Icy moons in the Solar System

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Course overview

1. Motivation - why do we study icy moons.
   History of exploration - telescope observations, spacecraft missions.
   Surface characteristics - composition, age, and morphology.

2. Interior structure - layered models: from gravity, shape, composition.
   Hydrosphere structure - H_2O phase diagram, presence of oceans.
   Preferred models for selected satellites.

3. Thermal evolution - heat sources, heat transfer.
   Dynamics of the different planetary layers.
   Melting/crystallization, anti-freezers.
   Implications for the long-term stability of subsurface oceans.

4. Selected applications.
   Overview of future missions.

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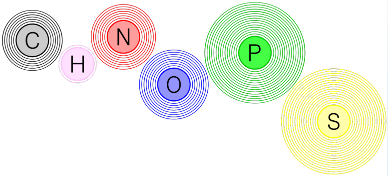
Habitability requirements

1. material: C, H, N, O, P, S (~98% of bio molecules on Earth)
2. solvent to speed up reactions - liquid water, ...?
3. energy source to sustain metabolism
4. stable environment



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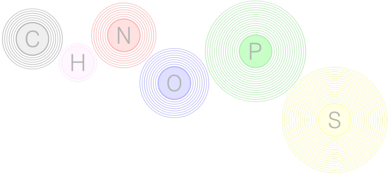



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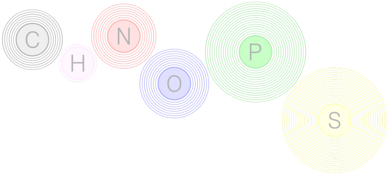
→ goal of heat transfer modeling is to characterize the thermal conditions within the moon on long time scales





Thermal evolution of icy moons

(*Matson* +, 2009)

- moons' thermal budget:

Q^+ heat from internal sources - their relative importance depends on:

- time when they occur
- how long they continue to supply heat

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- release of gravitational energy due to **differentiation**
- despinning

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Q^- total heat lost through the moon's surface

- on long time scales: $Q^+ < Q^- \rightarrow$ moons are cooling
- thermal evolution modeling - characterisation of thermal conditions within the moons' interior

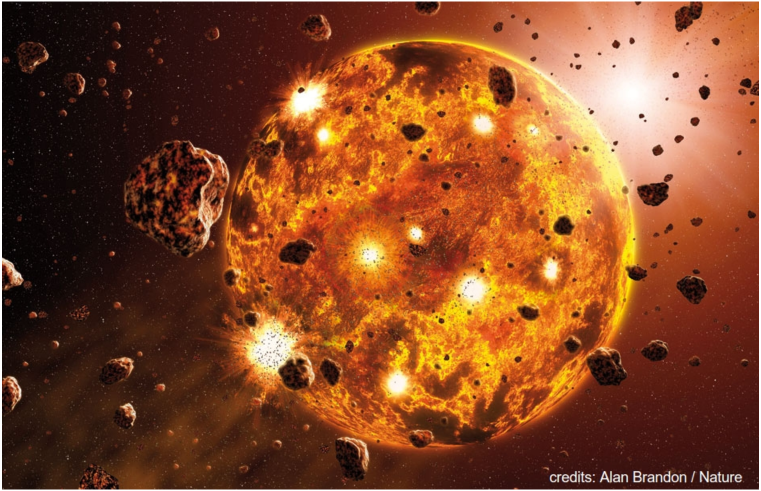
Accretion and differentiation heating

(*Matson* +, 2009)

- mass falling within the gravitational field

$\rightarrow$ potential energy changes to kinetic energy

$\rightarrow$ heat is dissipated



Accretion heating

(*Matson* +, 2009)

- incoming material impacts satellite's surface
- heating occurs during satellite formation
- temperature profile after accretion (*Squyres* +, 1988):

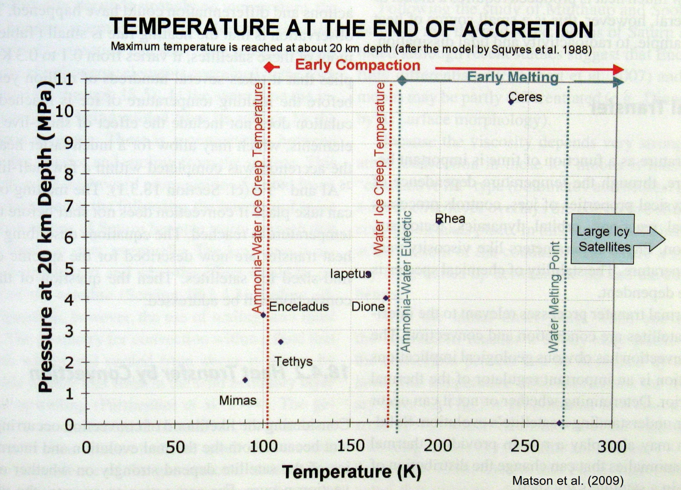
$$T(r) = \frac{h}{c_p}\left(\frac{4\pi}{3}\rho G r^2 + \frac{\langle\nu\rangle^2}{2}\right) + T_i$$

- $h \in \langle 0, 1\rangle$..fraction of mechanical energy turned into heat
- c_p..specific heat
- ρ..ice-rock mixture density
- G..universal gravitational constant
- r..instantaneous radius
- $\langle\nu\rangle$..mean encounter velocity of planetesimal with growing satellite
- T_i temperature of the planetesimals

Accretion heating

(*Matson* +, 2009)

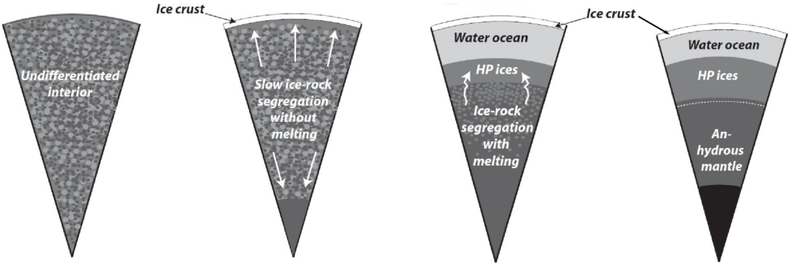
- for T_i~75 K
- small–midsized satellites: ΔT~20–100 K
- large satellites: ΔT high enough for melting to occur



