Dynamics of black holes in dense stellar systems

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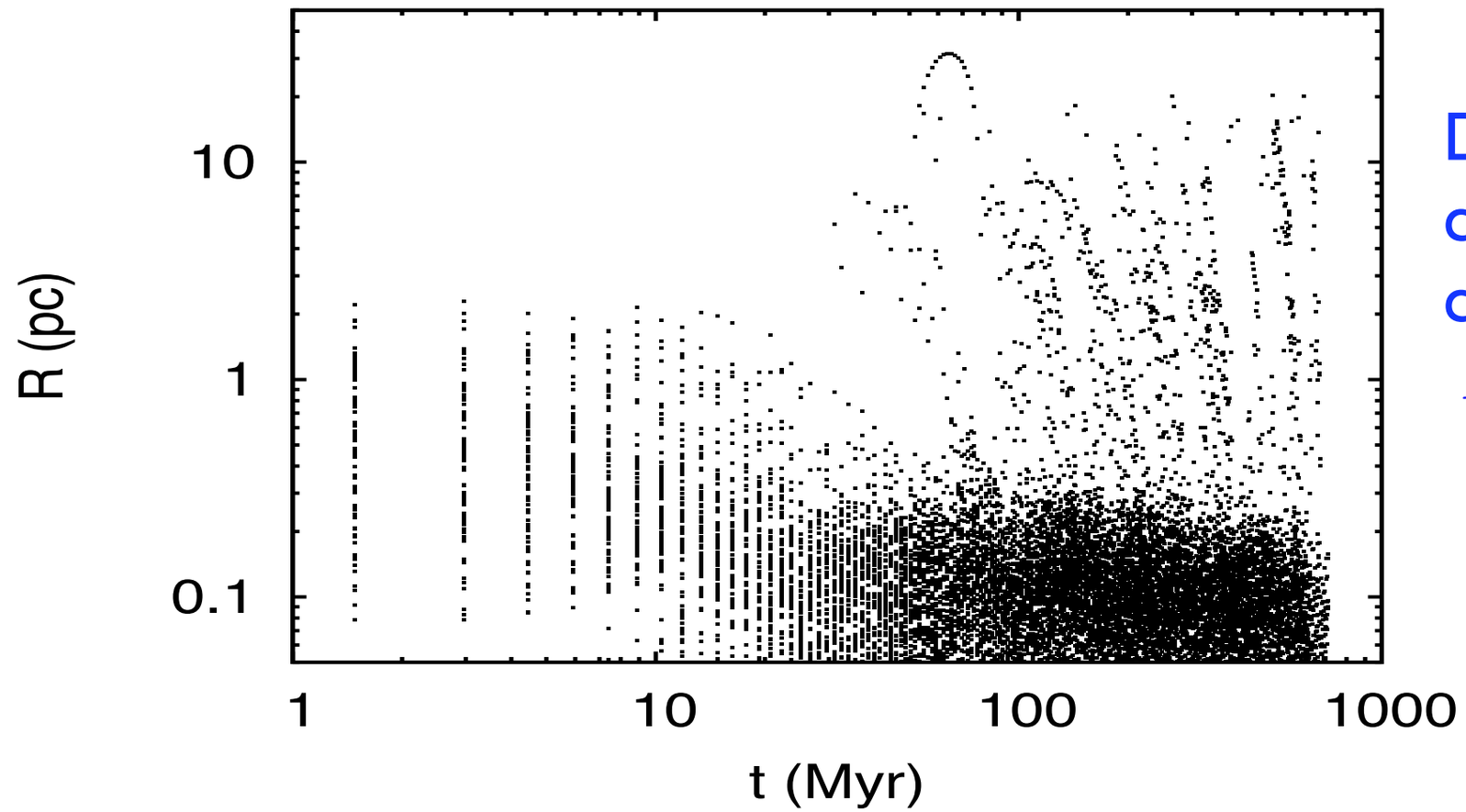
Charles University of Prague**

Lecture 2

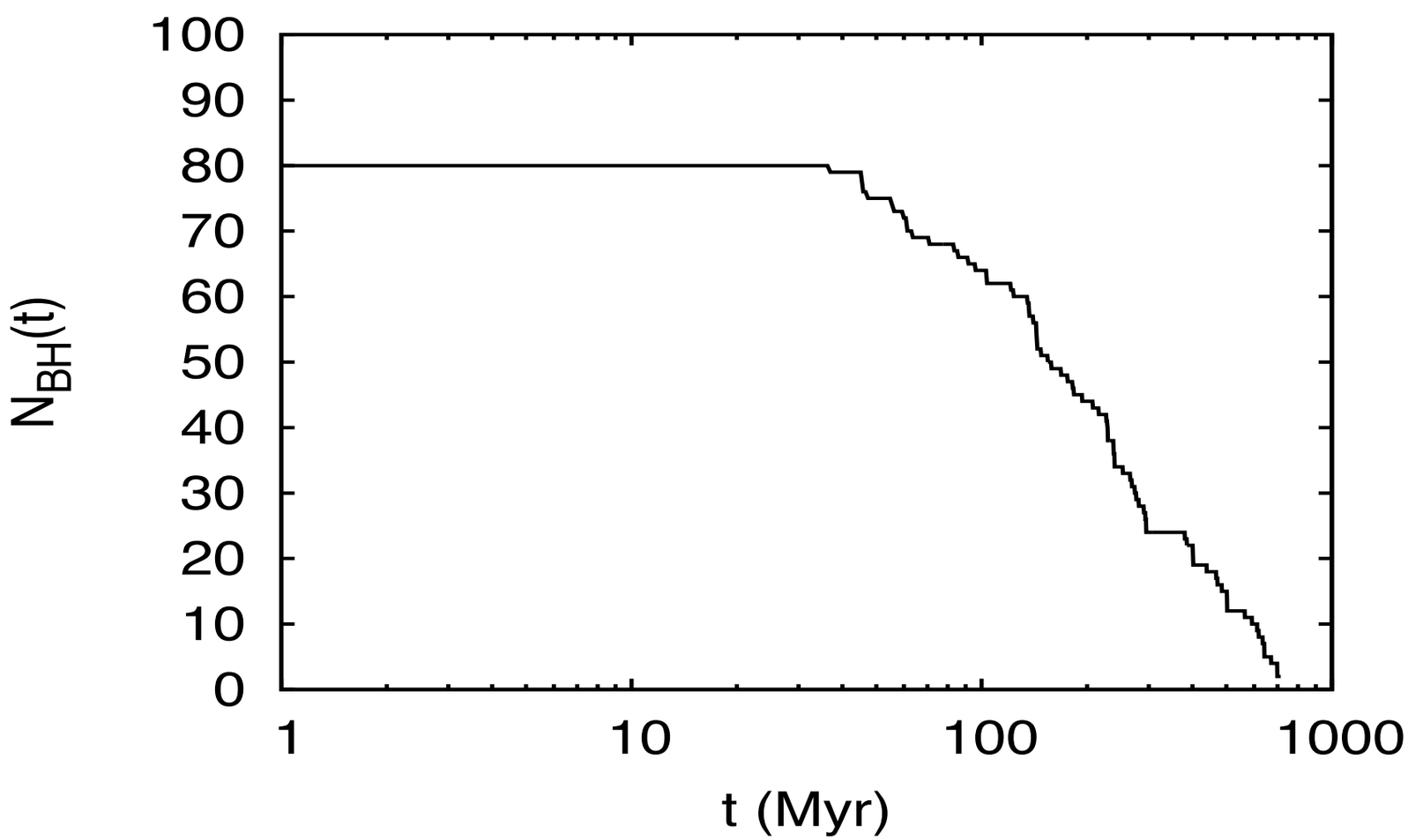
UNIVERSITÄT BONN

DFG Deutsche Forschungsgemeinschaft



Direct N-body computation of $N = 4.5 \times 10^4$ cluster, $r_h(0) = 1$ pc, $N_{BH} \approx 100$ (full retention).

Banerjee, S., et al., 2010, MNRAS, 402, 371



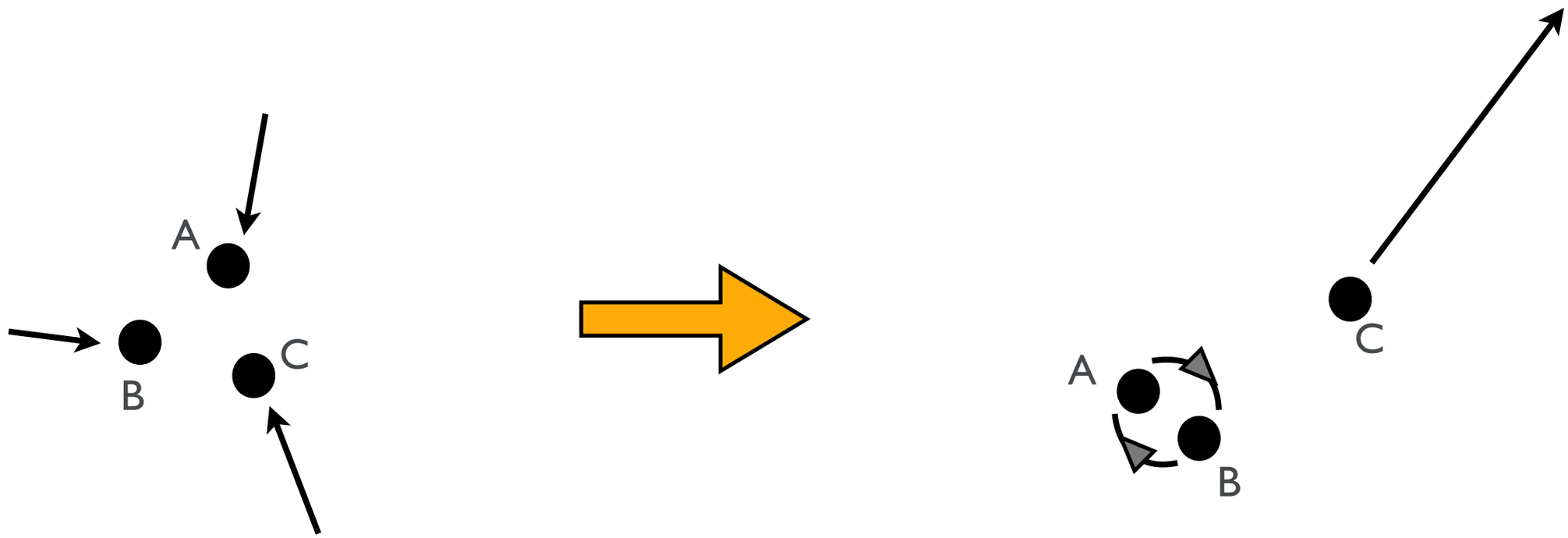
Two phases: (a) initial segregation: $N_{BH} \approx$ const (b) formation of BH-core: N_{BH} depletes due to super-elastic dynamical encounters.

BH-core (or "Dark core") phase have potential for a wide variety of physical phenomena

Dynamical formation of BH-BH binaries

3-body binary formation in dense BH-core:

in close encounter among 3 BHs, two of them get bound while third escape with the excess K.E.



BH-BH binaries from primordial binaries

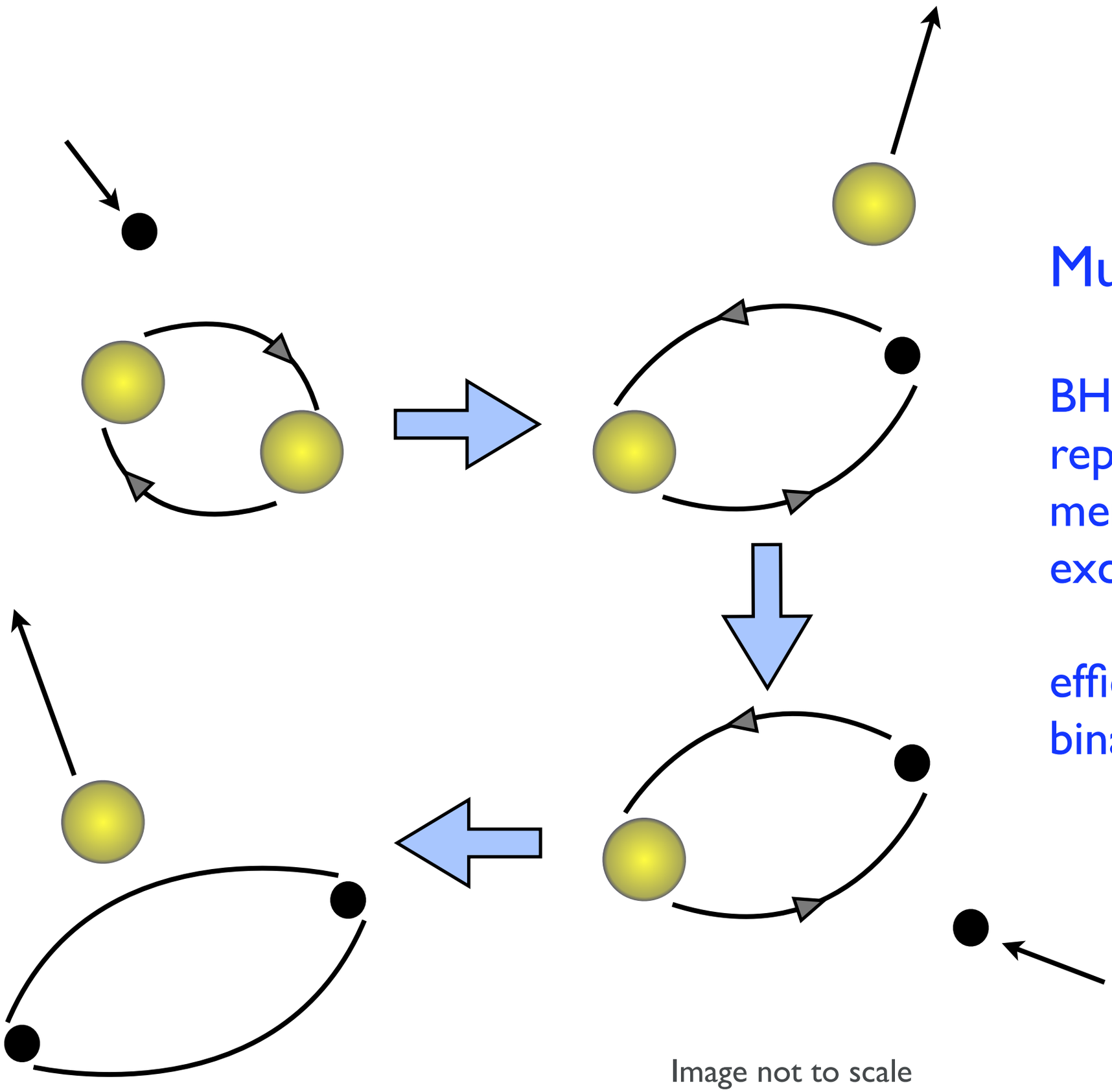


Image not to scale

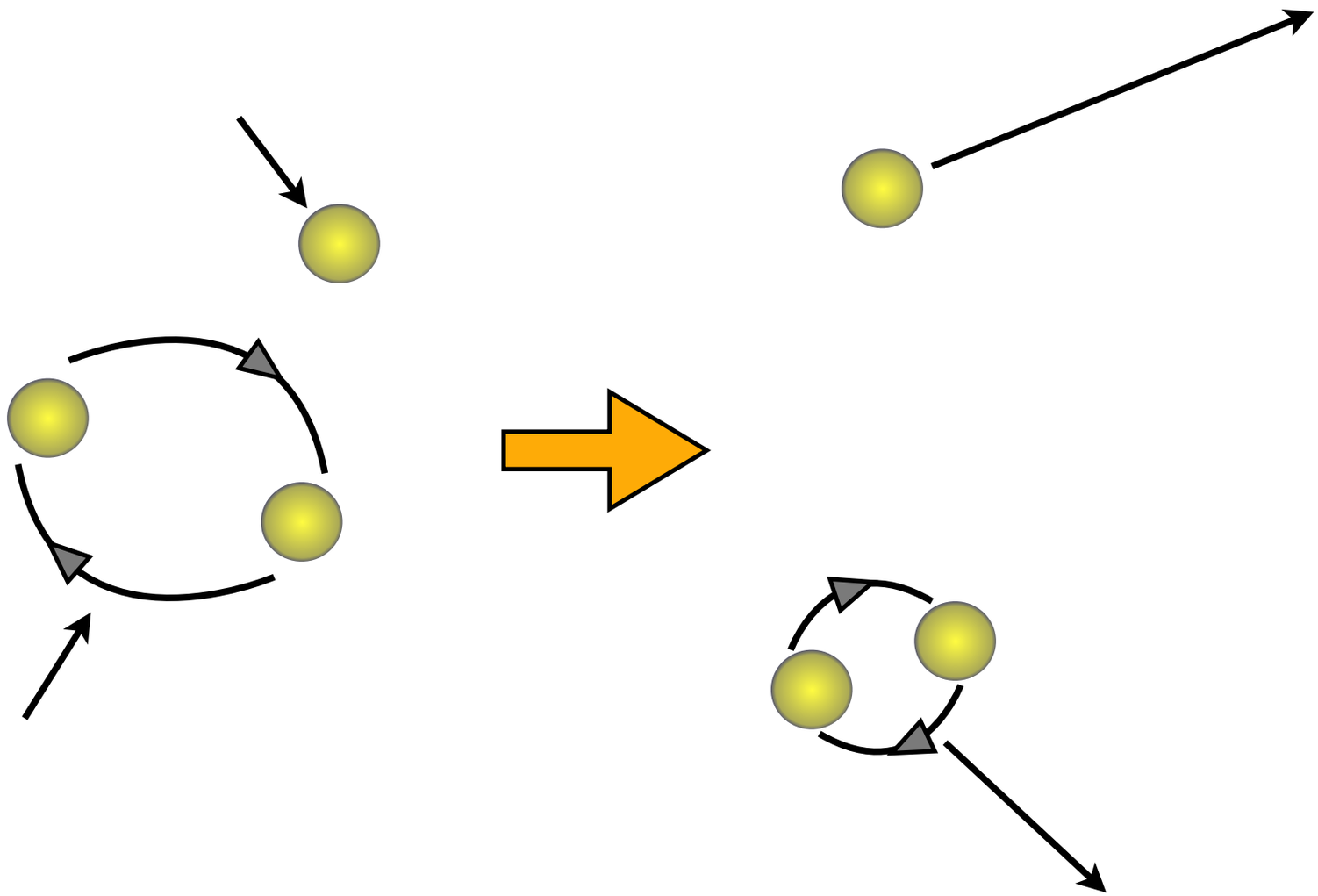
Multiple exchange:

