Diffusive Shock Acceleration

The most popular way to accelerate particles in the Universe

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Syllabus

- **Lecture 1:** Introduction. Cosmic Rays. Gamma rays. Synchrotron radiation
- **Lecture 2:** Derivation of the [illegible] power law of accelerated particles.
- **Lecture 3:** The Fokker-Planck equation and its solutions. Phenomenology of efficient accelerators.

Table of contents - Lecture 2

The multi-messenger spectrum…

…of the **non-thermal Universe**

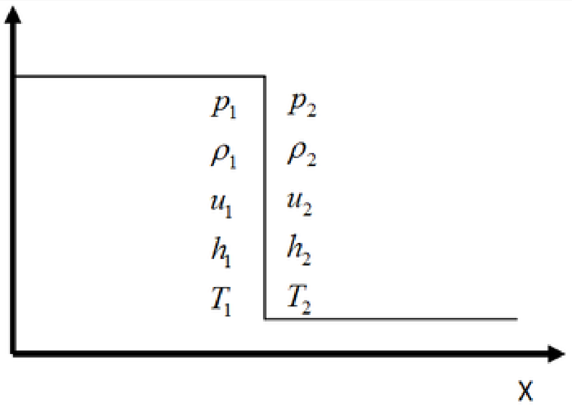
- Cosmic Rays
- Photons (rays)
- Neutrinos ( )

Shocks

Collisionless shocks

Astrophysical shocks[1] are collisionless

- Shock transition region: ⌞ Coulomb collisions length



[1] Supersonic plasma

We set a perturbation in all the parameters, $\psi = \psi_0 + \tilde{\psi}$, and then $\tilde{\psi}$ satisfies the wave equation $\nabla^2\tilde{\psi} - \frac{1}{c^2}\frac{\partial^2\tilde{\psi}}{\partial t^2} = 0$

- Conservation of mass

$$\tilde{\rho}_1 v_1 = \tilde{\rho}_2 v_2$$

- Conservation of momentum

$$\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2$$

- Conservation of energy

$$\frac{\gamma_{ad} P_1}{(\gamma_{ad} - 1)\rho_1} + \frac{1}{2}v_1^2 = \frac{\gamma_{ad} P_2}{(\gamma_{ad} - 1)\rho_2} + \frac{1}{2}v_2^2$$

γ_{ad} : adiabatic index

- Density and velocity

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$$

- Pressure

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1}$$

- Temperature ($T_2 = p_2/(R\rho_2)$)

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma-1)][(\gamma-1)M_1^2+2]}{(\gamma+1)^2 M_1^2}$$

Mach number $M_1 = u_1/c_s = u_1/\sqrt{\gamma p_1/\rho_1}$

Strong shocks

In the limit of strong shocks ($Ú_0$ 1) and $_{9L}$ = 5=3

$$\lim_{Ú_1 \, ! \, 1} \frac{2}{1} = \frac{9L + 1}{9L \quad 1} = 4$$

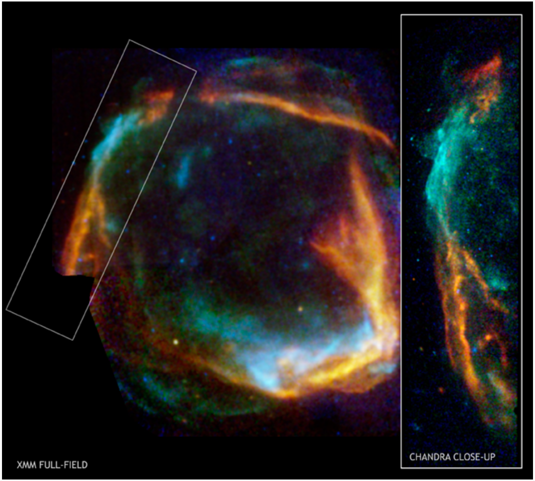
Therefore ...

$$_2 = 4 \, _1 \qquad 9jL \qquad ,_2 = \frac{,_1}{4}$$

$$\dot{E}_2 = \frac{3}{4} \, {}_0 ,_1^2 \qquad 9jL \qquad \imath_2 \quad 2 \quad 10 \quad ^9 ,_1^2 \; b$$

Shocks are very common in the Universe!

Shocks are the main source of Cosmic Rays!