Differentiation heating

(*Matson* +, 2009)

- shrinking and differentiation of a satellite releases gravitational energy in form of heat
  - heat produced ~ difference in gravitational potential of mass distribution before and after
  - small source compared to radioactivity and tides



Radiogenic heating

(*Matson* +, 2009; *Robuchon* +, 2010)

- radionuclides incorporated during accretion, amount $\propto$ rock content

$$H_r = \rho x_s \sum_{i=1}^{n} C_0 H_{0,i} \exp(-\lambda_i t)$$

- x_s..silicates mass fraction
- C_0..initial concentration of radiogenic elements
- $H_{0,i}$..initial power produced by radiogenic decay per unit mass
- λ_i..decay constant, $t_{1/2} = \frac{\ln 2}{\lambda}$..half-life, t..time since formation

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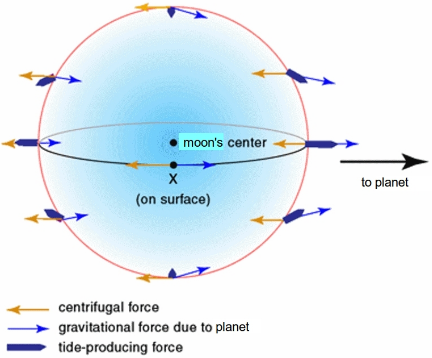
	element	$t_{1/2}$ [Myr]	H_0 [W kg^{-1} of elements]	C_0 [ppb]
long-lived radioactive isotopes (LLRI)	^{238}U	4468	94.65×10^{-6}	26.2
	^{235}U	703.81	568.7×10^{-6}	8.2
	^{232}Th	14030	26.38×10^{-6}	53.8
	^{40}K	1277	29.17×10^{-6}	1104
short-lived radioactive isotopes (SLRI)	^{26}Al	0.716	0.341	600
	^{60}Fe	1.5	0.071	200
	^{53}Mg	3.7	0.027	25.7

Tidal force

($Sotin$ +, 2009)

- gravitational force $\vec{F}_g$ from planet different at each point of the moon
- centrifugal force $\vec{F}_c$ constant
- tidal force: $\vec{F}_t = \vec{F}_g - \vec{F}_c$



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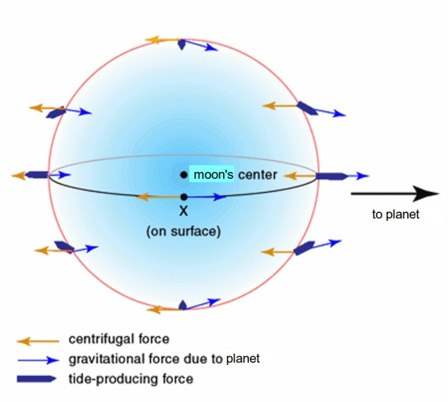

- circular orbit ($e = 0$):
  - every point in moon remains fixed wrt parent planet

→ static tides → relaxed long time ago into 3-axial hydrostatic shape

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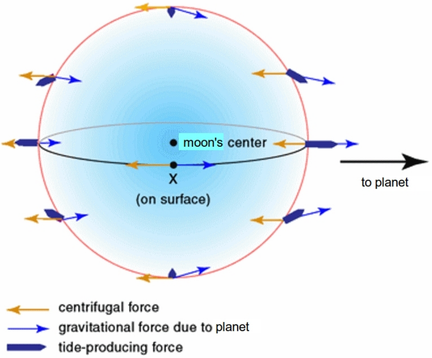
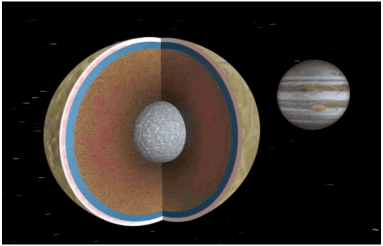


- circular orbit ($e = 0$):
  - every point in moon remains fixed wrt parent planet
  - → static tides → relaxed long time ago into 3-axial hydrostatic shape
- ellipsoidal orbit ($e > 0$): most satellites
  - → orbital tides

Diurnal (orbital) tides

(*Sotin* +, 2009)

- $e > 0 \rightarrow$ fixed points in the moon move relative to parent planet:



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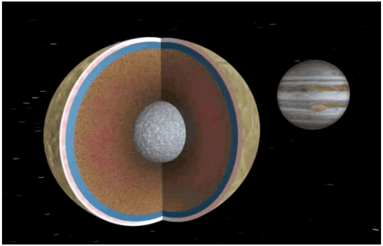
(i) distance between moon and planet changes with time

$\rightarrow$ change in amplitude of planet's gravitational force on the moon

$\rightarrow$ **radial tide** (tidal bulge larger at pericenter than at apocenter)



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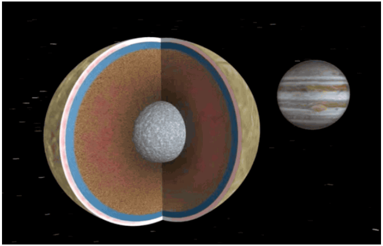
(ii) satellite at pericenter/apocenter orbits slightly faster/slower

$\rightarrow$ tidal force pattern rocks back and forth (east-west)

$\rightarrow$ **librational tide**

($e = 0$, $\varepsilon \neq 0$: rocking back and forth in north-south direction)

- smaller displacement (by a factor of $1/e$) than static tide



Tidal potential

(*Sotin* +, 2009)

$$V_T(\vec{r}) = \frac{GM^*}{R^*} \sum_{j=2}^{\infty} \left(\frac{r}{R^*} \right)^j P_j(\cos \gamma)$$

- M^* mass of parent planet, R^* distance from parent planet
- P_j Legendre polynomial
- $\frac{r}{R^*} \ll 1 \rightarrow$ only $j = 2$ term is kept

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  - γ angle between point on moon's surface: $\vec{r} = (r, \theta, \phi)$ and parent planet: $\vec{R}^* = (R^*, \theta', \phi')$ (varies in time)
- $\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi')$