BHs being more massive replace stellar binary members in successive exchange encounters;

efficient with primordial binaries

Encounter/collisional hardening

Dynamically formed BH-BH binaries are "hard":
total binding energy greater than mean stellar K.E.



Heggie's Law: "hard binary hardens" --- encounter/collisional hardening

(hardening => increase of binding energy, i.e., shrinking of semi-major-axis)

Both intruder star & binary get recoiled with larger total K.E.

N.B. : soft binary softens, hence easily dissociated

Consequence of "negative specific heat" of a single binary

Image not to scale

Encounter/collisional hardening

Statistical effect over many encounters: theoretically predicted & verified through numerical experiments. [Heggie, D.C, 1975, MNRAS, 173, 729]

Encounters with hard binaries "super-elastic":
hard binaries supply K.E. to encountering stellar environment as they shrink --- *energy source*.

"Binary burning": has profound consequences on cluster's dynamical evolution; halt's core collapse.

Hardening rate $\dot{a} \sim a^2$ (roughly), rate decreases as binary shrinks: too close binaries ($P \lesssim 10^3$ days typically for globular clusters) behave essentially as single stars.

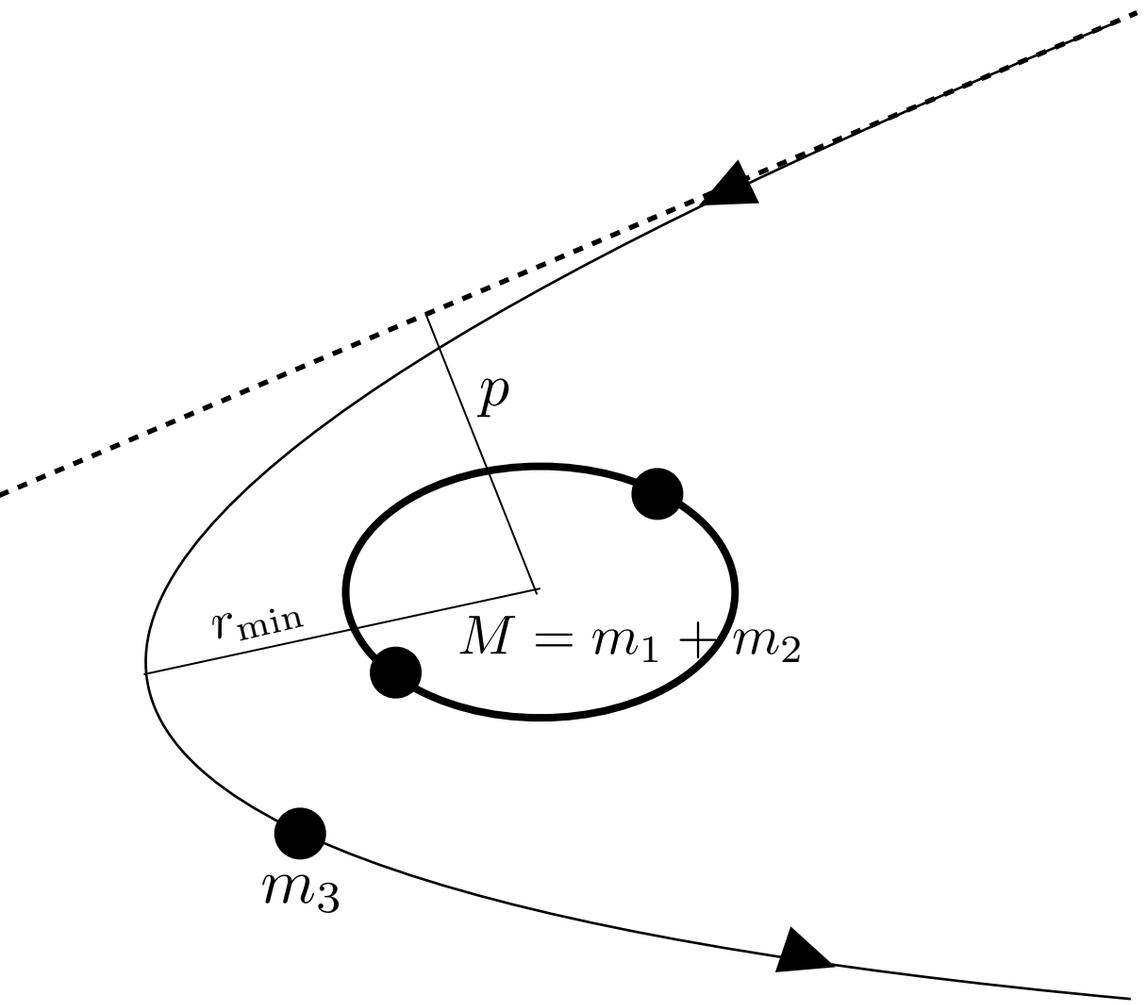
Rate of collisional hardening for distant ($d >> a$) encounters :

$$\dot{a}_{\rm coll} \approx -2.36 \times 10^{-7} \left(\frac{\langle m \rangle^3}{m_1 m_2} \right) \left(\frac{\rho}{v} \right) a^2 \; R_\odot \; {\rm Gyr}^{-1}$$

Shull (1979), Heggie & Hut (2003), Banerjee & Ghosh (2006)

Much stronger hardening is possible through close ($d \sim a$) encounters!

Gravitational focussing



$$\sigma_{\rm foc} = \pi r_{\rm min}^2 \left\{ 1 + \frac{2G(M+m_3)}{r_{\rm min} v^2} \right\}$$

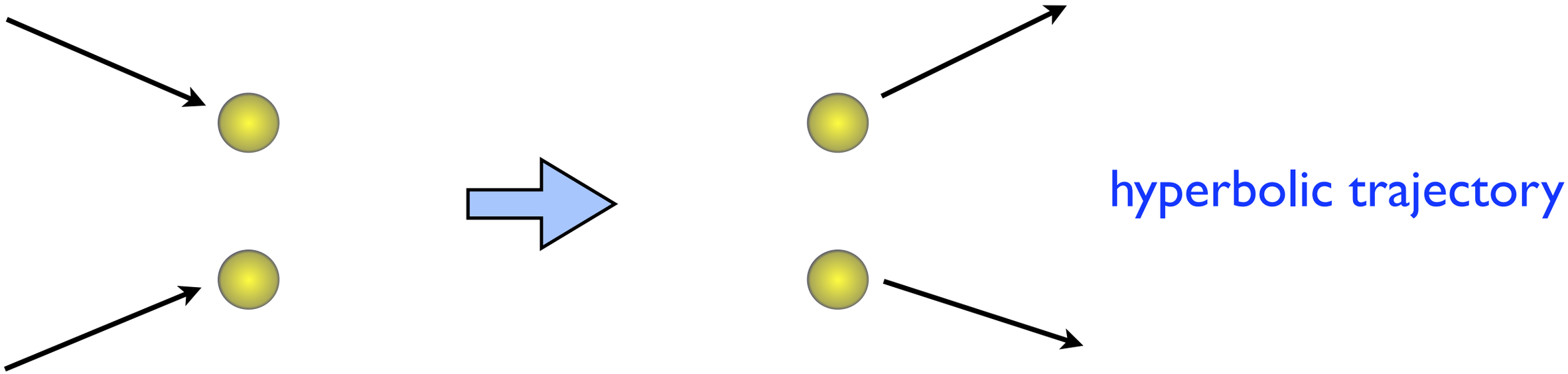
(Spitzer Eqn. 6-15)

$$\frac{1}{\tau_{\rm foc}} = n \sigma_{\rm foc} v$$

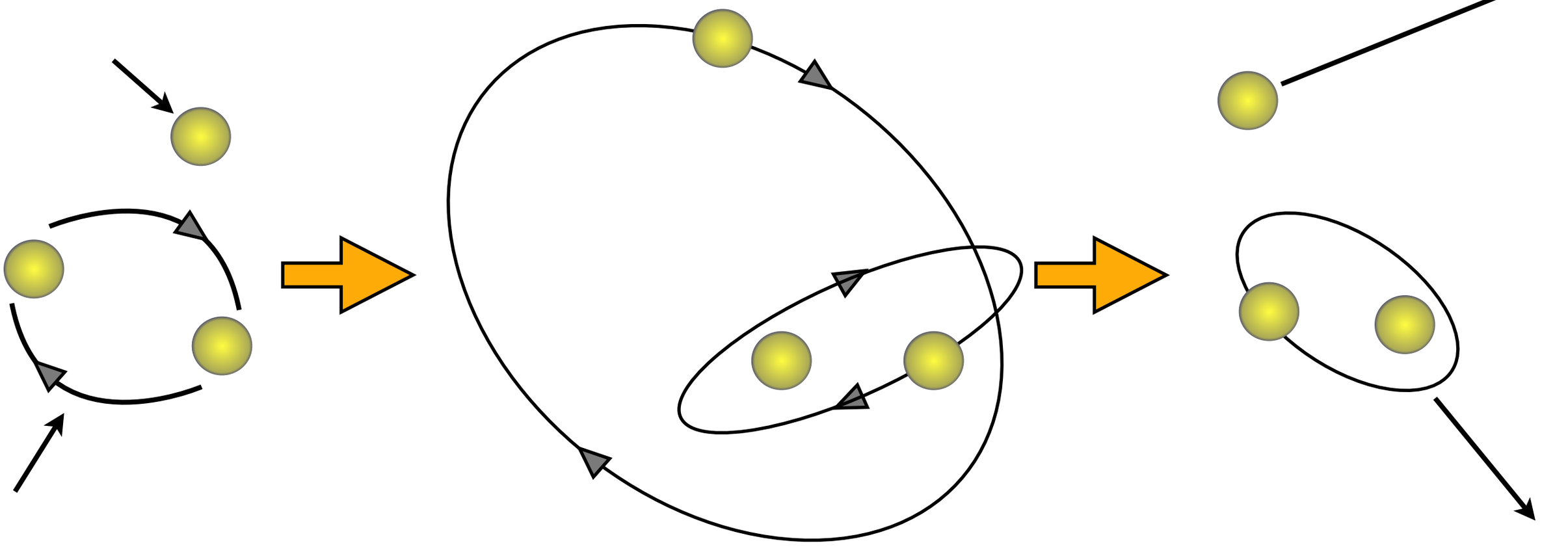
$$m_1, m_2, m_3 \sim 10 M_\odot, \quad n \sim 10^4 \ {\rm pc}^{-3}, \quad v \sim 10 \ {\rm km \ s}^{-1} :$$

