Diffusive Shock Acceleration

The most popular way to accelerate particles in the Universe

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Syllabus

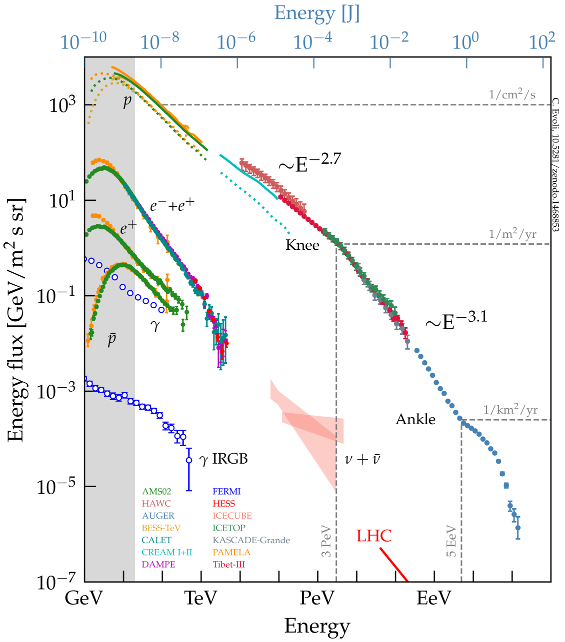
- **Lecture 1:** Introduction. Cosmic Rays. Gamma rays. Synchrotron radiation
- **Lecture 2:** Derivation of the *Universal* power law of accelerated particles
- **Lecture 3:** The Fokker-Planck equation and its solutions. Phenomenology of efficient accelerators

Table of contents - Lecture 2

The multi-messenger spectrum...

...of the **non-thermal Universe**

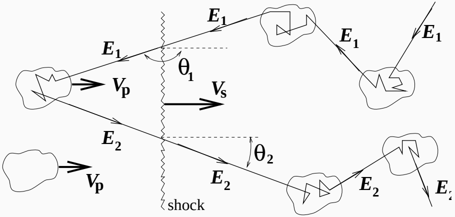
- Cosmic Rays
- Photons (γ rays)
- Neutrinos (ν)



Diffusive Shock Acceleration

Basics

- Isotropization allows particles to cross the shock more efficiently
- Every times the CR crosses the shock, a net energy gain is received
- The resulting spectrum of particles is independent of the the diffusion regime
- The acceleration efficiency depends on the scattering efficiency



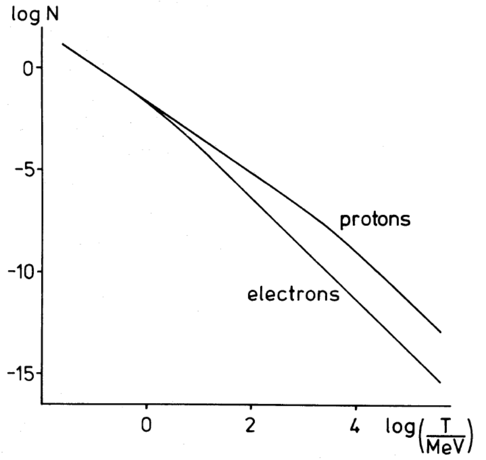
Derivation of the Universal power law (70's)

Microscopic approach

- Bell 1978a,b

Macroscopic approach (Fokker-Planck equation)

- Axford, Leer & Skadron 1977
- Krymskii 1977
- Blandford & Ostriker 1978



Bell 1978b

Tony Bell's approach

Evaluation of the number of particles located at the shock versus the number of particles that escape downstream. Only particles that do not escape are qualified for one more cycle of acceleration

After k cycles[1],

- the energy of particles is increased by

$$E = E_0 \left(1 + \left\langle \frac{\Delta E}{E} \right\rangle_{\text{cycle}}\right)^k \sim E_0 \left(1 + \frac{4}{3}\left(\frac{r-1}{r}\right)\frac{V_{\text{sh}}}{c}\right)^k$$

- the number of particles is reduced by

$$n = n_0(1 - P_{\text{esc}})^k = n_0 \left(1 - \frac{4}{r}\frac{v_{\text{sh}}}{c}\right)^k$$