Tidal potential

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$$V_T(\vec{r}) \sim \frac{GM^*}{R^*}\left(\frac{r}{R^*}\right)^2 P_2(\cos\gamma) \sim \frac{3GM^*R^2}{2a^3}\left(\frac{r}{R}\right)^2 [T_0 + T_1]$$

- M^* mass of parent planet, R^* distance from parent planet
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- γ angle between point on moon's surface: $\vec{r} = (r, \theta, \phi)$ and parent planet: $\vec{R}^* = (R^*, \theta', \phi')$ (varies in time)
- $\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi')$
- keeping only terms to first order in e:

$$T_0 = \frac{1}{6}(1 - 3\cos^2\theta) + \frac{1}{2}\sin^2\theta\cos(2\phi)$$

$$T_1 = \frac{e}{2}\left[\left((1 - 3\cos^2\theta) + 3\sin^2\theta\cos(2\phi)\right)\cos(nt) + 4\sin^2\theta\sin(2\phi)\sin(nt)\right]$$

a..semimajor axis, n..mean motion, R..moon radius, t..time wrt pericenter

- T_0: static tidal potential (time independent)
- T_1: diurnal tidal potential (smaller than T_0 by e)
- $\sim$ $\cos(nt)$: radial tide, $\sin(nt)$: librational tide

Tidal heating

(*Sotin* +, 2009)

- periodic forcing:
  - different materials in the moons' interior deform differently
  - elastic + anelastic deformation → time delay of response wrt forcing
  - $\sigma = \sigma_0 \exp[i\omega t] \rightarrow \varepsilon = \varepsilon_0 \exp[i(\omega t - \delta)]$ (δ..phase lag)

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- peak energy reached during one cycle: $E = \sigma_0 \varepsilon_0 / 2$
- Q quality factor (specific dissipation function): $Q = \frac{1}{\tan \delta}$
- average heating rate due to eccentricity tides:

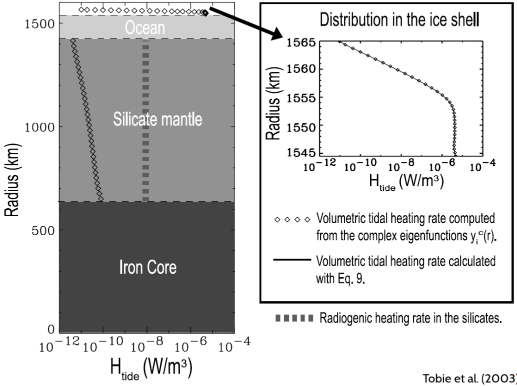
$$\frac{dE}{dt} = \frac{21}{2} \frac{k_2}{Q} \frac{(nR)^5}{G} e^2$$

  - orbital characteristics: n mean motion, e eccentricity
  - interior structure char.: k_2 tidal Love number, Q quality factor

Distribution of heating

(*Tobie*, 2003; *Tobie* +, 2003; *Sotin* +, 2009)

- Europa:
- tidal heating in ice shell: $H_t^i \sim 10^{-6}$ W m^{-3}
- radiogenic heating in silicates: $H_r^s \sim 10^{-8}$ W m$^{-3} \lesssim 100 \times H_t^i$
- tidal heating in silicates negligible wrt to H_r^s



Tobie et al. (2003)

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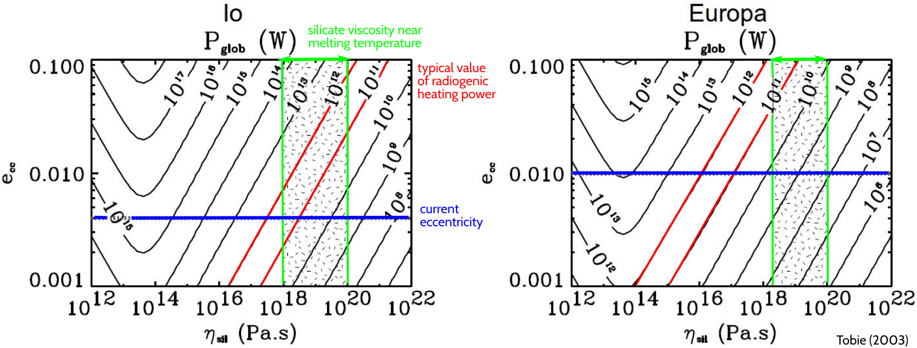
- Io: tidal heating in silicates similar to radiogenic heating



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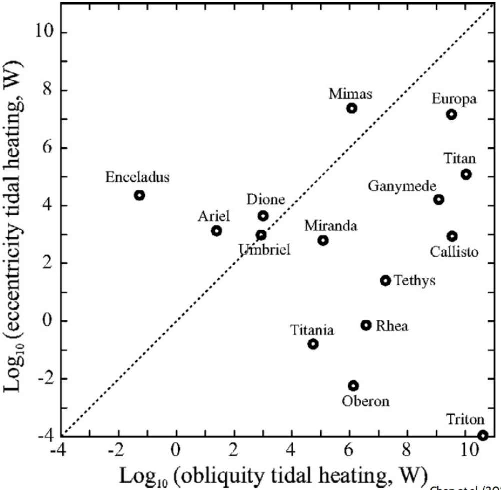
→ $P_t^i \sim 10^{12}$ W