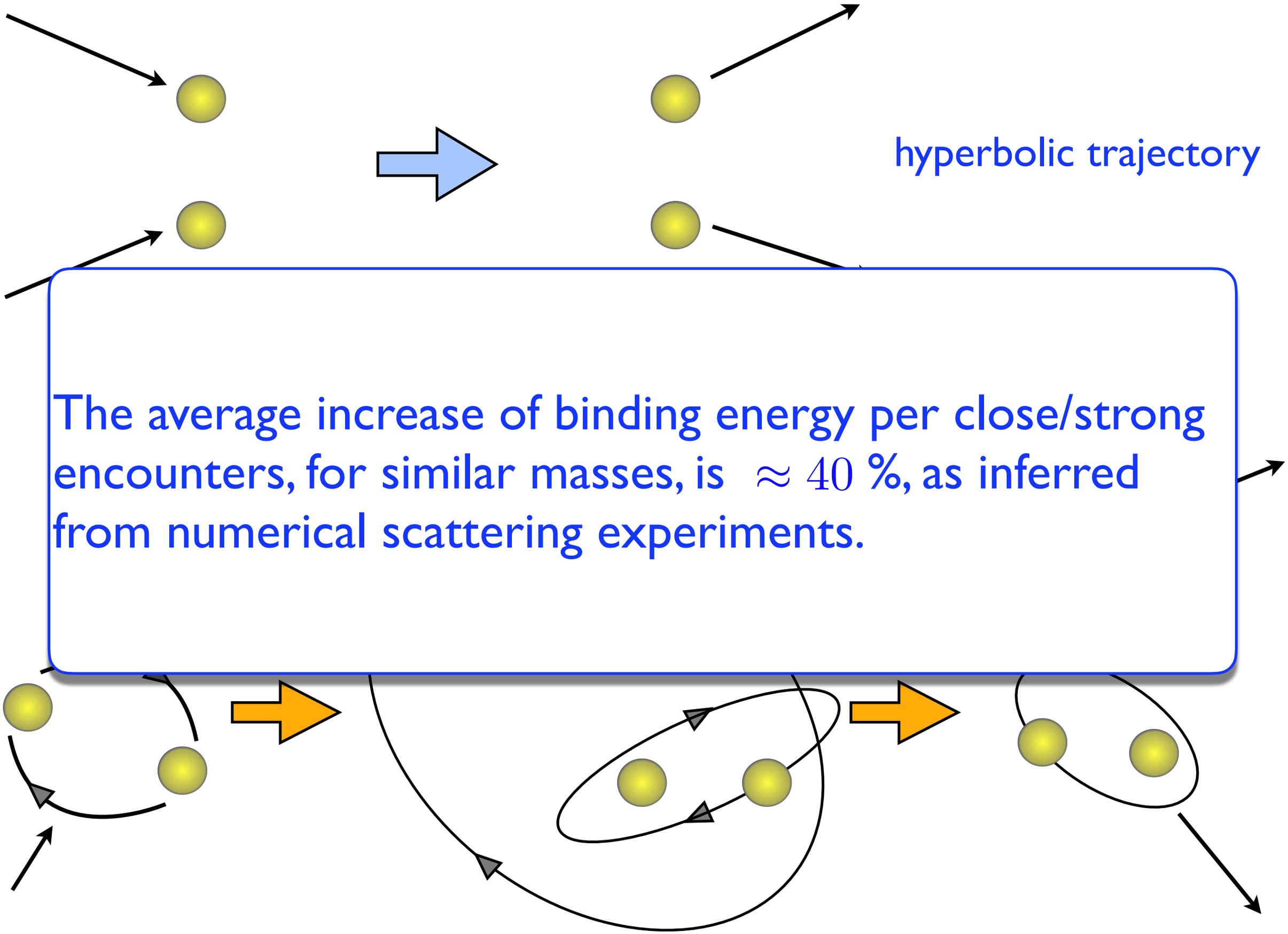
$$\tau_{\rm foc} \lesssim {\rm Myr}$$

Image not to scale

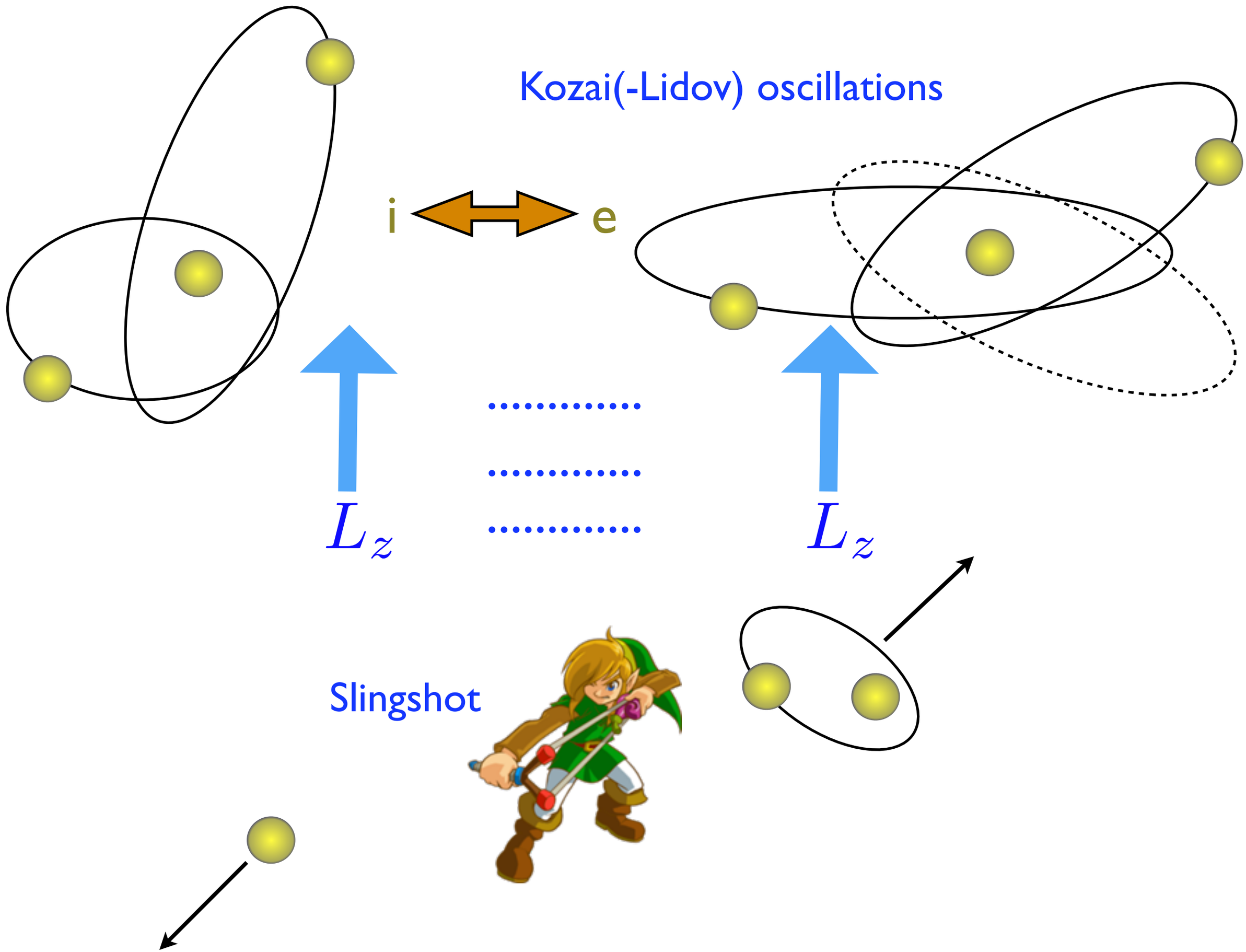


Intermediate (hierarchical) triple



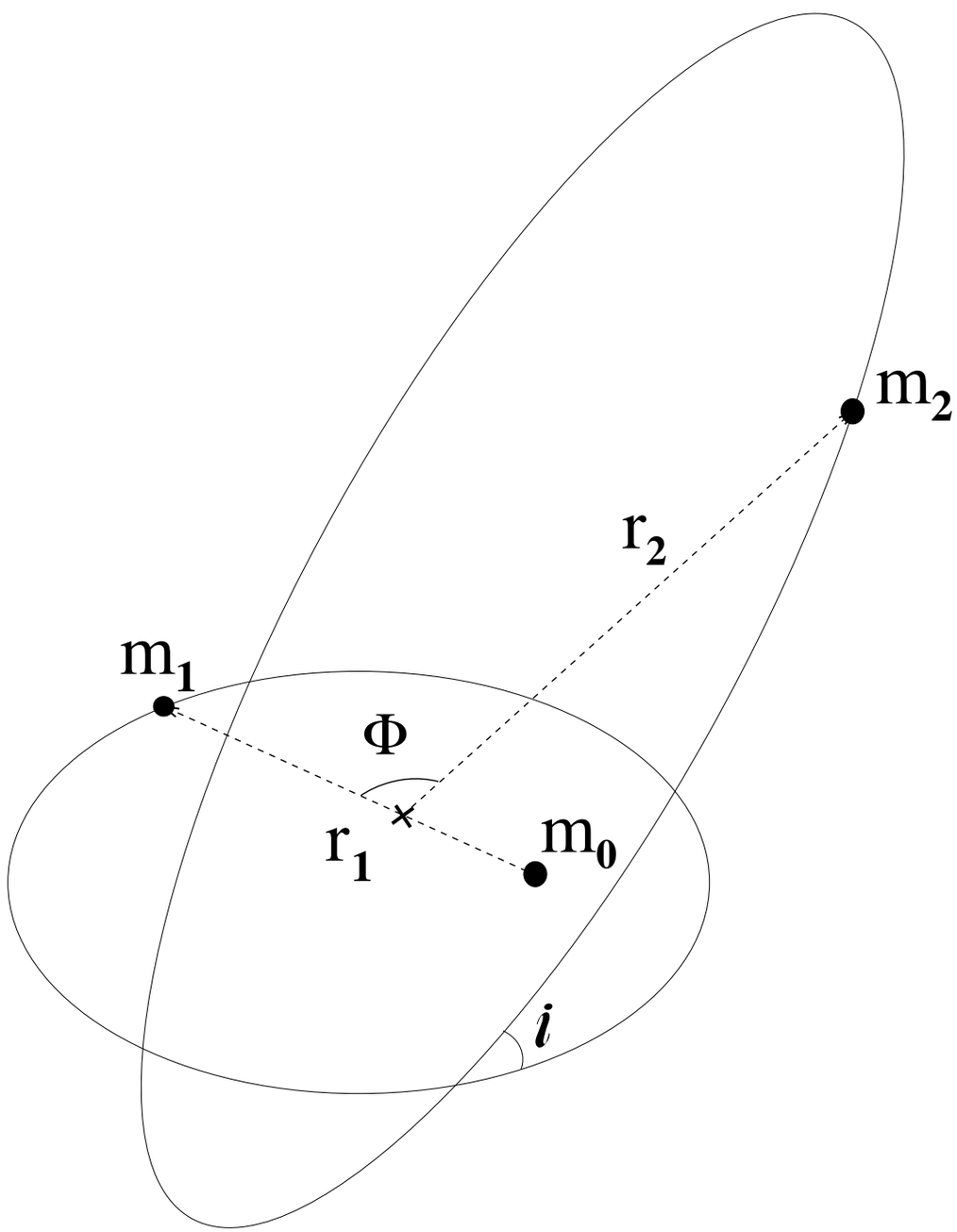


The average increase of binding energy per close/strong encounters, for similar masses, is ≈ 40 %, as inferred from numerical scattering experiments.


Kozai(-Lidov) oscillations
i
e
L_z
L_z
Slingshot

$$P_e \approx 2\pi\sqrt{\frac{a_1^3}{G(m_0+m_1)}}\left(\frac{m_0+m_1}{m_2}\right)\left(\frac{a_2}{a_1}\right)^3(1-e_2^2)^{3/2}$$

[1 => inner orbit, 2 => outer orbit]



$$e_{\max} \approx \sqrt{1-(5/3)\cos^2 i_0} \quad (e_{1_0}=0)$$

E.g.,

$$a_1 = 1\ \mathrm{AU}, a_2 = 100\ \mathrm{AU}, e_2 = 0$$

$$m_0 = m_1 = m_2 = 10 M_\odot$$

$$\Rightarrow P_e \approx 0.4\ \mathrm{Myr}$$

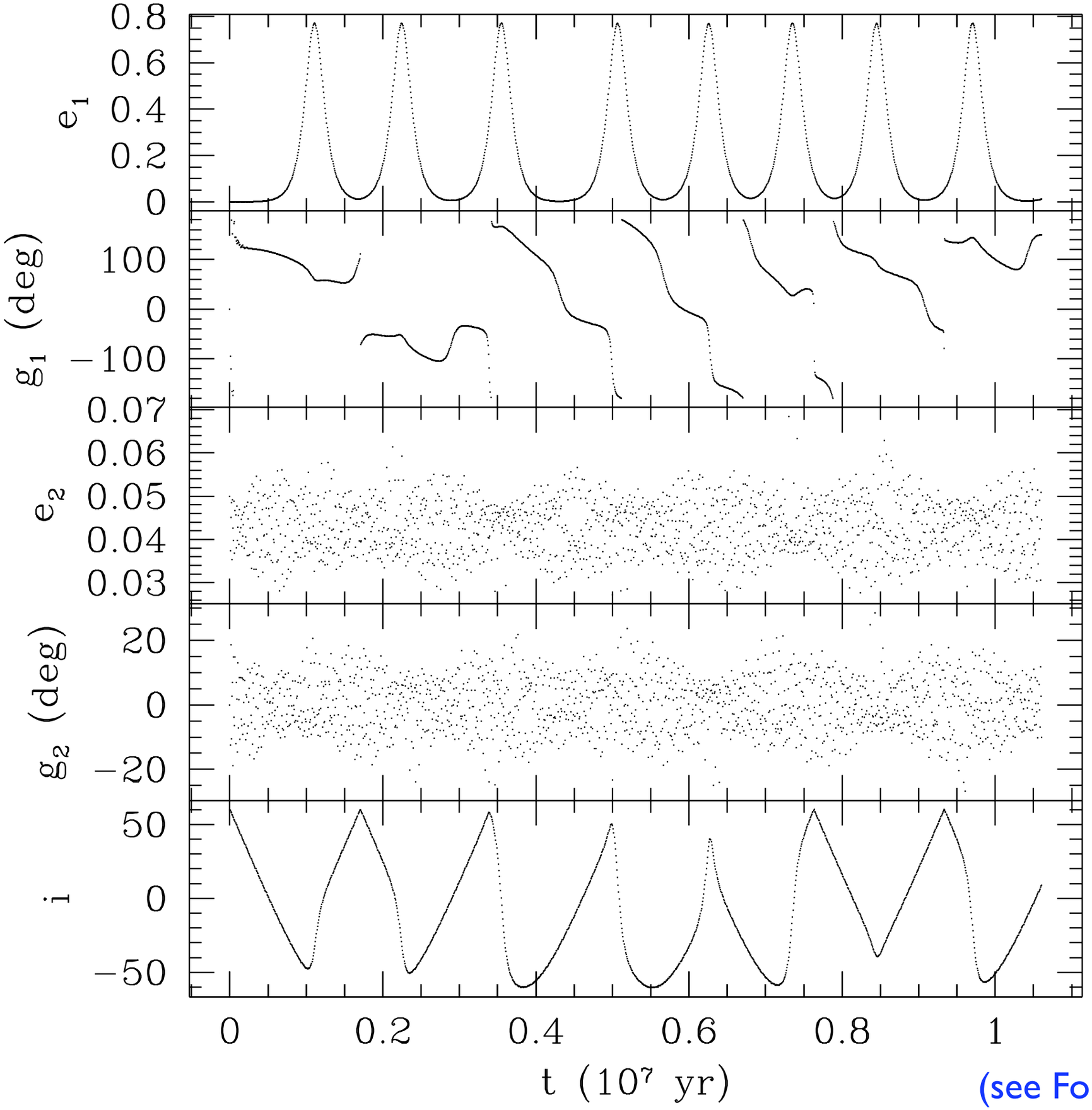
(similar to dynamical encounter time in a dense cluster)

Kozai oscillation formulae

see

Kiseleva et al. 1998, MNRAS, 300, 292

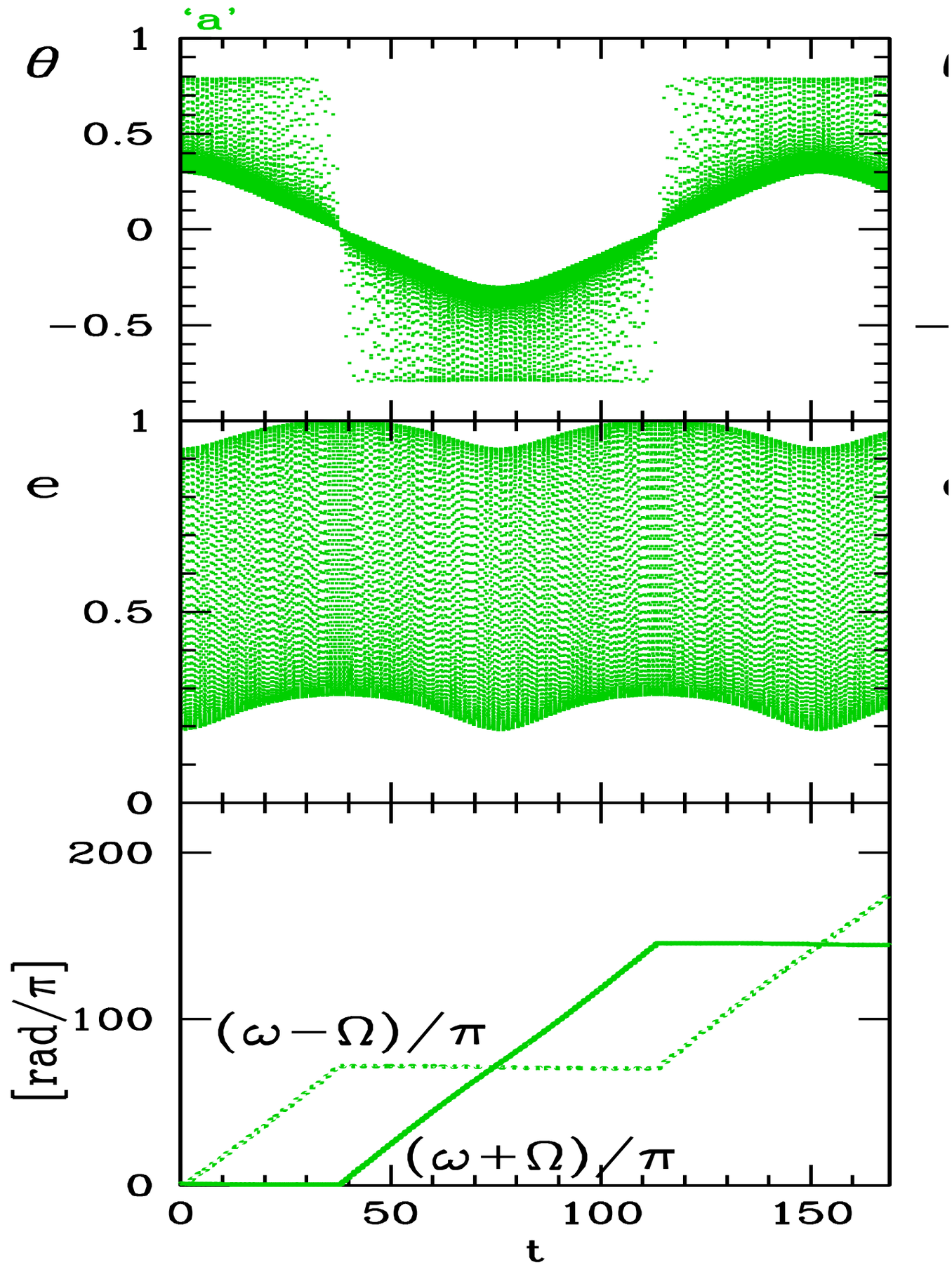
Ford et al. 2000, ApJ, 535, 385,



Standard
Kozai(-Lidov) Mechanism
(SKM)