[1] Shock with velocity v_{sh} and compression factor r

The spectrum of accelerated particles

Therefore[2] ...

$$\frac{\log(n/n_0)}{\log(E/E_0)} = \frac{\log(1 - \frac{4}{r}\frac{v_{\mathrm{sh}}}{c})}{\log(1 + \frac{4}{3}\frac{r-1}{r}\frac{v}{c})} \sim \frac{-\frac{4}{r}\frac{v_{\mathrm{sh}}}{c}}{\frac{4}{3}\frac{r-1}{r}\frac{v_{\mathrm{sh}}}{c}} = -\frac{3}{r-1} \Rightarrow \frac{n}{n_0} = \left(\frac{E}{E_0}\right)^{\frac{-3}{r-1}}$$

The differential spectrum is $n(E)dE \propto E^{\frac{r+2}{r-1}}dE$

[2] If $x \sim 0$ then, $\log(1+x) \sim x - x^2/2 + \ldots$ and $\log(1-x) \sim -x + x^2/2 - \ldots$

The Vlasov-Fokker-Planck equation

The distribution of relativistic particles can be described by

$$\frac{\partial f}{\partial t} + (u+v)\frac{\partial f}{\partial z} - \frac{1}{3}\frac{\partial u}{\partial z} p \frac{\partial f}{\partial p} - \frac{\partial}{\partial z}\left(D\frac{\partial f}{\partial z}\right) = 0$$

By setting $f = f_0 + f_1$ and requiring:

- Steady state solution far upstream $\Rightarrow f = f_0(p)e^{\frac{3u_1\nu}{c^2}z}$
- Steady state solution downstream $\Rightarrow f = f_0(p)$
- Continuity at the shock $\Rightarrow \frac{c}{3}f_1 + \frac{u_1}{3}p\frac{\partial f_0}{\partial p} = \frac{u_2}{3}p\frac{\partial f_0}{\partial p}$

We obtain

$$u_1 f_0 + \left(\frac{u_1}{3} - \frac{u_2}{3}\right) p\frac{\partial f_0}{\partial p} = 0 \Rightarrow f_0 \propto p^{-\frac{3r}{r-1}}$$

Strong shocks: $r = 4 \Rightarrow f_0 \propto p^{-4}$ **or equivalently** $f_0 \propto E^{-2}$

Evolution of the non-thermal spectrum

Once a distributon of non-thermal particles is injected in the shock downstream region, it evolves due to energy losses and eventually escape of the particles from the system.

The diffusion equation is:

$$\frac{\partial n}{\partial t} - D\,\nabla^2\,n + \frac{\partial}{\partial E}(b\,n) + \frac{n}{T} = Q(E, \vec{r}, t)$$

- $Q(E, \vec{r}, t)$ is the injection function (source term)
- b are cooling losses
- T is escape time or lifetime
- D is the diffusion coefficient

It’s solved using the Green method.

Steady state ($\partial/\partial t = 0$)

$Q(E) = KE^{-p}$

$$
\begin{aligned}
\frac{\partial}{\partial E}[b(E)\,n(E)] &= -Q(E) \\
\int \mathrm{d}[b(E)\,n(E)] &= -\int Q(E)\,\mathrm{d}E \\
\int_E^\infty \mathrm{d}[b(E)\,n(E)] &= -\int_E^\infty K\,E^{-p}\mathrm{d}E \\
-b(E)n(E) &= -\left.\frac{KE^{-p+1}}{-p+1}\right|_E^\infty = -\frac{KE^{-(p-1)}}{(p-1)} \\
n(E) &= \frac{K\,E^{-(p-1)}}{(p-1)b(E)}
\end{aligned}
$$

Non steady state

Injection during a period t_0

$$Q(E) = \begin{cases} KE^{-p}, & t < t_0 \\ 0 & t > t_0 \end{cases}$$

Radiative losses $b(E) = A\,E^2$ (e.g. synchrotron radiation, inverse Compton scattering)