- $H_r^s \sim 10^{-8}$ W m^{-3}

→ $P_r^s \sim 10^{11}$ W

- $P_e^o \sim 1.5 \times 10^7$ W
- $P_o^o \sim 3.1 \times 10^9$ W



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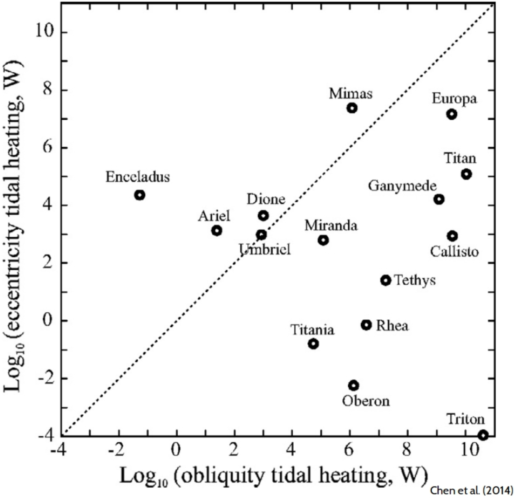
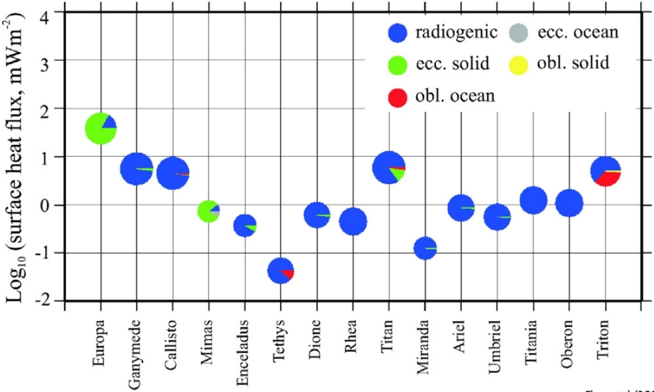
→ tidal dissipation in ocean likely not important wrt radiogenic & solid-body tidal heating



Heat sources - importance

(*Chen* +, 2014)

- relative contributions from different heating sources
- most satellites:
- ocean tidal heating likely not important wrt radiogenic / solid tides
- Triton: ocean tidal heating may play a role in its thermal budget



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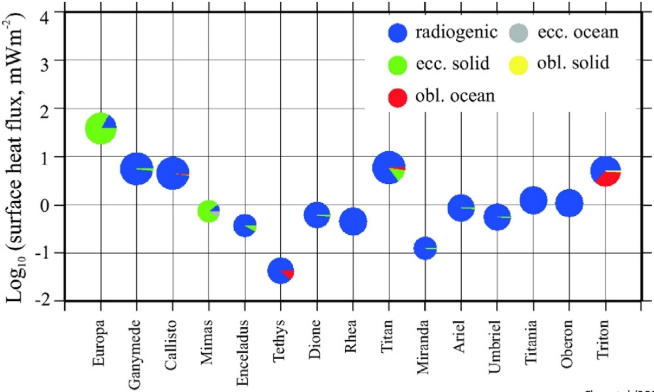
- Enceladus: strong tidal heating in the porous core (*Choblet* +, 2017)



Chen et al. (2014)

Heat sources - importance

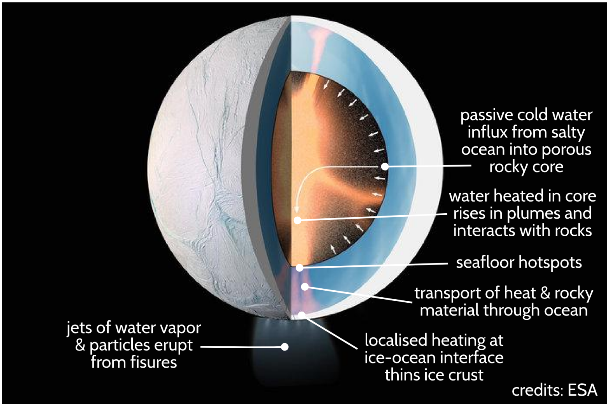
nature astronomy

LETTERS

https://doi.org/10.1038/s41550-017-0289-8

Powering prolonged hydrothermal activity inside Enceladus

Gaël Choblet[1*], Gabriel Tobie[1], Christophe Sotin[2], Marie Běhounková[3], Ondřej Čadek[3], Frank Postberg[4,5] and Ondřej Souček[6]



Effect on orbital evolution

(*Sohl* +, 1995; *Hussmann & Spohn*, 2004)

- decrease in eccentricity due to dissipation:

$$\frac{de}{dt}\frac{e}{1-e^2} = \frac{-a}{GM^* M}\frac{dE}{dt}$$

→ dissipation would circularize the moons' orbits

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→ dissipation would circularize the moons' orbits

- forced eccentricity due to resonances maintained over long periods:
- Laplace 1:2:4 resonance between Io, Europa, Ganymede
- Enceladus-Dione 2:1 resonance

Effect on orbital evolution

(*Sohl* +, 1995; *Hussmann & Spohn*, 2004)

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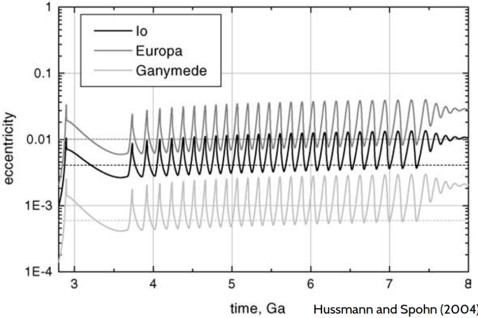

Hussmann and Spohn (2004)

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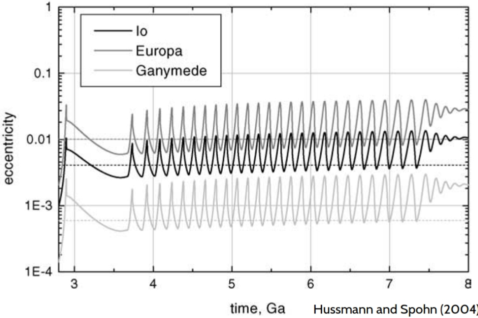


Hussmann and Spohn (2004)