$(e_{1_0} = 0)$

(see Ford et al. 2000, ApJ, 535, 385)



Eccentric
Kozai(-Lidov) Mechanism
(EKM)

$(e_{1_0} > 0)$

Lithwick & Naoz 2011, ApJ, 742, 94

Kozai osc. of BH triple / inner BH binary

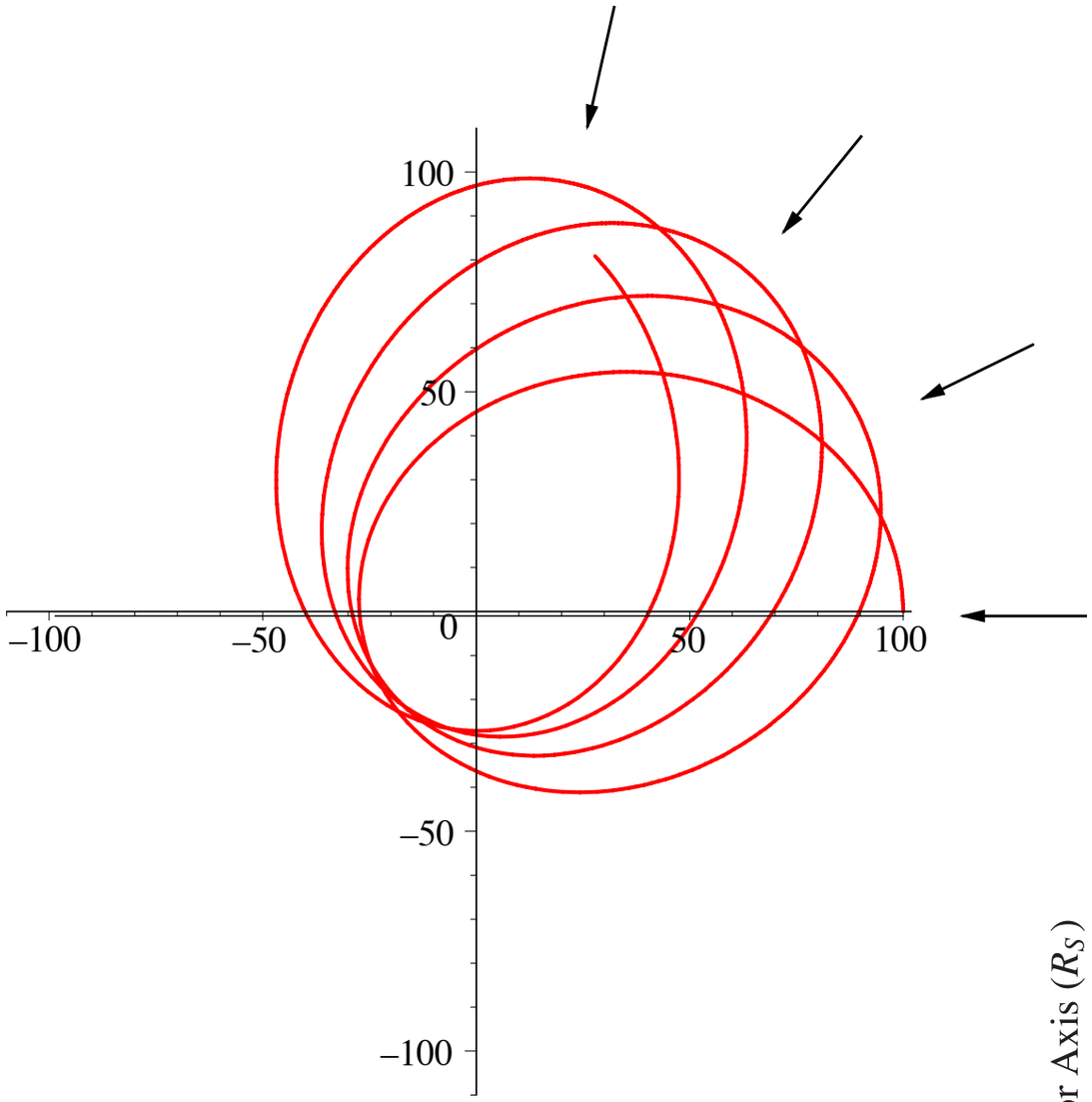
High inner eccentricity => gravitational-wave radiation

Recall Peters' formula of orbit-shrinkage via GW

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64G^3}{5c^5} m_1 m_2 (m_1 + m_2) a^{-3} (1 - e^2)^{-\frac{7}{2}}$$

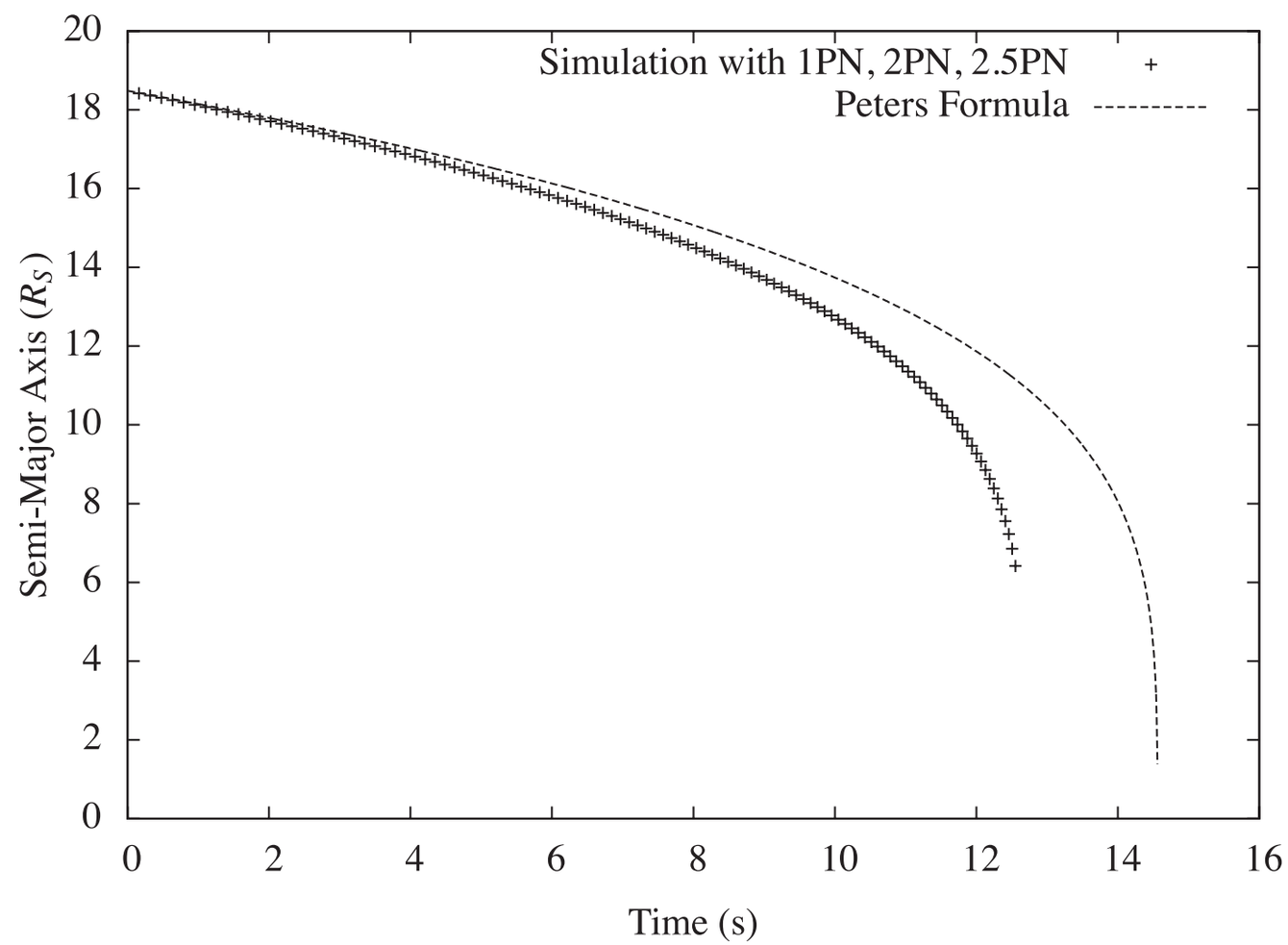
$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304G^3}{15c^5} m_1 m_2 (m_1 + m_2) a^{-4} e (1 - e^2)^{-\frac{5}{2}}$$

Inner orbit in a BH-BH-BH becomes relativistic via Kozai mechanism



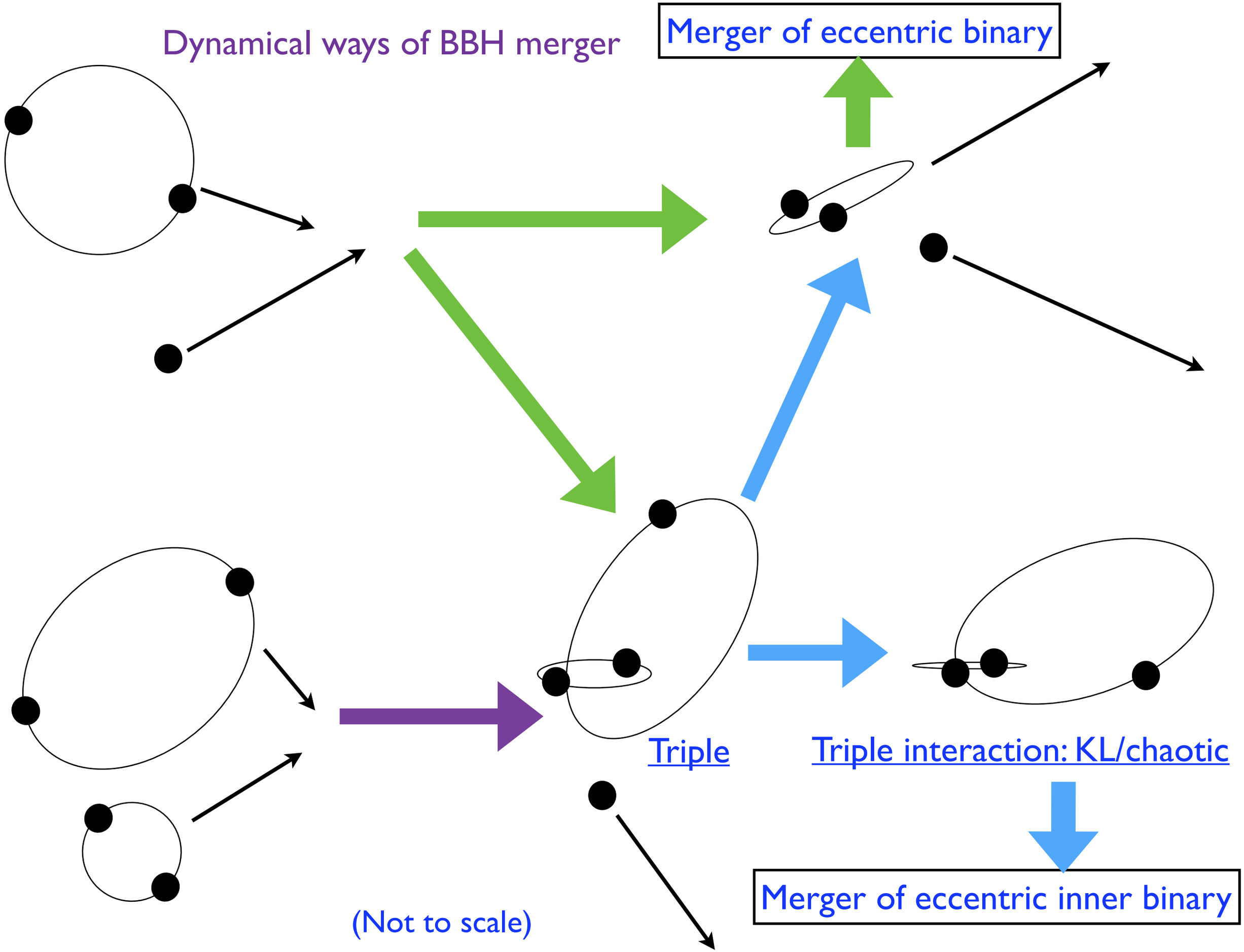
GR precession partially "detunes" Kozai cycle and delays onset of GW in-spiral for inner binary