RCW 86 (Chandra and XMM-Newton X-ray data) - J. Vink

Diffusive Shock Acceleration

Diffusive Shock Acceleration (DSA) - Fermi I

- Cosmic rays are isotropic on either side of the shock due to small angle scattering off magnetic field fluctuations
- Isotropization allows particles to cross the shock more efficiently
- Every times the CR crosses the shock, a net energy gain is received
- The resulting spectrum of particles is independent of the diffusion regime
- The acceleration efficiency depends on the scattering efficiency

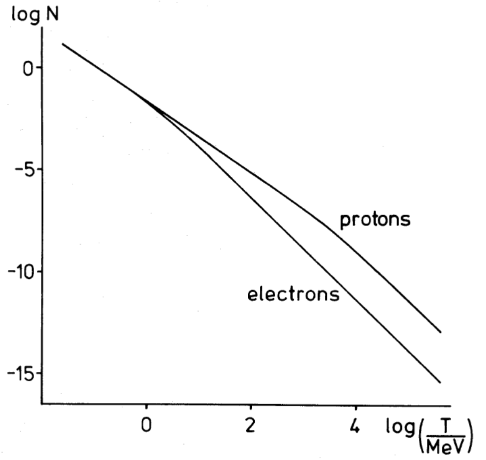
Derivation of the Universal power law (70's)

Microscopic approach

- Bell 1978a,b

Macroscopic approach
through the Fokker-Planck equation

- Axford, Leer & Skadron 1977
- Krymskii 1977
- Blandford & Ostriker 1978

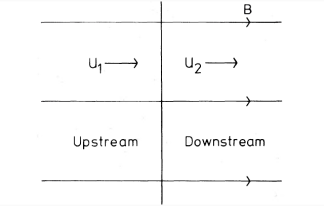


Bell 1978b

Tony Bell's approach

Evaluation of the number of particles located at the shock versus the number of particles that escape downstream. Only particles that do not escape are reaady for one more cycle of acceleration

- u_1 : Upstream velocity
- u_2 : Downstream velocity
- $v_s = u_1$: Shock velocity



Flux of particles passing from upstream to downstream with velocity v:

$$\phi = n_0 \frac{1}{4} \int (u_1 + v\cos(\theta))\, 2\, d\cos(\theta) \approx n_0 \frac{v}{4}$$

Return probability

- P_{esc} is the probability to escape downstream
- $P_{ret} = 1 - P_{esc}$ is the probability to cross the shock back to the upstream

Flux of particles passing from upstream to downstream: $n\frac{v}{4}$

Flux of particles escaping downstream: $n u_2$

$$P_{esc}\, n \frac{v}{4} = n u_2 \Rightarrow P_{esc} = u_2 \frac{4}{v}$$

Note that if $v \to c$ and $u_2 \ll c$, then $P_{esc} \to 0$ and $P_{ret} \to 1$

Energy transformation

- Energy transformation

$$E' = \gamma(E + p_x V \cos(\theta))$$

Non relativistic shock, $\gamma \approx 1$ and $E = pc$. Therefore,

$$E' - E = \Delta E = p V \cos(\theta) \Rightarrow \frac{\Delta E}{E} = \frac{V}{c}\cos(\theta)$$

- Fractional energy change when the particle goes from upstream to downstream

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mathrm{upstream} \to \mathrm{downstream}} = \frac{2}{3}\frac{V}{c}$$

- Fractional energy change when the particle goes from upstream to downstream and back to the upstream (1 cycle)

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mathrm{cycle}} = 2\left\langle \frac{\Delta E}{E} \right\rangle_{\mathrm{upstream} \to \mathrm{downstream}} = \frac{4}{3}\frac{V}{c}$$

Spectrum of particles

After k cycles, the energy of particles is increased by

$$\_ = \_{0}(1 + \frac{=}{\_}_{\text{H´HeO}})^{z}$$

$$\ln \frac{=}{\_{0}} = z \ln \left(1 + \frac{=}{\_}_{\text{H´HeO}} \right!$$

where

$$z = \frac{\ln(\_=\_{0})}{\ln\ 1 + \frac{=}{\_}} ;$$

and

$$\frac{\_}{\_}_{\text{H´HeO}} \quad \frac{4}{3} \quad \frac{1}{} \quad \frac{\hat{s}}{s}$$

Spectrum of particles

$$
\begin{aligned}
Æ \quad &/ \quad (1 \quad \dot{E}_{0\ \mathrm{H}})^{\zeta} \\
Æ \quad &= \quad \zeta(1 \quad \dot{E}_{0\ \mathrm{H}})^{\frac{\ln(\_/\_0)}{\ln(1+ \quad \_/\_)}}; \qquad G = \mathrm{H§0} \\
\ln Æ \quad &= \quad G^{0} + \frac{\ln(\_=\_0)}{\ln(1 + \quad \_=\_)} \ln(1 \quad \dot{E}_{0\ \mathrm{H}}); \qquad G^{0} = \ln(\mathfrak{s}) = \mathrm{H§0}
\end{aligned}
$$

Finally

$$Æ = \mathfrak{s}^{00} \quad (\qquad 1) \ln(\_);$$

where

$$= 1 \quad \frac{\ln(1 \quad \dot{E}_{0\ \mathrm{H}})}{\ln(1 + \quad \_=\_)} = 1 \quad \frac{\ln(1 \quad \frac{4\ \hat{s}\ \backslash}{,})}{\ln(1 + \frac{4}{3} \frac{(\quad 1)}{} \frac{\hat{s}\ \backslash}{\mathfrak{s}})}:$$