- Titan: large eccentricity, no resonance → tidal dissipation not likely

Amount of internal heat for different bodies

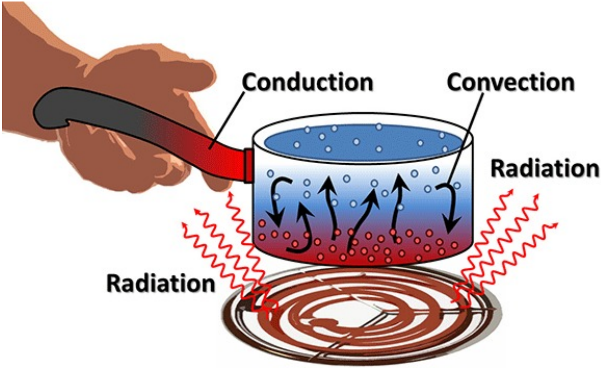
    

6371 km	1822 km	2575 km	1561 km	252.3 km
5.97 10^{24} kg	0.0894 10^{24} kg	0.1345 10^{24} kg	0.048 10^{24} kg	0.000108 10^{24} kg
5525 kg/m^3	3528 kg/m^3	1881 kg/m^3	2970 kg/m^3	1608 kg/m^3
2/3 Silicates and 1/3 iron	Silicates	Ice and silicates	Ice and silicates	Ice and silicates
42 TW **82 mW/m^2**	**108 TW** **2589 mW/m^2**	**750 GW** **9 mW/m^2**	**> 1 TW** **33 mW/m^2**	**20 GW** **25 mW/m^2**

Radioactive power is proportional to the mass
Other internal heat sources include tidal dissipation, cooling, and latent heat

Heat transfer mechanisms

- **conduction**: heat transfer via direct molecular collision
- $P = A\,k\nabla T$ (P..power, A..area, k..thermal conductivity, T..temp.)



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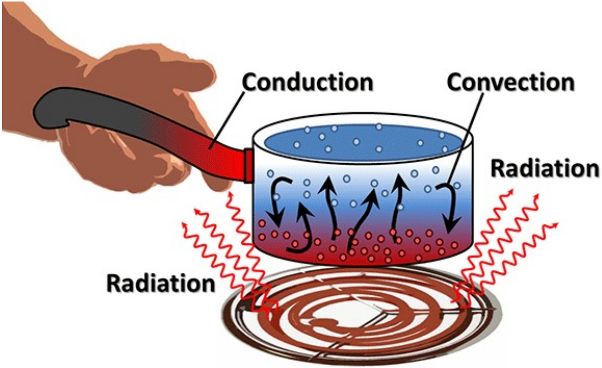

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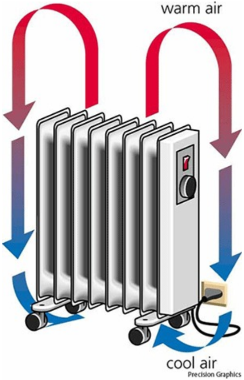
  - warm air expands & rises → cooler air sinks & becomes heated

→ convection current



Heat transfer mechanisms

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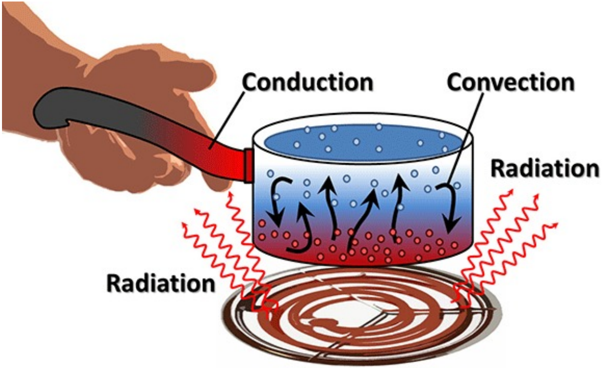
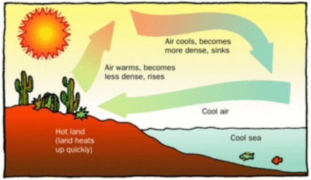
- **convection**: heat transfer due to molecular expansion
  - warm air expands & rises $\rightarrow$ cooler air sinks & becomes heated

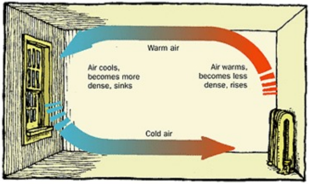
$\rightarrow$ convection current
- **radiation**: heat transfer due to emission of elmg. waves
  - Stefan-Boltzmann law: $P = A\varepsilon\sigma T^4$ (ε..emissivity, σ..S-B const.)



Thermal convection - examples

- air currents





Thermal convection - examples

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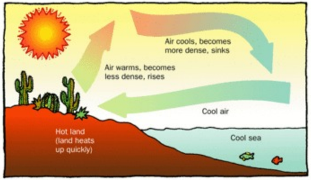
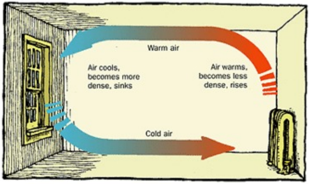
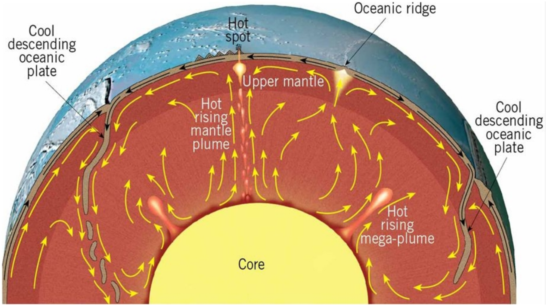




- flow in Earth's mantle → plate tectonics



Equation of motion

$$\rho \frac{d\boldsymbol{v}}{dt} = \nabla \cdot \tau + \boldsymbol{f}$$

- ρ..density, $\boldsymbol{v}$..velocity
- $\boldsymbol{\tau}$..Cauchy stress tensor
- $\boldsymbol{f}$..volume force

Equation of motion

$$\rho \frac{d\boldsymbol{v}}{dt} = \nabla \cdot \tau + \boldsymbol{f}$$

$$\rho \frac{d\boldsymbol{v}}{dt} = -\nabla p + \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f}$$

- ρ..density, $\boldsymbol{v}$..velocity
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- time derivative in a moving continuum: $\frac{d\boldsymbol{v}}{dt} = \frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}$

Navier-Stokes equations

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$$\boxed{\rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = -\nabla p + \nabla \cdot \left[\eta (\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^{\mathrm{T}}) \right] + \rho \boldsymbol{g}, \quad \nabla \cdot \boldsymbol{v} = 0}$$