<u>In-spiral still happens as seen in numerical studies</u>

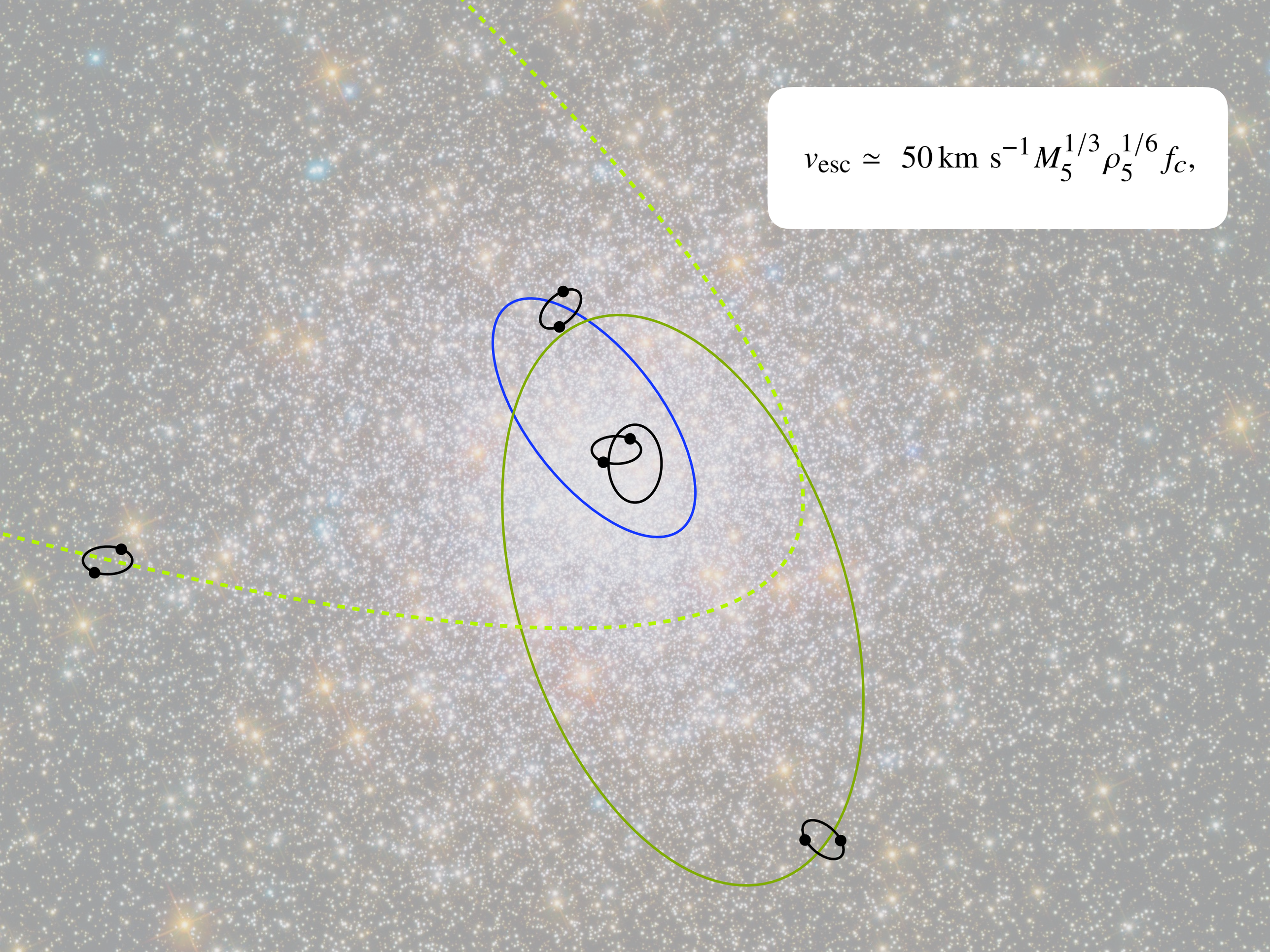


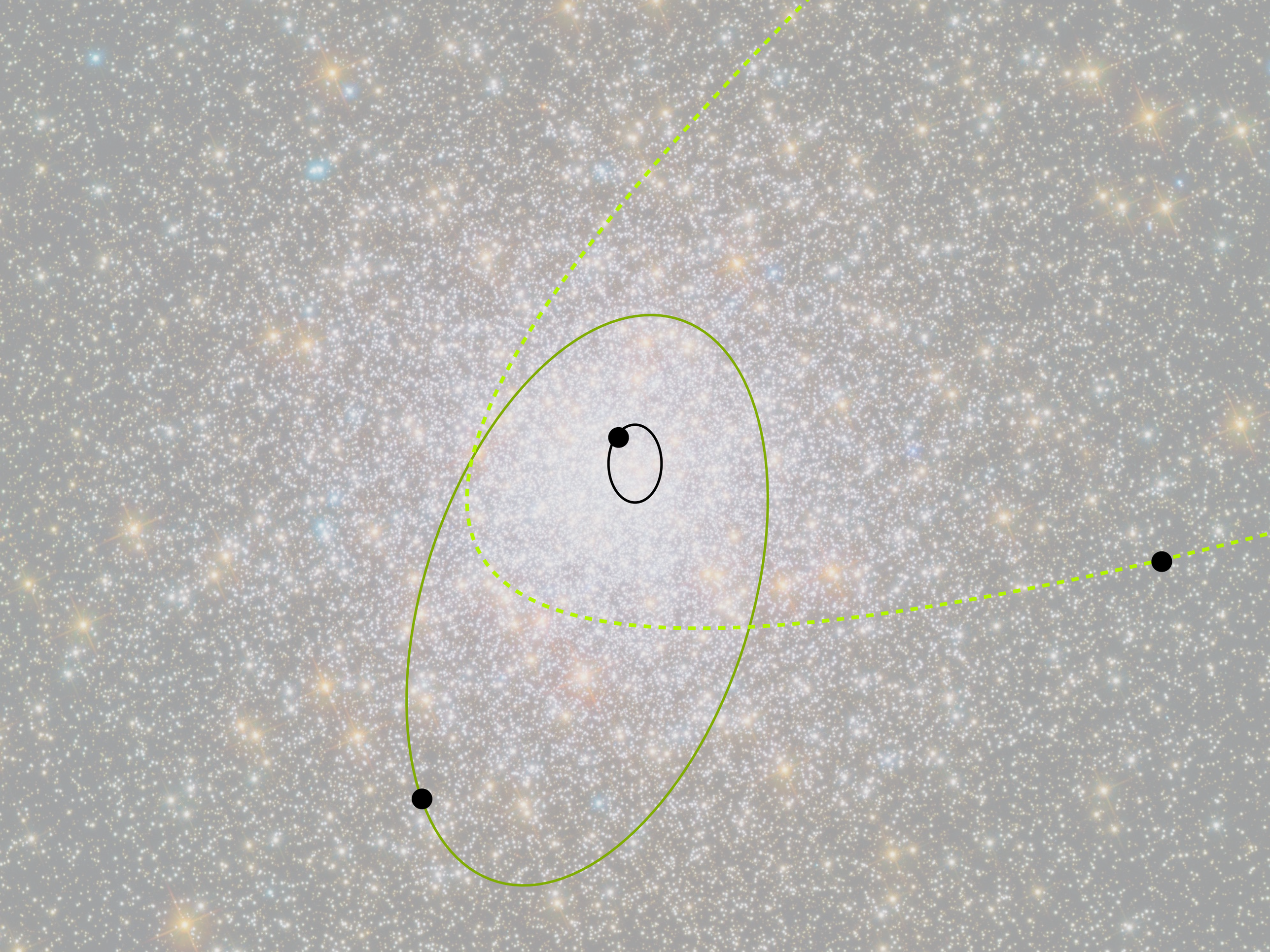
In-spiral process itself becomes quicker

From Brem P., Amaro-Seoane P., Spurzem R., 2013, MNRAS, 434, 2999


Dynamical ways of BBH merger
Merger of eccentric binary
Triple
Triple interaction: KL/chaotic
Merger of eccentric inner binary
(Not to scale)

$$v_{\rm esc} \simeq 50\,{\rm km\ s^{-1}}\, M_5^{1/3}\, \rho_5^{1/6}\, f_c,$$





Simulate star cluster in a computer: direct N-body calculations

Solve Newtonian equation of motion for N bodies (N^2xN operation)

$$\ddot{x}_{k,i} = -G \sum_{j=1, j\neq i}^{N} \frac{m_j}{r_{ij}^3} \vec{r_{ij}} \cdot \hat{k}$$

Computationally expensive but fully self-consistent treatment of all types of dynamical interactions.

No symmetry required

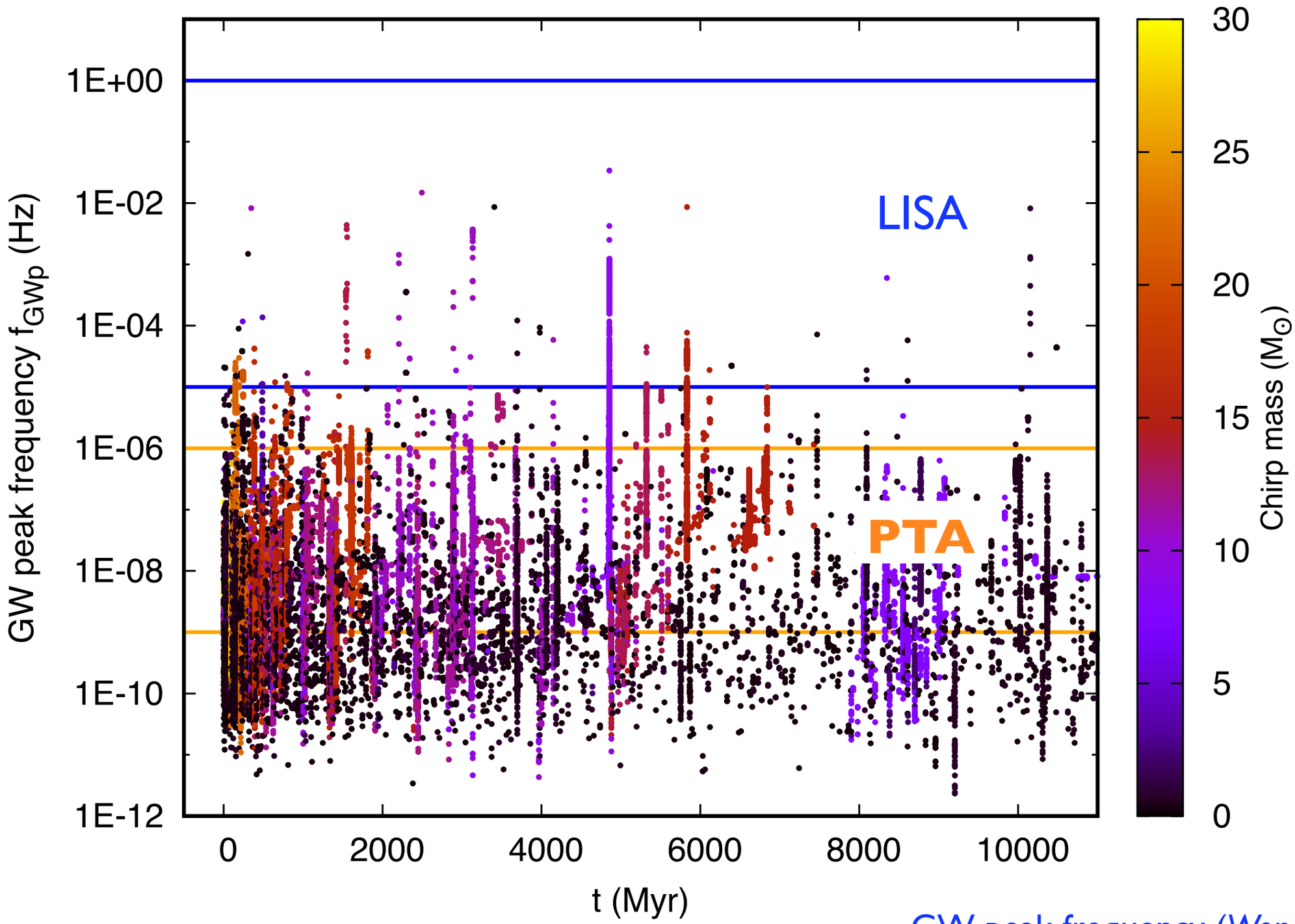
Up to several 10^5 bodies doable in GPU-based/parallel architecture.

Sverre Aarseth's (IoA, Cambridge) N-body code "NOBDY6/7".

What is NBODY6/NBODY7?

- A versatile, direct, object-by-object N-body integration program initiated and still being developed by Sverre Aarseth (for at least 20 years!) [Fortran-based code]
- No assumption on initial conditions or symmetry of the N-body system (or of the problem under study)
- No approximation while dealing with internal or external forces
- Can handle (in principle!) an arbitrary population of compact subsystem (binary and above)
- Incorporates analytic (population-synthesis type) recipes of stellar and binary evolution
- Incorporates general relativity (post-Newtonian)
- Incorporates any form of static or time-varying external gravitational field (host-galactic potential, embedding gas potential)
- Publicly available!

Binaries inside cluster undergoing close (Chain) interactions



Outcome of a direct N-body model calculation (NBODY7).

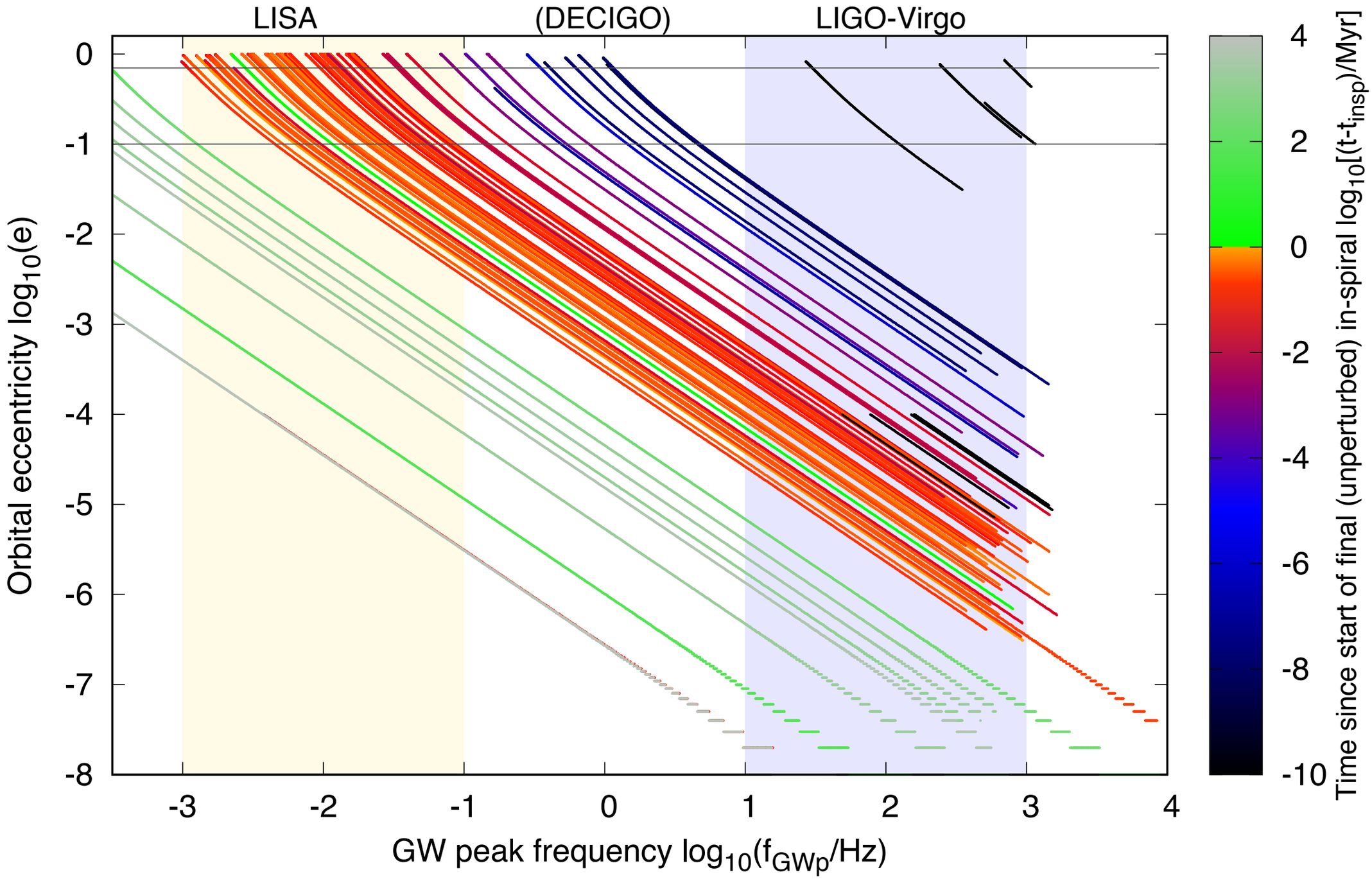
$$N \sim 10^4; r_h \sim \text{ pc}$$

See Banerjee, S., 2018, MNRAS 481, 5123

GW peak frequency (*Wen 2003*):

$$f_{\rm GWp} = \frac{\sqrt{G(m_1+m_2)}}{\pi} \frac{(1+e)^{1.1954}}{\left[a(1-e^2)\right]^{1.5}}.$$

Multi-band in-spiral of *stellar-mass* BBHs: from relativistic, direct N-body simulations



Banerjee, S.: in prep. NBODY7 computations. $10^4 M_\odot \lesssim M_{cl}(0) \lesssim 10^5 M_\odot$; $1\mathrm{pc} \lesssim r_h(0) \lesssim 3\mathrm{pc}$; $Z_\odot/100 < Z < Z_\odot$ (approx. 50 models)

see also: Sesana A., 2016, PRL, 116, 231102,
Chen X., Amaro-Seoane P., 2017, ApJ, 842, L2
Banerjee, S., 2018, MNRAS 473, 909