If ⌞ 0:

$$\ln(1+⌞) \quad ⌞ \quad \frac{⌞^2}{2} + ::: \qquad 9jL \qquad \ln(1 \quad ⌞) \qquad ⌞ + \frac{⌞^2}{2} \quad :::;$$

therefore

$$1 \quad \frac{\frac{4\hat{s}}{\quad}}{\frac{4}{3}\frac{(\quad 1)}{\quad}\frac{\hat{s}}{s}}$$

$$1 + \frac{3}{(\quad 1)}$$

$$1 \quad =) \qquad = \frac{1+3}{1}$$

$$= \frac{+2}{1}$$

$$Æ(\_) \;/\; \_$$

Strong shock

$$= 4 \quad =) \qquad = 2$$

Maximum energy of particles

Acceleration timescale

The spectrum of accelerated particles doesn't depend of the diffusion regime. However.... the acceleration time does

- Particles moving in a turbulent magnetic field diffuse on a times-scale $?_{L\_TT} = \hat{g}^2/P$, where $\hat{g}$ is the diffusion length
- The diffusion coefficient is $P = \quad \mathfrak{s}/3$, where   is the mean free path
- P is a big unknown in CR physics. We assume that $P \ / \ \_$
- Bohm diffusion regime[2]: $P_{?n\backslash h} = d_X \mathfrak{s}/3$

[2] $d_X = \_/\iota@$ is the Larmor radius of a relativistic particle

Cycle timescale

Balance between away from the shock upstream and advection downstream creates a CR precursor located at $\hat{x} = \frac{D_u}{u}$ upstream of the shock

- Flux of particles passing from upstream to downstream: $n_0 \frac{c}{4}$
- Number of CR in the precursor per unit area: $n_0 \hat{x}$
- Average time a particle spend upstream: $t_u \approx \frac{n_0 \hat{x}}{n_0 c/4} = 4 \frac{D_u}{u c}$

Similarly...

- Average time a particle spend downstream: $t_d \approx 4 \frac{D_d}{u_d c}$

Cycle time between upstream and downstream:

$t_{cycle} = t_u + t_d = 4 \left(\frac{D_u}{u_u} + \frac{D_d}{u_d} \right) \frac{1}{c}$

Acceleration timescale

- Energy gain per cycle:

$$\left\langle\frac{-}{-}\right\rangle_{\mathrm{H\acute{}HeO}} = \frac{4}{3}\frac{›}{ʂ} \quad \frac{›}{ʂ}$$

- Acceleration timescale

$$ʔ_{\mathrm{9HH}} = ʔ_{\mathrm{H\acute{}HeO}} \left\langle\frac{-}{-}\right\rangle_{\mathrm{H\acute{}HeO}} \quad 4\frac{P_{«} + 4P_{\mathrm{L}}}{,^{2}}$$

by assuming Bohm diffusion: $P_{«} = P_{?n\backslash h} = ȡ_{X}ʂ{=}3 = (\_{=}\iota@)ʂ{=}3$, and therefore P_{L} $P_{«}{=}4$ (@-compression at the shock)

$$ʔ_{\mathrm{9HH}} = \frac{8}{3}\frac{-}{@›^{2}\backslash}$$

Maximum energy

we balance $?_{9HH}$ with cooling and dynamical timescales

- Lifetime of the source
- Radiative cooling

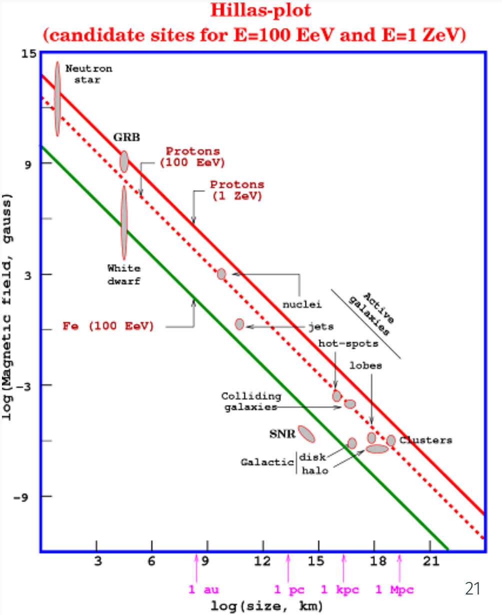
The Hillas energy

Larmor radius ($đ_X = \frac{—}{\hat{W}L@}$) = size of the source ($\hat{I}$)

$$\left(\frac{@}{100\mu\mathrm{W}}\right) = \frac{1}{\hat{W}}\left(\frac{—}{100\ \mathrm{NO}\text{ -}}\right)\left(\frac{\hat{I}}{\mathrm{c\,qH}}\right)^{-1}$$

Hillas upper-limit on the maximum energy:

$$\left(\frac{—[}{100\ \mathrm{NO}\text{ -}}\right) = \hat{W}\left(\frac{ž}{\mathrm{s}}\right)\left(\frac{@}{100\mu\mathrm{W}}\right)\left(\frac{\hat{I}}{\mathrm{c\,qH}}\right)$$



Questions?