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- time derivative in a moving continuum: $\frac{d\boldsymbol{v}}{dt} = \frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}$
- incompressible viscous fluid: $\nabla \cdot \boldsymbol{v} = 0$, $\boldsymbol{\sigma} = \eta(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^{\mathrm{T}})$: η..viscosity

Heat transfer equation

$$\rho c_{\mathrm{p}}\left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T\right) = -\nabla \cdot \boldsymbol{q} + H$$

- c_{p}..heat capacity at constant pressure
- T..temperature
- $\boldsymbol{q}$..heat flux
- H volumetric heat sources

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- $\boldsymbol{q}$..heat flux: Fourier law: $\boldsymbol{q} = -k\nabla T$
- H volumetric heat sources

Thermal convection

$$
\begin{aligned}
\nabla \cdot \boldsymbol{v} &= 0 \\
\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) &= -\nabla p + \nabla \cdot \left[\eta\left(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^{\mathrm{T}}\right)\right] + \rho \boldsymbol{g} \\
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Thermal convection

- Equation of State (EoS):
  $\rho = \rho(P, T) = \rho_0(P)(1-\alpha(T-T_0)) \sim \rho_0(1-\alpha(T-T_0))$

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$$\rho c_{\mathrm{p}}\left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T\right) = \nabla \cdot (k \nabla T) + H$$

Dimensionless formulation

- scales:

$$\boldsymbol{r} = D\boldsymbol{r}', \quad t = \frac{D^2}{\kappa}t', \quad \boldsymbol{v} = \frac{\kappa}{D}\boldsymbol{v}', \quad \pi = \frac{\eta_0\kappa}{D^2}\pi', \quad \eta = \eta_0\eta',$$
$$T = T_\mathrm{s} + \Delta T T', \quad k = k_0 k', \quad H = \frac{k\Delta T}{D^2}H'$$

- D..characteristic domain dimension
- $\kappa = \frac{k_0}{\rho_0 c_\mathrm{p}}$..thermal diffusivity
- η_0..reference viscosity
- k_0..reference thermal conductivity
- $\Delta T = T_\mathrm{b} - T_\mathrm{s}$..temperature contrast across domain (**b**ottom, **s**urface)

Dimensionless formulation

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\nabla \cdot \boldsymbol{v} &= 0 \\
Pr^{-1}\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) &= -\nabla \pi + \nabla \cdot \left[\eta\left(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^T\right)\right] + Ra\left(T - T_0\right)\boldsymbol{e}_\mathrm{r} \\
\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T &= \nabla \cdot \left(k \nabla T\right) + H
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Dimensionless formulation

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$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \nabla \cdot (k \nabla T) + H$$

- Prandtl number: momentum vs. thermal diffusivity

$$Pr = \frac{\eta_0}{\rho_0 \kappa}$$

Dimensionless formulation

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$$Pr = \frac{\eta_0}{\rho_0 \kappa}$$

material	η_0 [Pa s]	ρ_0 [kg m^{-3}]	κ m^2 s^{-1}	Pr	Pr^{-1}
silicates	10^{18}–10^{20}	3500	10^{-6}	10^{20}–10^{22}	×

Dimensionless formulation

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water	10^{-3}	1000	10^{-7}	10	✓

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water	10^{-3}	1000	10^{-7}	10	✓
ice	10^{14}–10^{16}	1000	10^{-6}	10^{17}–10^{19}	×

Dimensionless formulation

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- Rayleigh number: time scale for diffusive thermal transport vs. time scale for convective thermal transport

$$Ra = \frac{D^2/\kappa}{\eta_0/\Delta\rho D g} = \frac{\rho_0 \alpha g \Delta T D^3}{\eta_0 \kappa}$$

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- critical value: $Ra_c \sim 10^3$

Ra	$\lesssim 10^3$	$\lesssim 10^4$	$\lesssim 10^5$	$> 10^5$
regime	conduction	steady-state convection	periodic convection	chaotic convection

Dimensionless formulation

$$\nabla \cdot \boldsymbol{v} = 0$$

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material	α [1/K]	g [m/s^2]	D [m]	ΔT [K]	Ra	convection?
silicates	3×10^{-5}	1	10^5–10^6	10^3	10^3–10^8	×/✓

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silicates	3×10^{-5}	1	10^5–10^6	10^3	10^3–10^8	×/✓
water	10^{-4}	1	10^4–10^5	10^{-3}–10^{-2}	10^{18}–10^{22}	✓

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$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \nabla \cdot (k \nabla T) + H$$

- Rayleigh number: time scale for diffusive thermal transport vs. time scale for convective thermal transport

$$Ra = \frac{D^2/\kappa}{\eta_0/\Delta\rho D g} = \frac{\rho_0 \alpha g \Delta T D^3}{\eta_0 \kappa}$$

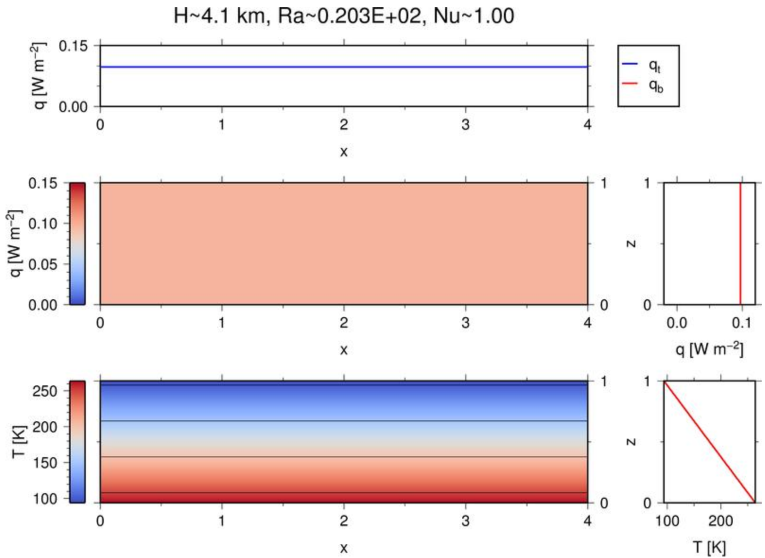
material	α [1/K]	g [m/s^2]	D [m]	ΔT [K]	Ra	convection?
silicates	3×10^{-5}	1	10^5–10^6	10^3	10^3–10^8	×/✓
water	10^{-4}	1	10^4–10^5	10^{-3}–10^{-2}	10^{18}–10^{22}	✓
ice	10^{-4}	1	10^3–10^5	100	1–10^8	×/✓