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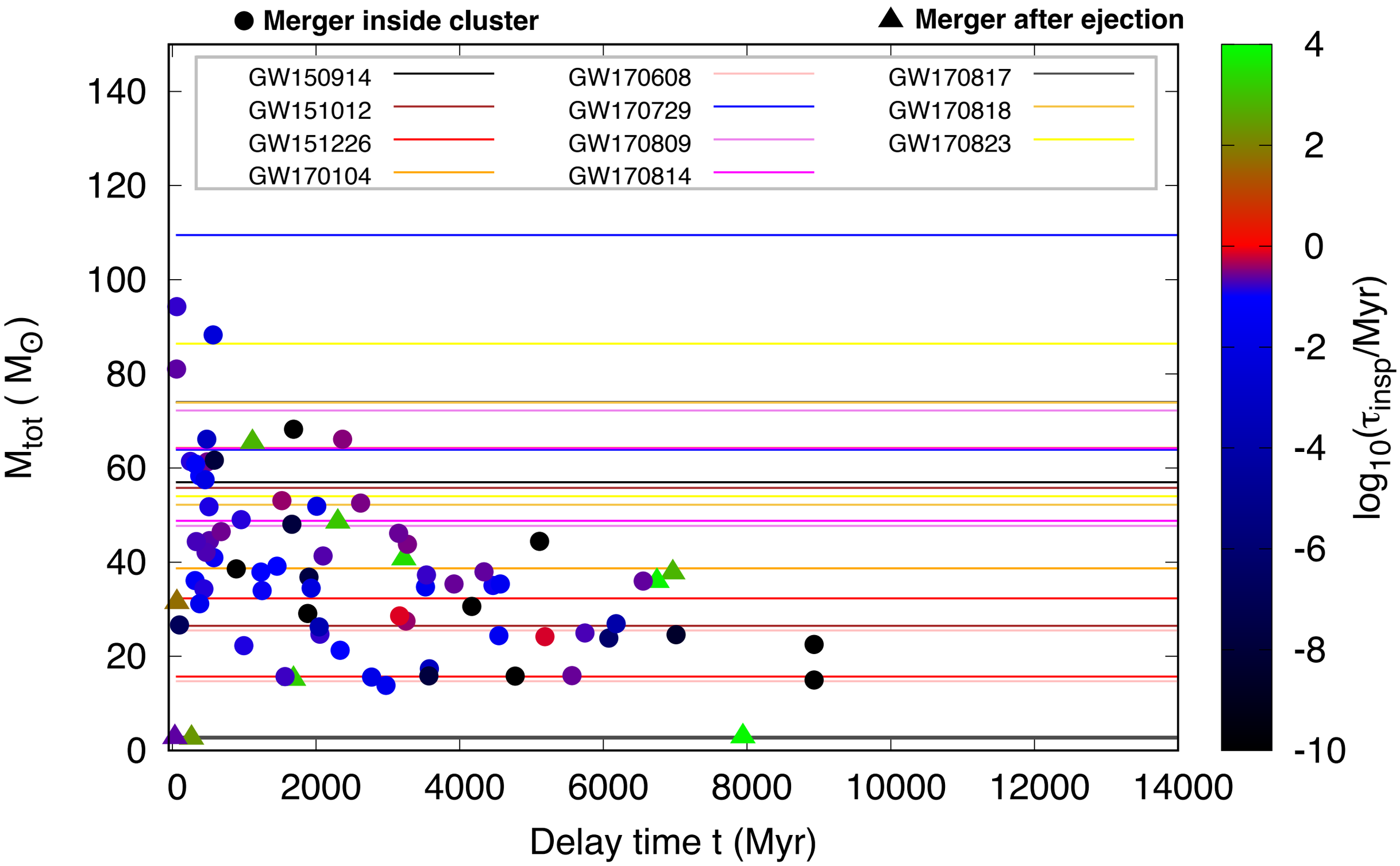
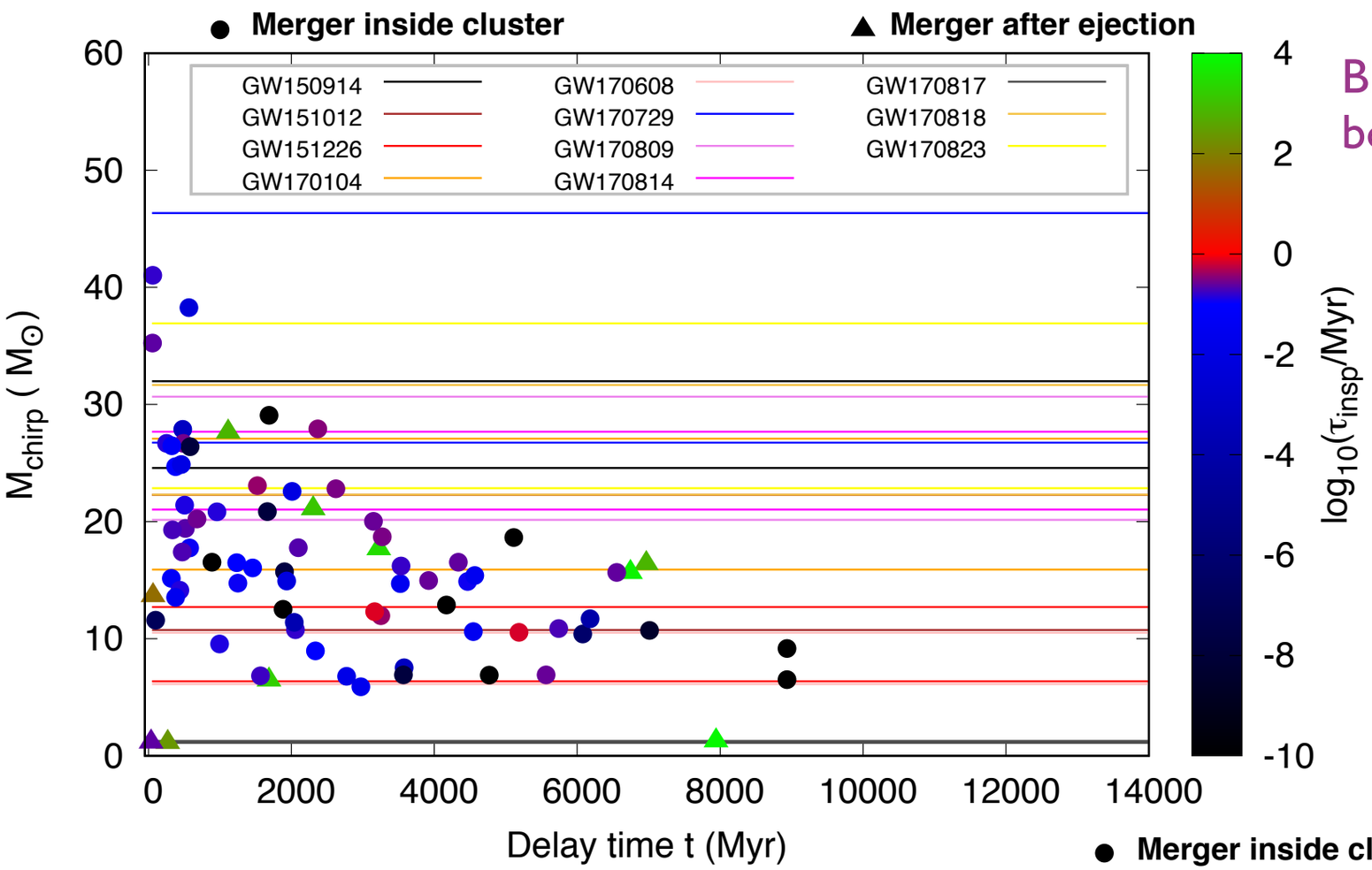
BBHs mergers from relativistic, direct N-body simulations



Banerjee, S.: in prep. NBODY7 computations. Approx. 50 long-term evolutionary models.

$$10^4 M_\odot \lesssim M_{cl}(0) \lesssim 10^5 M_\odot; \quad 1\mathrm{pc} \lesssim r_h(0) \lesssim 3\mathrm{pc}; \quad Z_\odot/100 < Z < Z_\odot$$



BBHs mergers from relativistic, direct N-body simulations

See also:

Banerjee, S., et al., 2010, MNRAS, 402, 371

Banerjee, S., 2017, MNRAS, 467, 524

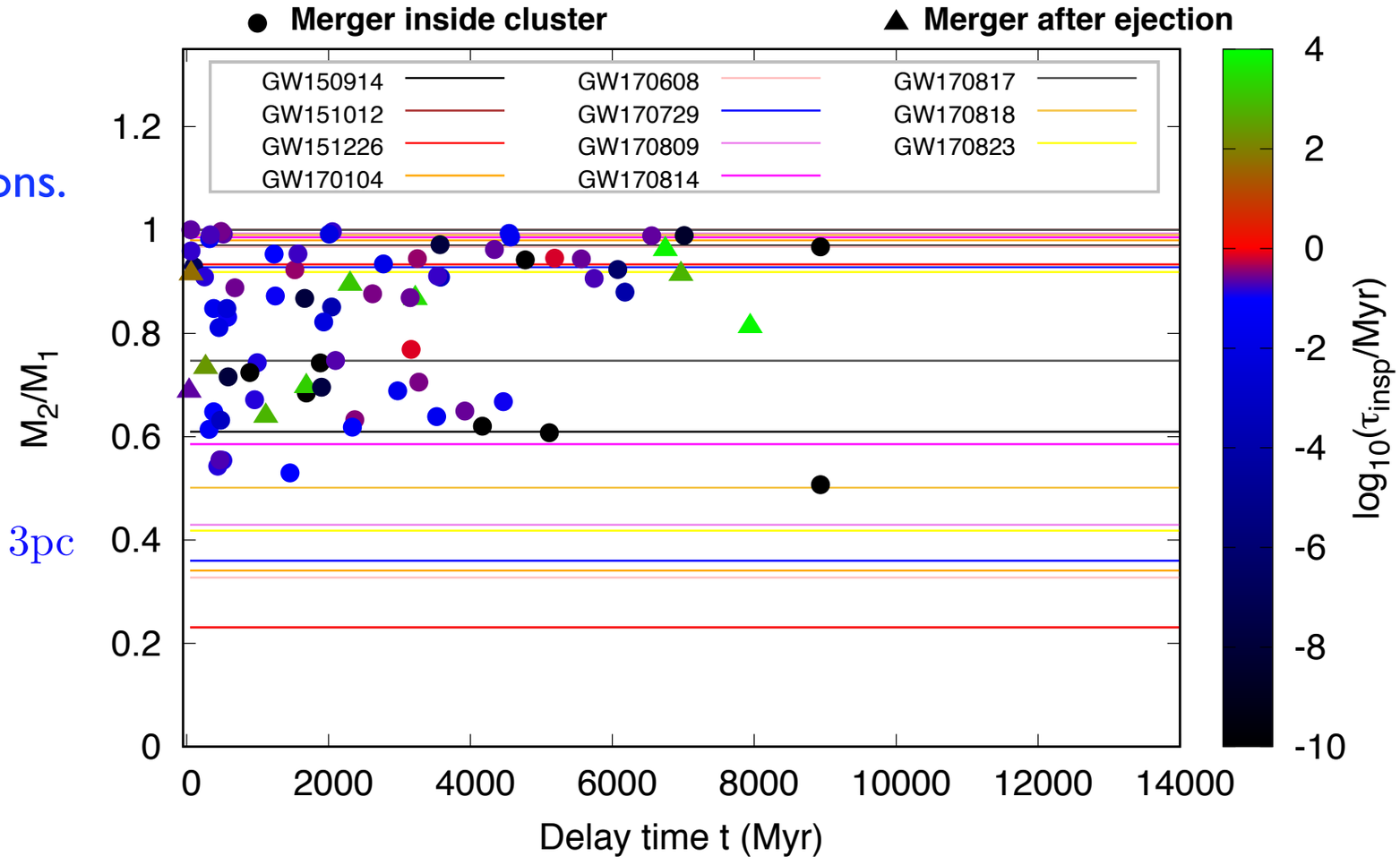
Banerjee, S., 2018, MNRAS 473, 909

Banerjee, S., 2018, MNRAS 481, 5123

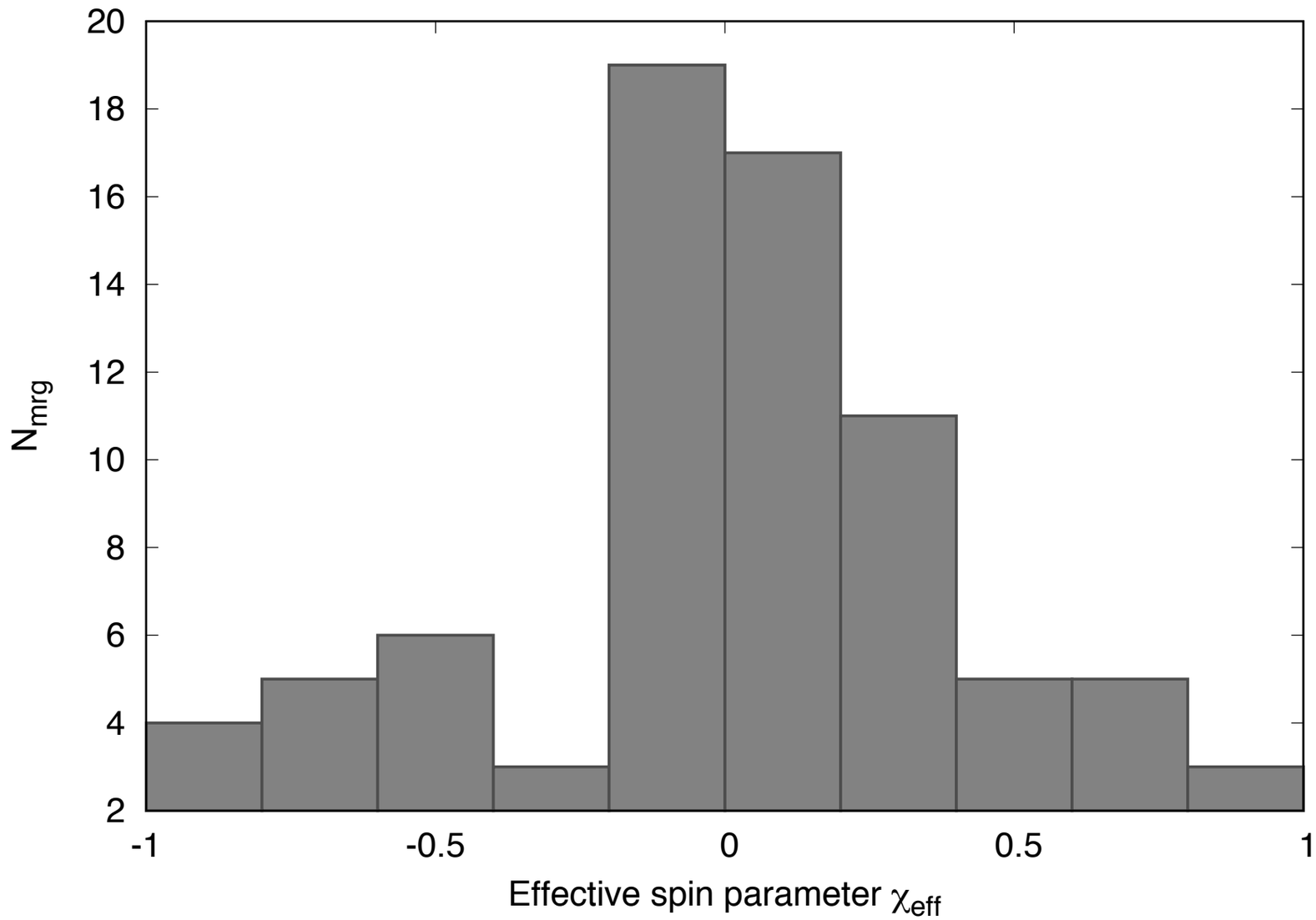
Banerjee, S.: in prep. NBODY7 computations. (approx. 50 models)

$$10^4 M_\odot \lesssim M_{cl}(0) \lesssim 10^5 M_\odot; \quad 1\text{pc} \lesssim r_h(0) \lesssim 3\text{pc}$$

$$Z_\odot/100 < Z < Z_\odot$$



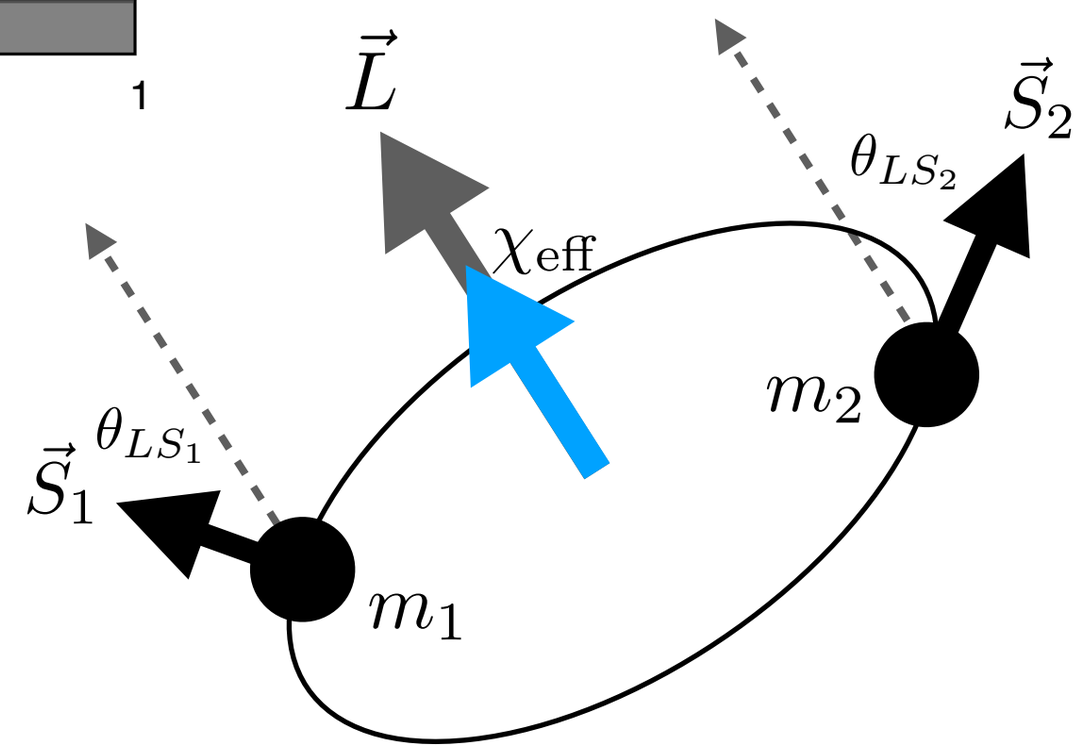
BBHs mergers from relativistic, direct N-body simulations: effective aligned spin magnitude parameter