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial E}[b(E)\,n(E)] + Q(E)$$

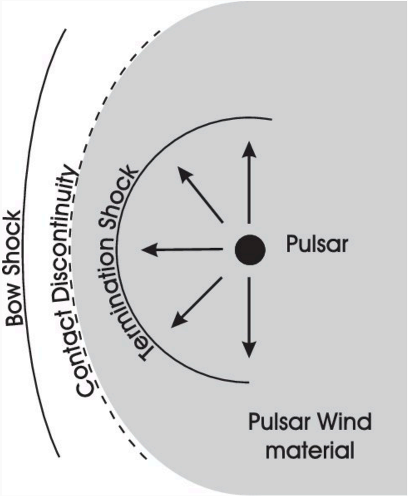
$$n(E) = \begin{cases} \dfrac{KE^{-(p+1)}}{A(p-1)}[1-(1-AEt)^{p-1}], & AEt_0 \leq 1 \\[2ex] \dfrac{KE^{-(p+1)}}{A(p-1)}, & AEt_0 > 1 \end{cases}$$

Supernova remnants

Basic characteristics

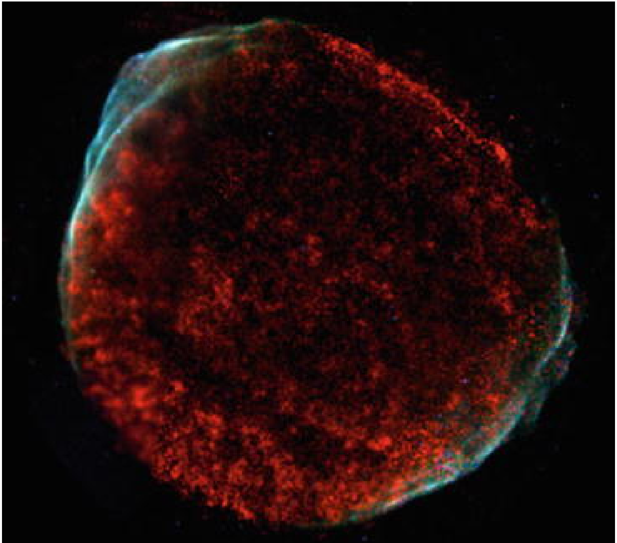
- The progenitor is a massive star
- The compact object is neutron star or a black hole
- Energy released $\sim 10^{51}$ erg
- Strong shock

$$\left(\frac{v_{\rm eyec}}{\rm km\ s^{-1}}\right) \approx 10^4 \left(\frac{E}{10^{51}\ \rm erg}\right)^{\frac{1}{2}} \left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}}$$

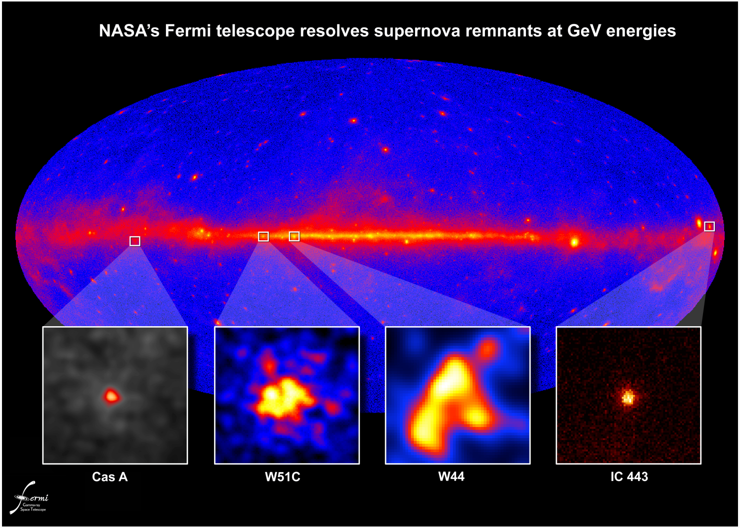


Do SNRs accelerate particles?