Conduction vs convection

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \nabla \cdot (k \nabla T) + H$$

- $\boldsymbol{v} \to 0$: heat transferred only by conduction (heat diffusion)
- $\boldsymbol{v} \neq 0$: heat transferred by conduction & advection $\to$ convection
- Nusselt number = convective vs conductive heat transfer
- measure of heat transfer efficiency
- $Nu = 1$: conduction
- $Nu > 1$: convection

Conduction vs convection



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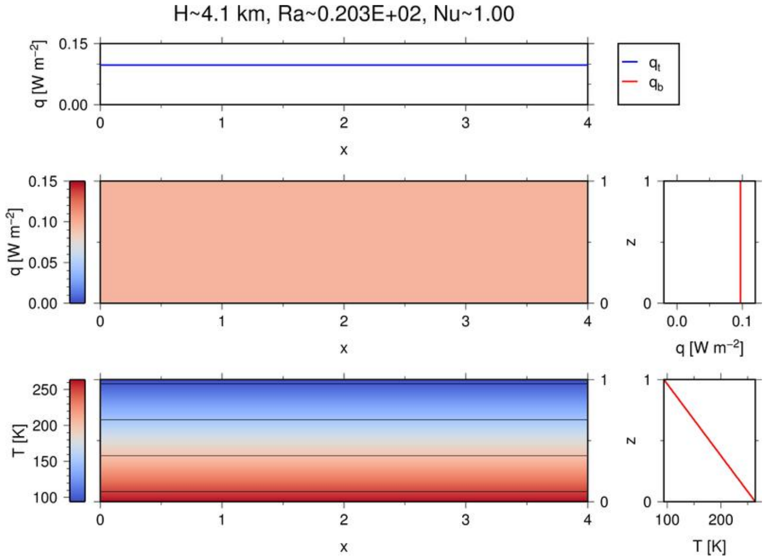

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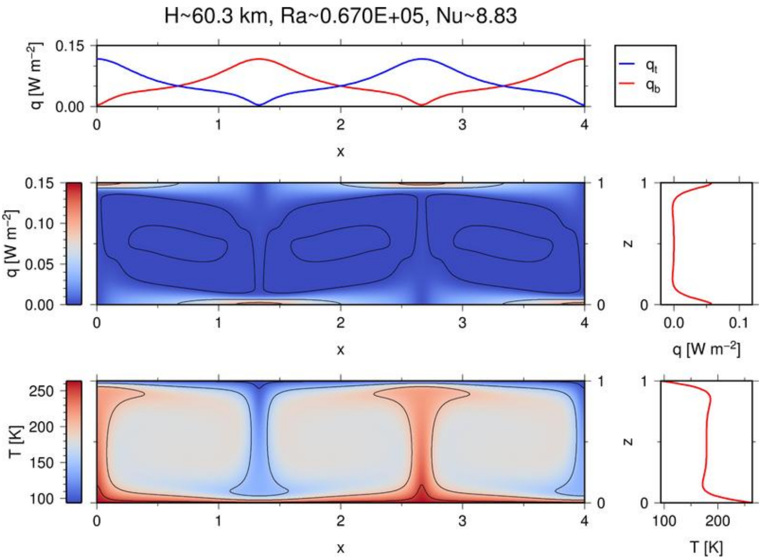

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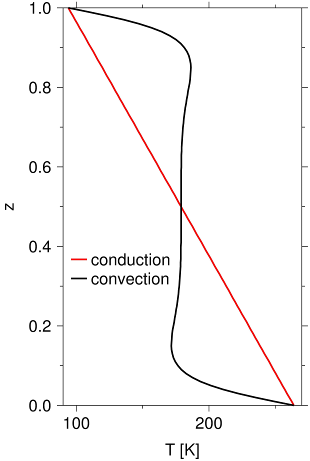
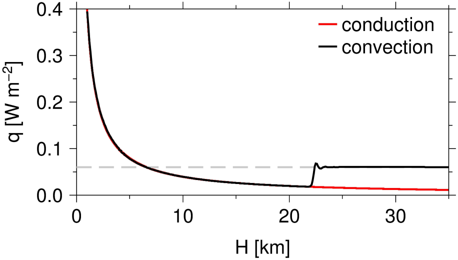


- heat flux of ~0.06 W m^{-2} obtained for:
- conductive ice layer with H~6.5 km
- convective ice layer with $H \gtrsim 22$ km