Banerjee, S.: in prep. NBODY7 computations. (approx. 50 models)

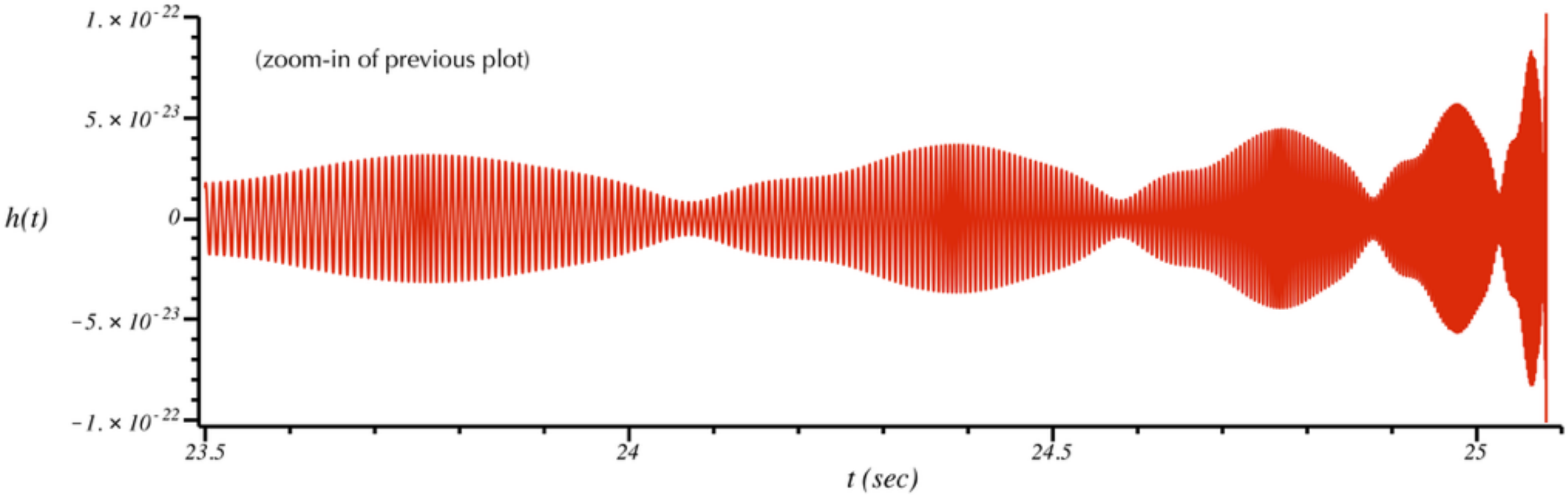
$$\vec{a} = \frac{c\vec{S}}{Gm^2}$$

$$\chi_{\rm eff} = \frac{m_1 a_1 \cos\theta_{LS_1} + m_2 a_2 \cos\theta_{LS_2}}{m_1 + m_2}$$



See also (Monte Carlo simulations):

Rodriguez, C.L., et al., 2016, ApJ, 832, L2
Morawski, J., et al., 2018, MNRAS 481, 2168

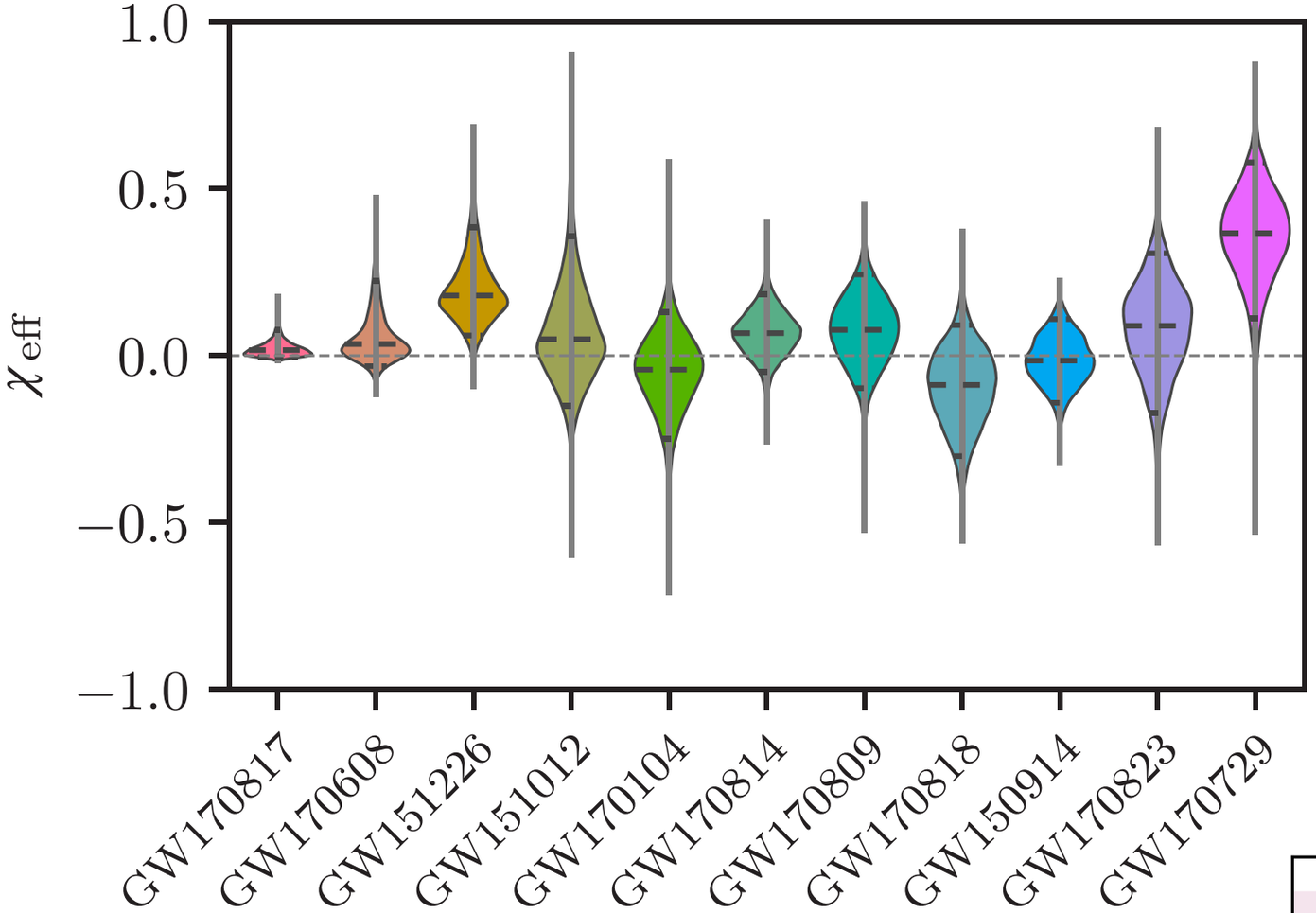


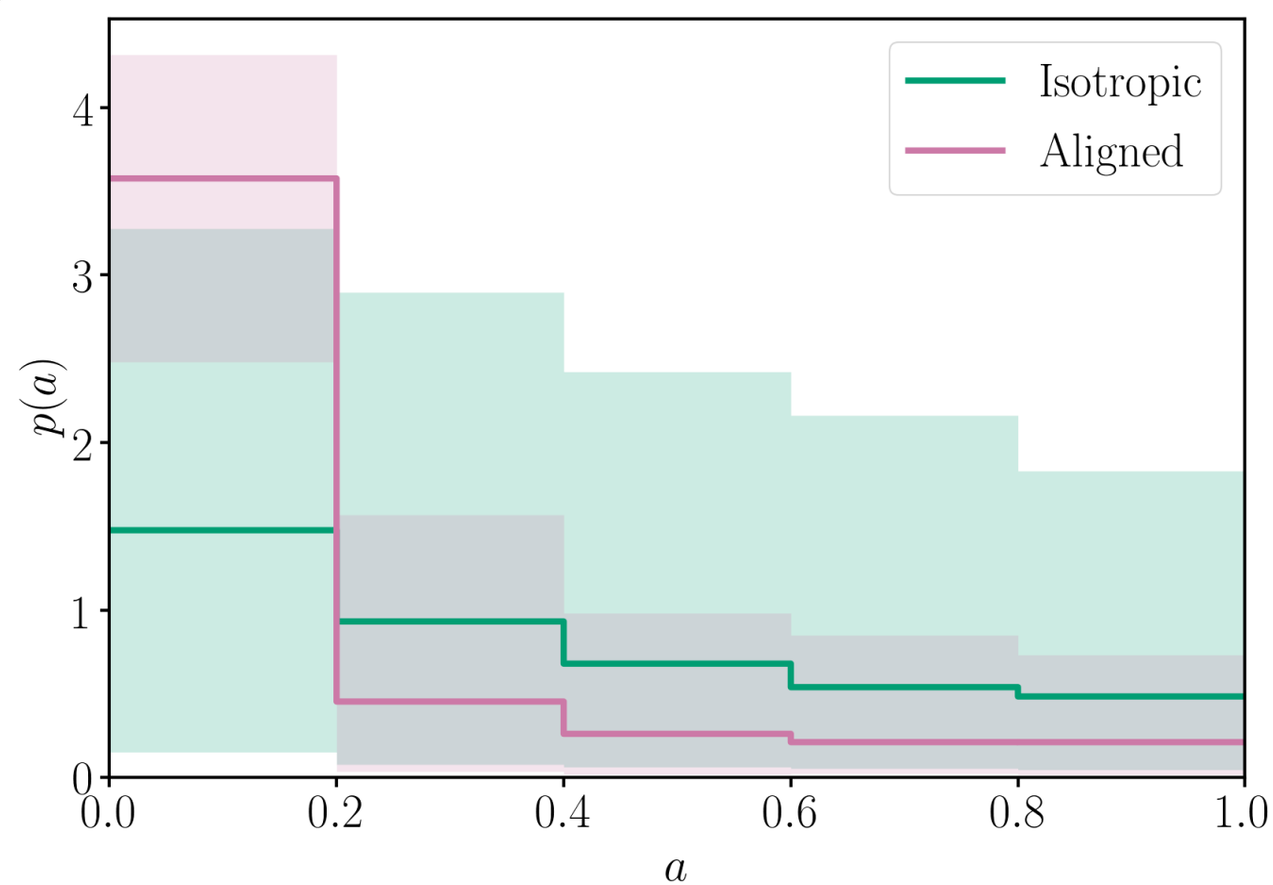
http://www.soundsofspacetime.org/spinning-binaries.html

Can measure initial spins through spin-orbit coupling if enough cycles of inspiral are measured.

Slide credit: Matt Benacquista

Effective alined spin magnitude from observations (LIGO O2)





From:

GWTC-1: Abbott, B.P., et al., 2019, Phys. Rev. X., 9, 031040

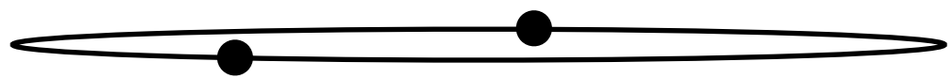
Abbott, B.P., et al., arXiv: 1811.12940

Dynamically-induced BBH mergers can happen over a wide range of timescales (eccentricities). Commonly used terminology:

$e = 1$

Mergers inside cluster

GW capture: GR merger *during* a dynamical interaction involving triple/multiplet or via GR energy loss *during a hyperbolic encounter*.

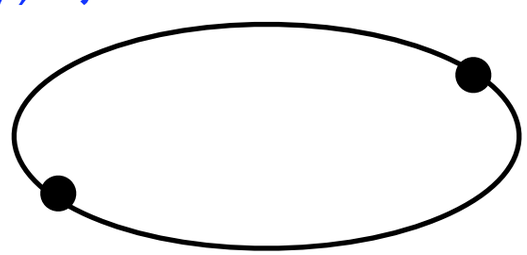


$$1 - e < 10^{-5}; \quad \tau_{\rm mrg} < \tau_{\rm orb} (\lesssim {\rm yr})$$

In-cluster mergers: GR merger of a BBH *in-between* two subsequent close interactions.

$$1 - e < 10^{-3}; \quad \tau_{\rm mrg} < \tau_{\rm foc} (\lesssim {\rm Myr})$$

Ejected mergers: GR merger after getting (dynamically) ejected from a cluster.

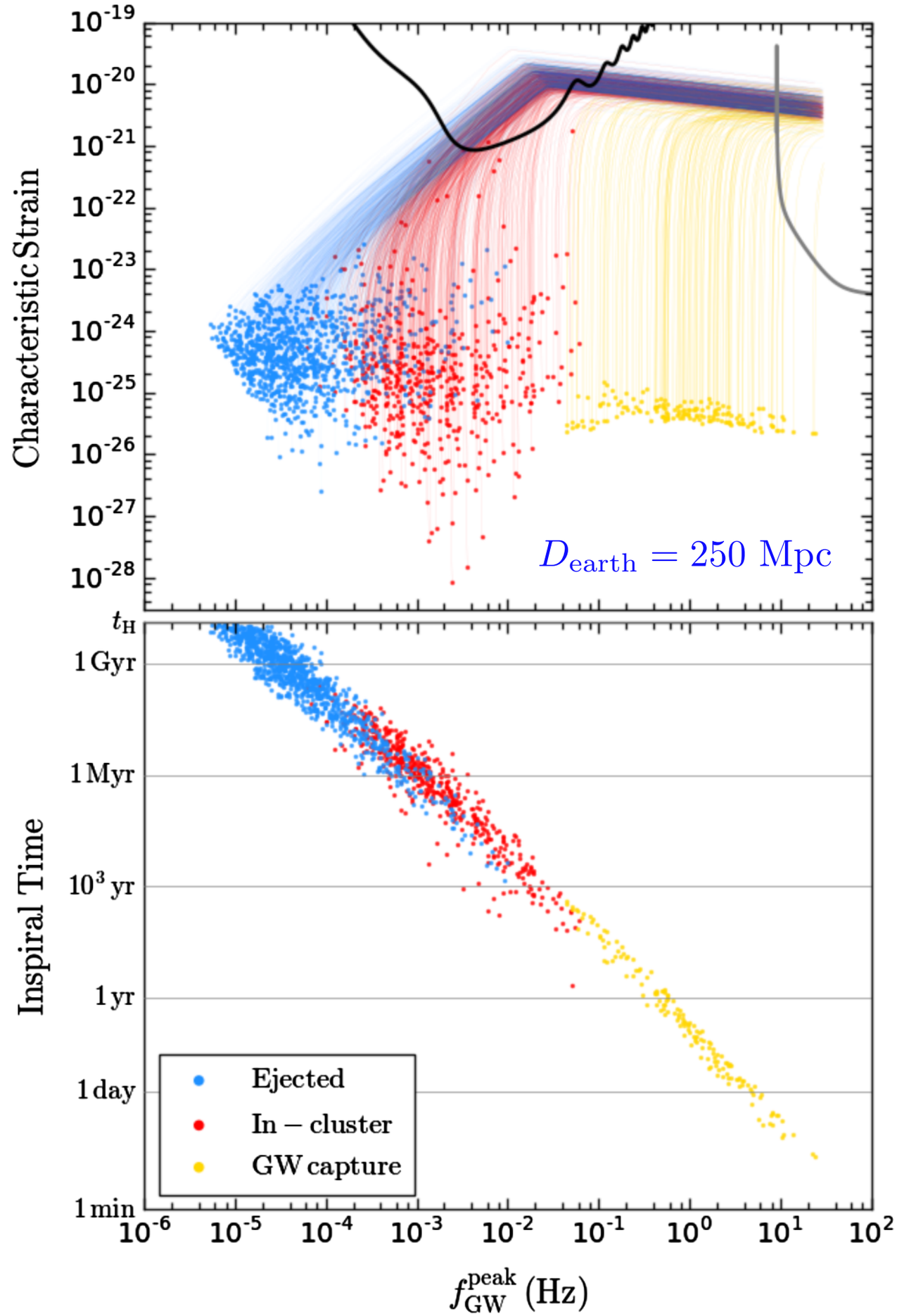


$$1 - e < 10^{-1}; \quad \tau_{\rm mrg} < \tau_{\rm Hubble} (\approx 13.7 \ {\rm Gyr})$$