- Synchrotorn emission from radio to X rays!
- TeV electrons are needed

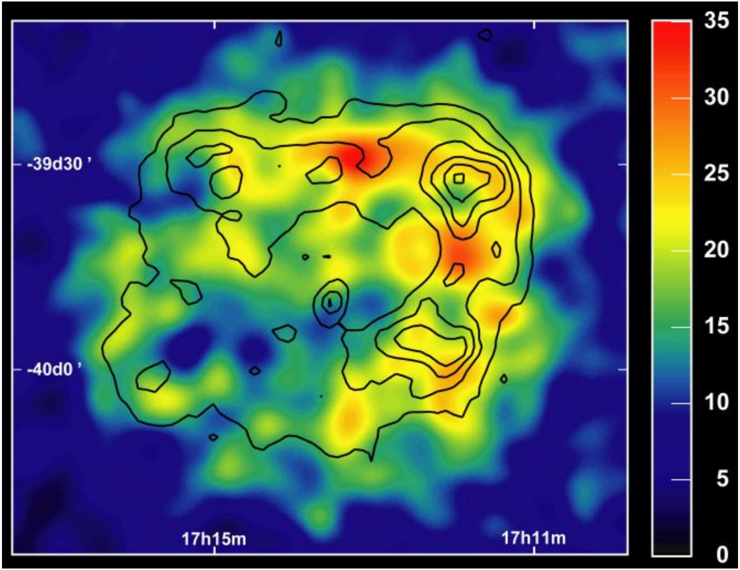


GeV gamma-ray emission



Fermi

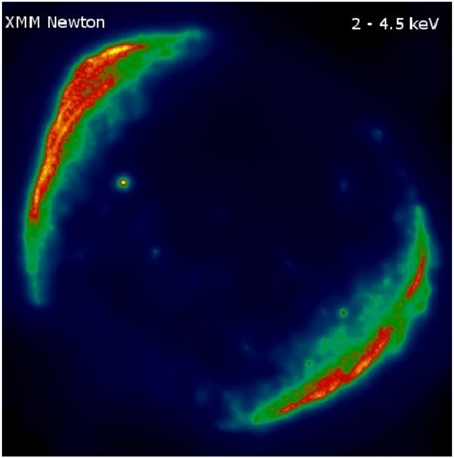
TeV gamma-ray emission



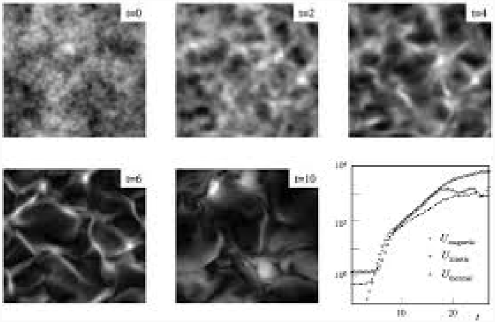
HESS collaboration

Magnetic field amplification

- Thin rims of synchrotron X-ray emission
- Fast synchrotron cooling ($t_{\mathrm{synchr}} \propto B^{-2}$)
- The magnetic field has to be much larger than the expedted value on the ISM ($\sim 1\mu$G)



- If CR acceleration is efficient, the CR-current should be taken into account in the MHD equations
- $\delta B >> B_0$

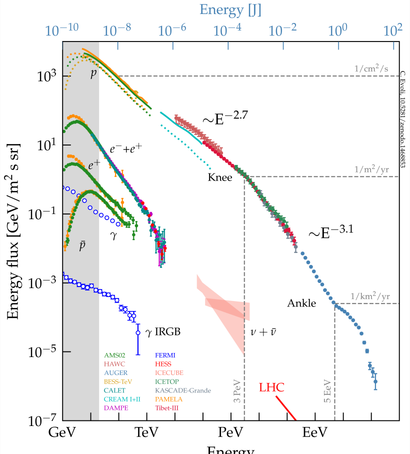


Bell & Lucek (2001), Bell (2004)

Acceleration up to the knee of the CR spectrum

Only very young SNR can accelerate 3 PeV protons (Bell et al. 2013)

$$E_{\max} \sim 3 \left(\frac{\eta_{\rm esc}}{0.1}\right) \left(\frac{n_e}{cm^{-3}}\right)^{\frac{1}{2}} \left(\frac{v_{\rm sh}}{30000\,{\rm km\,s^{-1}}}\right)^2 \left(\frac{R}{pc}\right) \ {\rm PeV}$$



