of radiation J_ν by

$$n_\nu = 4\pi J_\nu / (ch\nu)$$

Thus, the rate of radiative interactions (absorption, ionization, etc.) for an ion i is

$$\text{Number of absorptions by ion } i / (\text{cm}^3 \text{ sec}) = n_\nu n_i \langle \sigma_\nu c \rangle = n_i \int_{\nu_i}^{\infty} 4\pi J_\nu \sigma_\nu / h\nu d\nu$$

(typically $\sigma_\nu \sim 10^{-18} \text{ cm}^2$ for continuum radiative interactions)

Cross Sections