$e = 0$

Banerjee, S., et al., 2010, MNRAS, 402, 371
Banerjee, S., 2017, MNRAS, 467, 524
Bae, Yeong-Bok, et al., 2017, PhRvD, 96, id.084009
Rodriguez, Carl L., et al., 2018, PhRvLett, 120, id.151101
Kremer, K., et al., 2019, PhRvD, 99, id.063003

From Kremer, K., et al., 2019, PhRvD, 99, id.063003
(Monte Carlo simulations of model globular clusters)
See also: Samsing, J., D'Orazio, D.J., 2018, MNRAS, 481, 5445
Samsing, J., et al., 2014, ApJ, 784, 71



Compact-binary merger detection rate for LIGO

$$\mathcal{R}_{\rm LIGO} = \frac{4}{3}\pi D^3 \rho_{\rm cl} \mathcal{R}_{\rm mrg}, \qquad D = D_0 \left(\frac{M_{\rm ch}}{M_{{\rm ch},nsns}} \right)^{5/6},$$

$D =$ maximum distance for detection at mean orientation

At design sensitivity:

$D_0 \approx 300$ Mpc $\qquad D \gtrsim 1.5$ Gpc for BH-BH merger

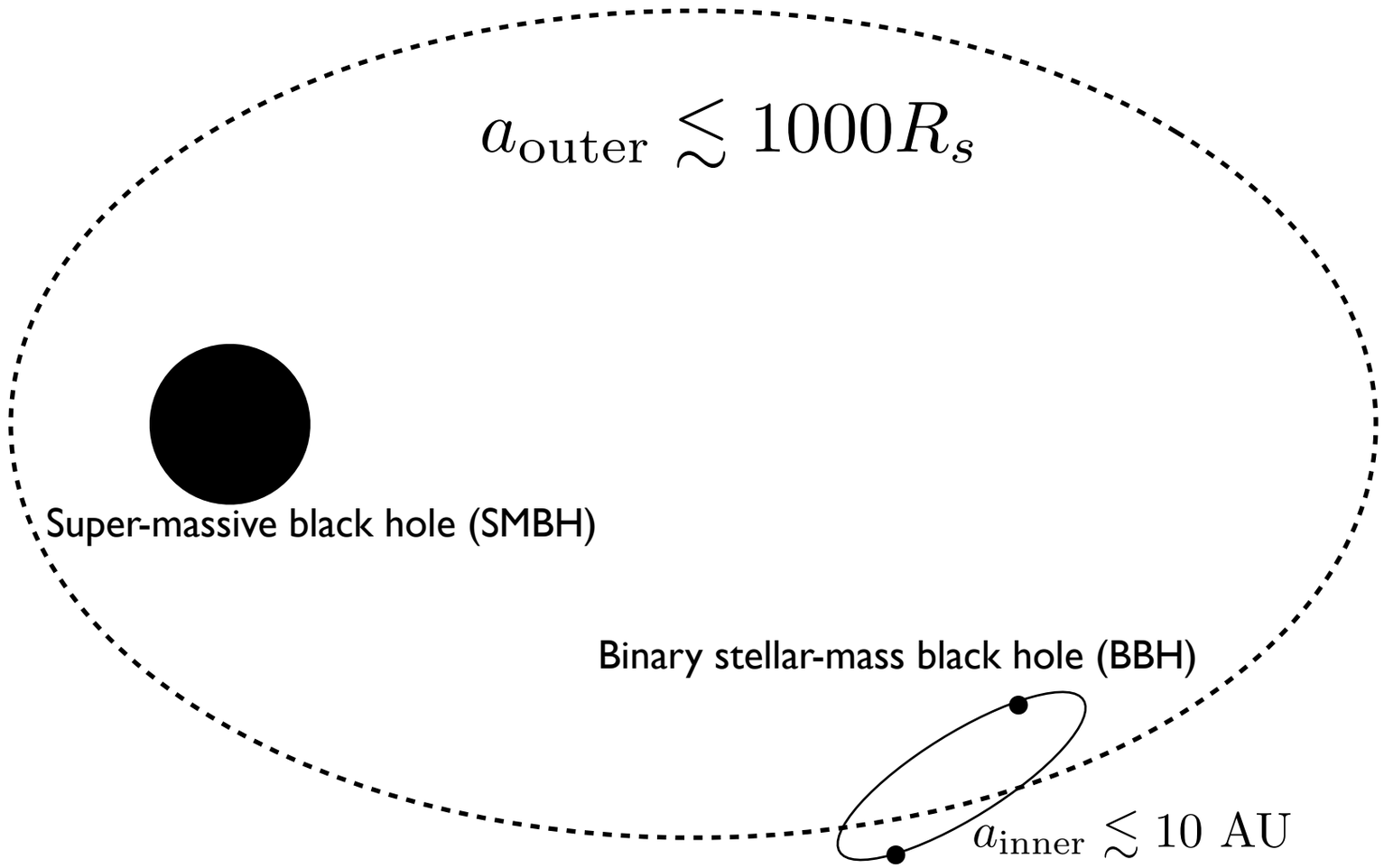
$\mathcal{R}_{\rm mrg} =$ merger rate per cluster, $\qquad \rho_{\rm cl} \approx 3.7\ {\rm Mpc}^{-3}$

See, e.g., Banerjee, S. et al., 2010, MNRAS, 402, 371

- ~ 10 to ~100 BBH coalescences per year by LIGO at design sensitivity, from dynamics in stellar clusters alone (Banerjee et al. 2010, Banerjee, S. 2017, *Rodriguez et al. 2015, Askar et al. 2016, Chatterjee et al. 2016*) (or $\lesssim 10\ {\rm Gpc}^{-3}\ {\rm yr}^{-1}$)

- *YMCs and open clusters potentially contribute to the BBH coalescence rate of the Universe to a similar extent as globular (and possibly nuclear) clusters* (Banerjee, S., 2018, MNRAS, 473, 909)

Stellar BBH inside a nuclear star cluster (NSC)



BBH in the vicinity of an SMBH: SMBH acts as the third, outer body, inducing Kozai-Lidov oscillations. This can potentially lead to the GR inspiral of the BBH.

Interactions with NSC stars must be taken into account.

See, e.g.,:

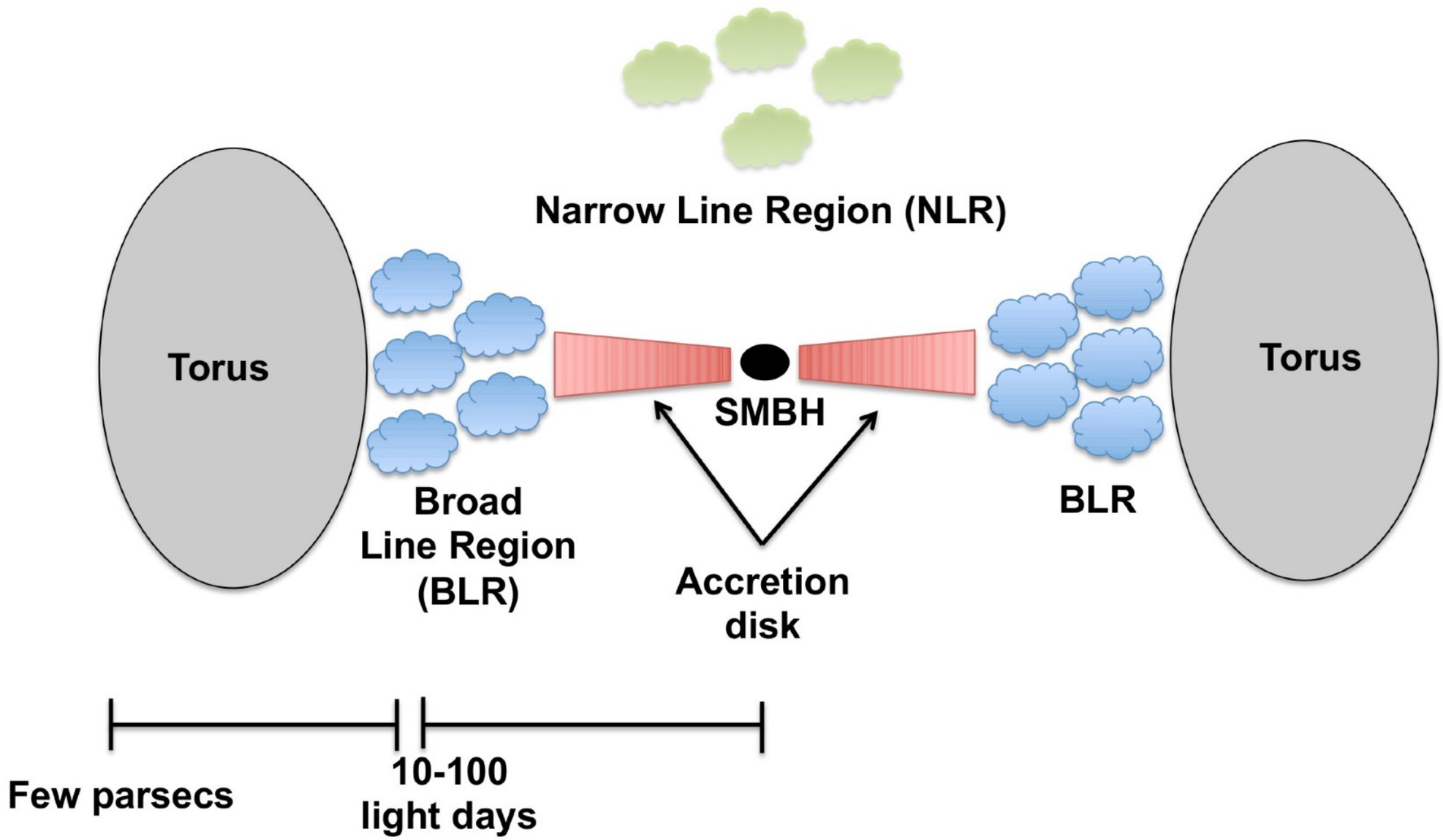
Hoang, B.-M., et al., 2018, ApJ, 856, 140

Antonini, F., Rasio, F.A., 2016, ApJ, 831, 187

Estimated full-sensitivity BBH merger rate:

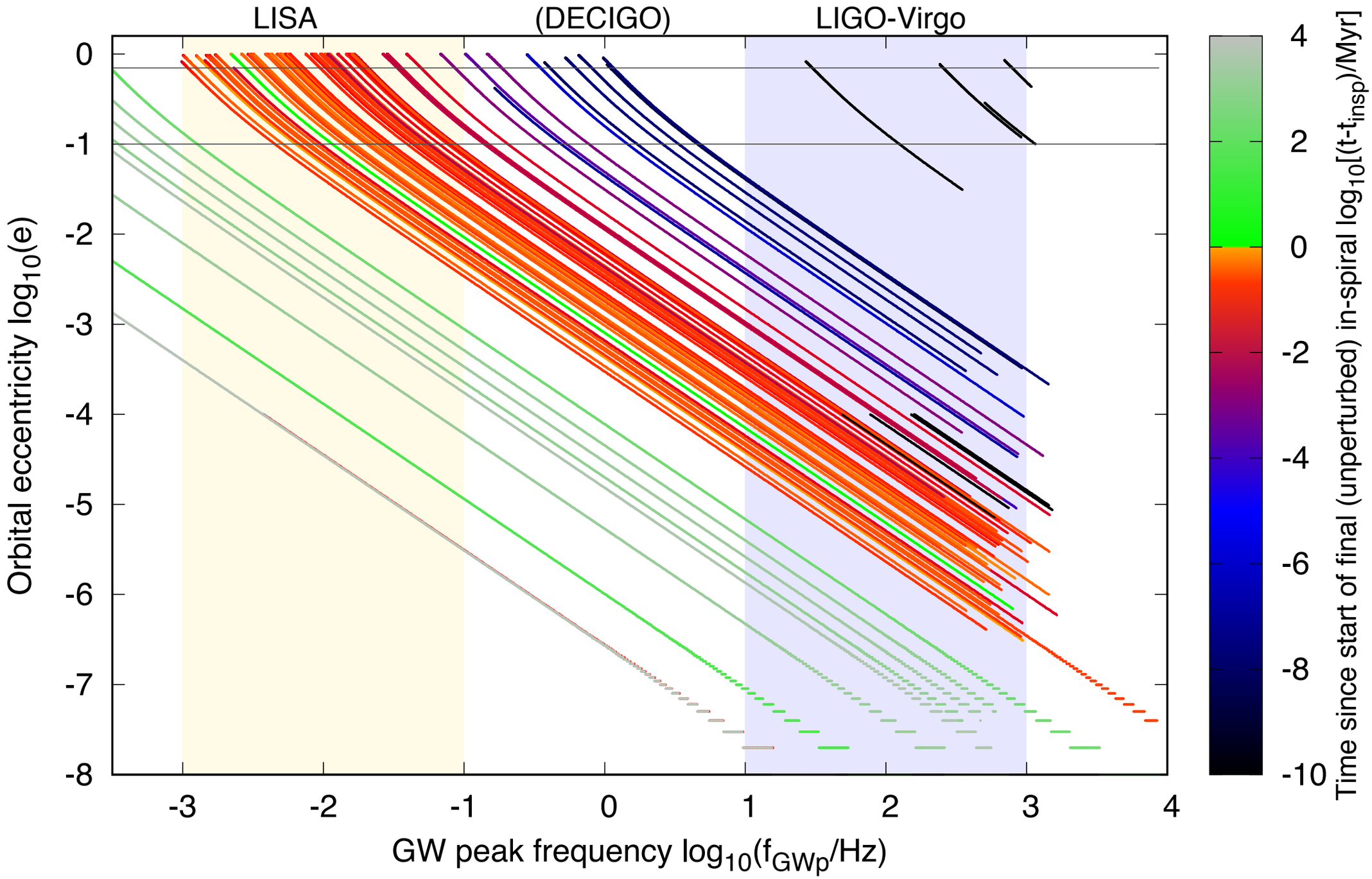
$$\sim 1 \ \mathrm{Gpc}^{-3} \ \mathrm{yr}^{-1}$$

Stellar BHs within accretion disks of active galactic nuclei (AGN)



See, e.g.,:
Secunda, A., et al., 2019, ApJ, 878, 85

Multi-band in-spiral of *stellar-mass* BBHs: from relativistic, direct N-body simulations



Banerjee, S.: in prep. NBODY7 computations. $10^4 M_\odot \lesssim M_{cl}(0) \lesssim 10^5 M_\odot$; $1\mathrm{pc} \lesssim r_h(0) \lesssim 3\mathrm{pc}$; $Z_\odot/100 < Z < Z_\odot$ (approx. 50 models)

see also: Sesana A., 2016, PRL, 116, 231102,
Chen X., Amaro-Seoane P., 2017, ApJ, 842, L2
Banerjee, S., 2018, MNRAS 473, 909

GW peak frequency (*Wen 2003*):

$$f_{\mathrm{GWp}} = \frac{\sqrt{G(m_1+m_2)}}{\pi} \frac{(1+e)^{1.1954}}{\left[a(1-e^2)\right]^{1.5}}.$$

Recap: observational signatures of dynamically-induced stellar-mass GR mergers

LISA band $10^{-3} - 10^{-1}$ Hz

Eccentric, (near-)persistent BBH sources. But till e < 0.7 from within the Local Group.

[DECIGO (and other future Decihertz detectors)] $10^{-1} - 10^{0}$ Hz

Low-eccentricity (e < 0.1), (near-)persistent BBH sources.
High-eccentricity (e > 0.5), highly variable BBH sources transitioning to LIGO band in days - years (the missing links!)

LIGO (and it successors) $10^{1} - 10^{3}$ Hz

Near-circular, transient BBH sources (mergers).
Rarely, eccentric (e>0.1) BBH mergers.
Spin-orbit misalignment including anti-alignment (approx. 50%).

These are **natural outcomes** of dynamics inside stellar clusters. No model adjustments, fine-tunings, or exotic physics required!