Energy budget of SNRs

CR energy density $\omega_{\rm CR} \sim 1\,{\rm eV\,cm^{-3}}$

Volume of our Galaxy: $V_{\rm G} = \pi\, R_{\rm G}^2\, h_{\rm G} \sim 4 \times 10^{66}\,{\rm cm}^3$

CR residence time $t_{\rm d} \sim \frac{3r^2}{D}$ (where $r = h_{\rm G}$ and $D \sim 10^{28}\ {\rm cm^2\ s^{-1}}$)

CR total power

$$\begin{aligned} W_{\rm CR} &= \frac{V_{\rm G}\ \omega_{\rm RC}}{t_{\rm d}} \sim \frac{4.1 \times 10^{66}\ {\rm cm^3}\, 10^{-12}{\rm erg\ cm^{-3}}}{10^{14}\ {\rm s}} \\ &\sim \sim 4.1 \times 10^{40} \qquad {\rm erg\ s^{-1}}. \end{aligned}$$

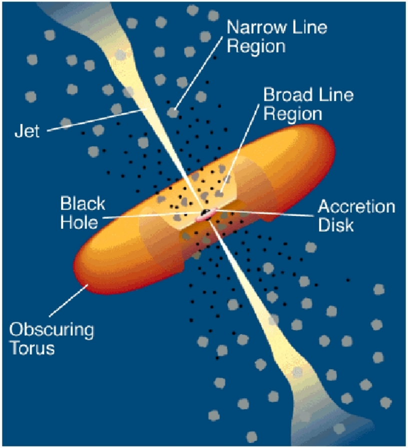
On average, there is 1 SN every 50 yr in our Galaxy

$$\begin{aligned} W_{\rm SN} &= \frac{10^{51}\ {\rm erg}}{50 \times 3.15 \times 10^{7}\ {\rm s}} \\ &\sim 6.3 \times 10^{41}\,{\rm erg\ s^{-1}} \end{aligned}$$

Active Galactic Nuclei

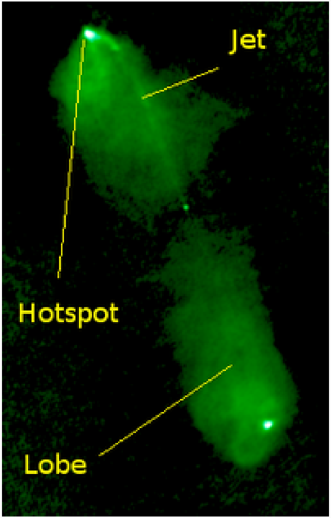
The standard model

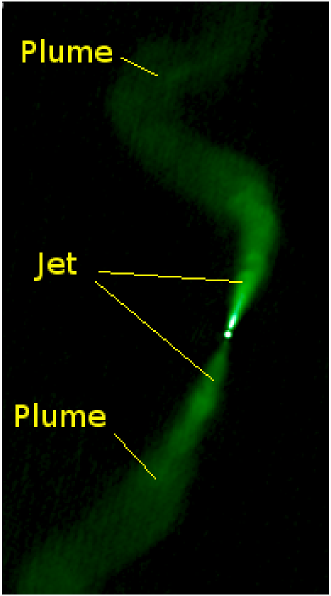
- $M_{\mathrm{SMBH}} \sim 10^6 - 10^9\, M_\odot$
- Matter is accreted from the accretion disc
- Relativistic bipolar jets



Urry & Padovani

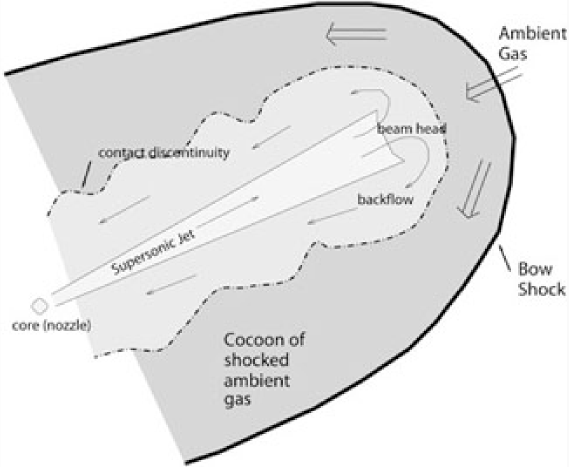
Radiogalaxies - FRI and FRII





Shocks in AGN jets

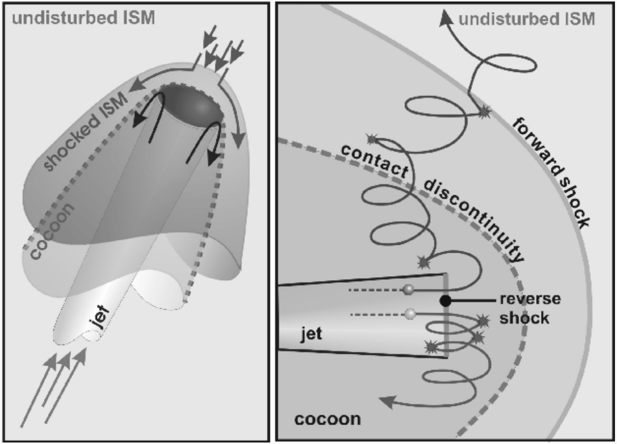
- Termination shocks
- Shocks in the backflow



Worral 2002

Reigning paradigm in the termination shocks

- $E_{e,\max}$ is determined by synchrotron losses ($t_{\rm acc} = t_{\rm synchr}$)
- Hadronic losses are minimal; protons are accelerated up to very high energies and escape ($E_{p,\max} = E_{\rm Hillas}$)

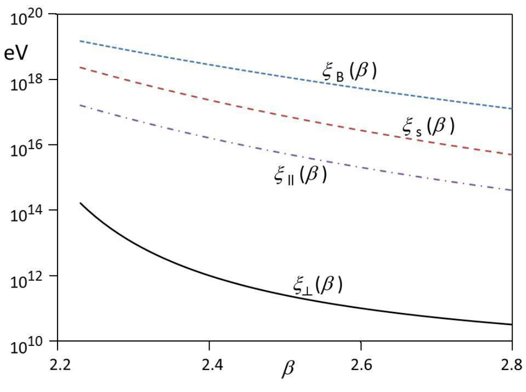


Heinz & Sunyaev (2002)

Relativistic shocks are inefficient accelerators

Three main effects limit the maximum energy to which CR can be accelerated by relativistic shocks (Bell et al. 2018):

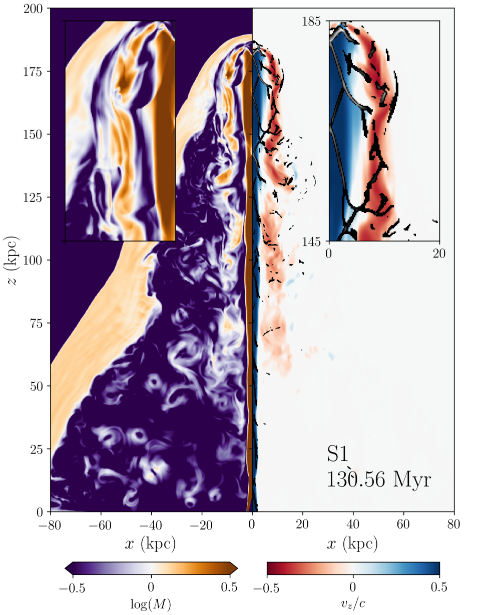
- Steep CR spectrum ($N \propto E^{-\beta}, \beta > 2$)
- Small-scale turbulence (s)
- Quasi-perpendicular shocks ($\mathsf{B_0} \perp \mathsf{v}_{\mathrm{sh}}$)



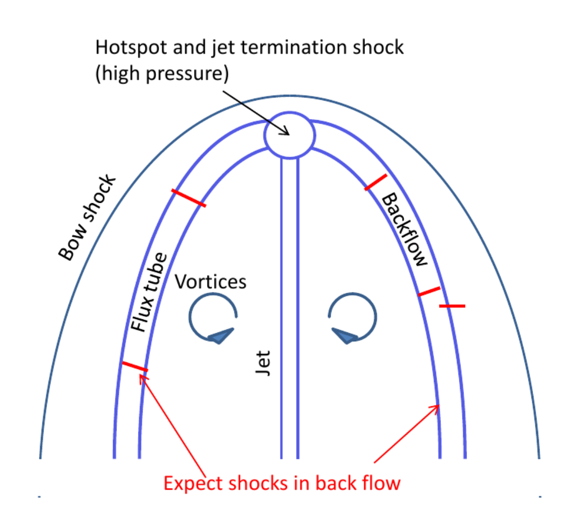
Shocks in the backflows

Numerical study of AGN jets looking for mildly relativistic shocks, with large B, and large L (Matthews et al. 2019)

- $\langle r_{\rm sh} \rangle \sim 2$ kpc
- $\langle v_{\rm sh} \rangle \sim 0.2c$
- $\langle B \rangle \sim 0.1$ mG



Flux tubes



Bell et al. (2019)

Fermi acceleration in flux tubes

In an infinite flux tube particles can only escape from the sides by diffusing across the magnetic field

$D_{\mathrm{Bohm}} = r_{\mathrm{g}} \frac{c}{3}$
$D_{\|} = D_{\mathrm{Bohm}} \omega_{\mathrm{g}} \tau_{\mathrm{scat}}$

$D_{\perp} = \frac{D_{\mathrm{Bohm}}}{\omega_{\mathrm{g}} \tau_{\mathrm{scat}}}$

$D_{\|} D_{\perp} = D_{\mathrm{Bohm}}^2$

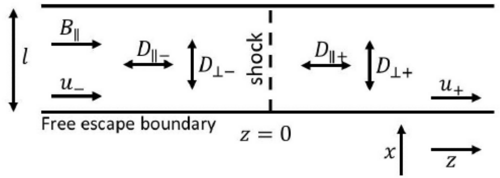


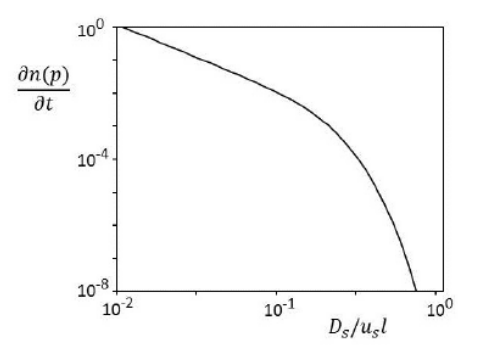
Figure 2. Flux tube geometry with a single stationary shock. The Ds are diffusion coefficients with subscripts $\|$ or $\perp$ denoting diffusion directed parallel or perpendicular to the flow. The subscripts $-$ or $+$ denote the upstream or the downstream of the shock respectively.

$$\frac{\partial f}{\partial t} + \frac{\partial (fu)}{\partial z} - \frac{\partial}{\partial z}\left(D_{\|}\frac{\partial f}{\partial z}\right) - \frac{\partial}{\partial z}\left(D_{\perp}\frac{\partial f}{\partial x}\right) - \frac{1}{3}\frac{\partial u}{\partial z}\frac{1}{p^2}\frac{\partial (p^3 f)}{\partial p} = 0$$

Maximum energy of protons

CRs can reach the Hillas energy even when Bohm diffusion is not achieved[3]

$$\boxed{E_{\mathrm{max}} = 0.6uBL \sim 0.6E_{\mathrm{Hillas}}}$$



Bell et al. (2019)

[3] $n(p) = \partial N/\partial p$

Summary and conclusions

Summary and conclusions

- Shocks are very common in astrophysical sources (e.g. supernova remnants, AGN jets)
- Strong shocks ($M \gg 1$) are efficient particle accelerators
- Shock accelerated particles follow a power-law energy distribution
- The universal power law ($N \propto E^{-2}$) can be derived following two approaches
- Young SNR ($v_{\mathrm{sh}} \sim 30000$ km s^{-1}) can accelerate PeV particles
- Backflows in AGN jets can accelerate UHECRs

Questions?