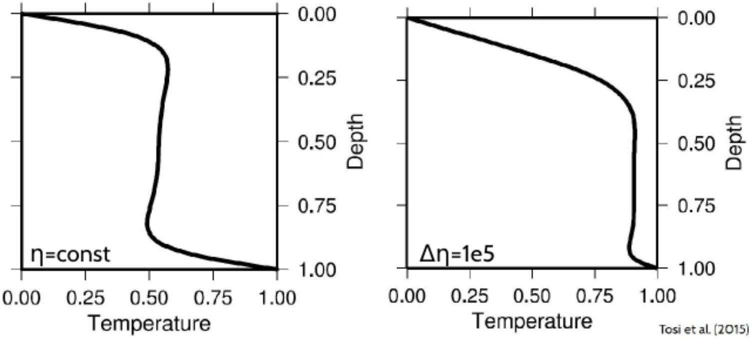
Material parameters

- viscosity - strongly depends on temperature as

$$\eta \propto \exp\left(\frac{A}{RT}\right)$$

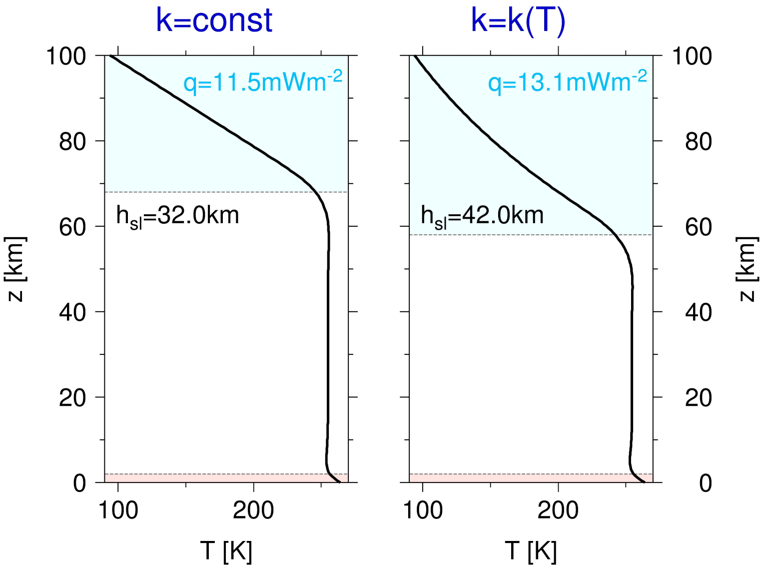
- cold temperatures @ surface

→ high viscosity → no flow → 'stagnant lid'



Material parameters

- thermal conductivity - function of temperature: $k \propto \frac{1}{T}$



Thermal evolution models

- numerical models - results strongly depend on model parameters:
- material (viscosity, conductivity, ...)
- internal structure (thickness of different layers)

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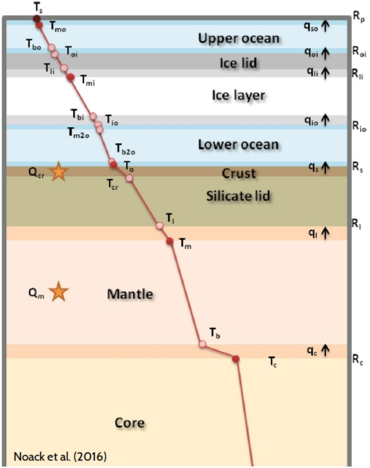

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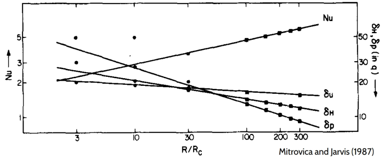

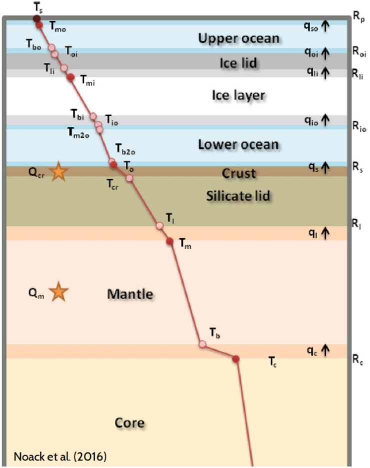

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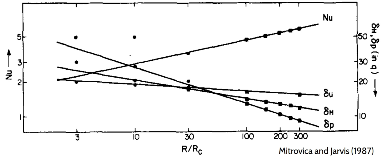

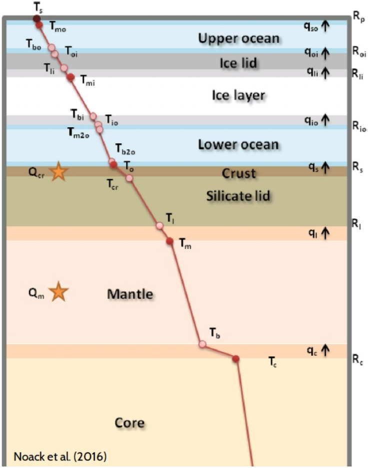

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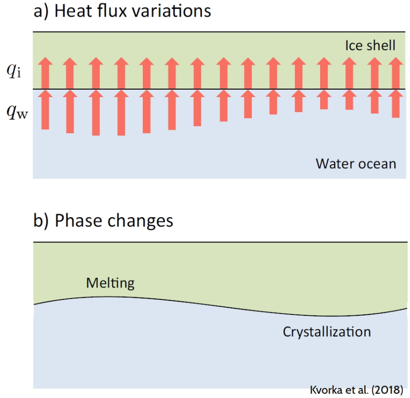
- jumps in q at phase change boundaries:

Stefan problem (melting/crystallization)

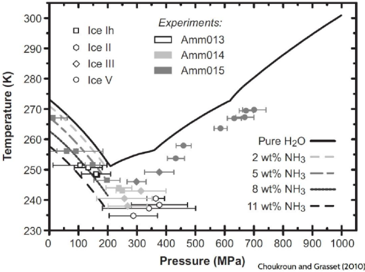
$$\boldsymbol{n} \cdot (\boldsymbol{q}_i - \boldsymbol{q}_w) = \Delta q_n = L \rho_i \frac{d\xi}{dt}$$

L..latent heat, $\xi(t)$..phase interface

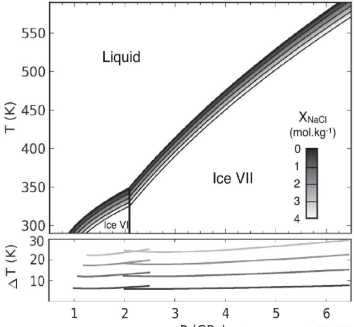


Effect of antifreezers

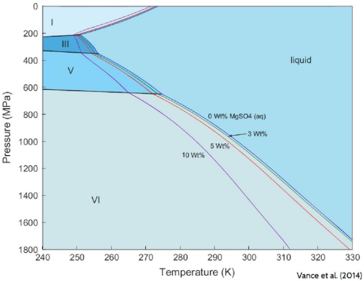
- NaCl, $MgSO_4$, NH_3 → significant reduction of melting temperature



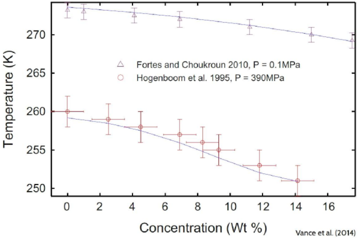
Choukroun and Grasset (2010)



Journaux et al. (2013)



Vance et al. (2014)



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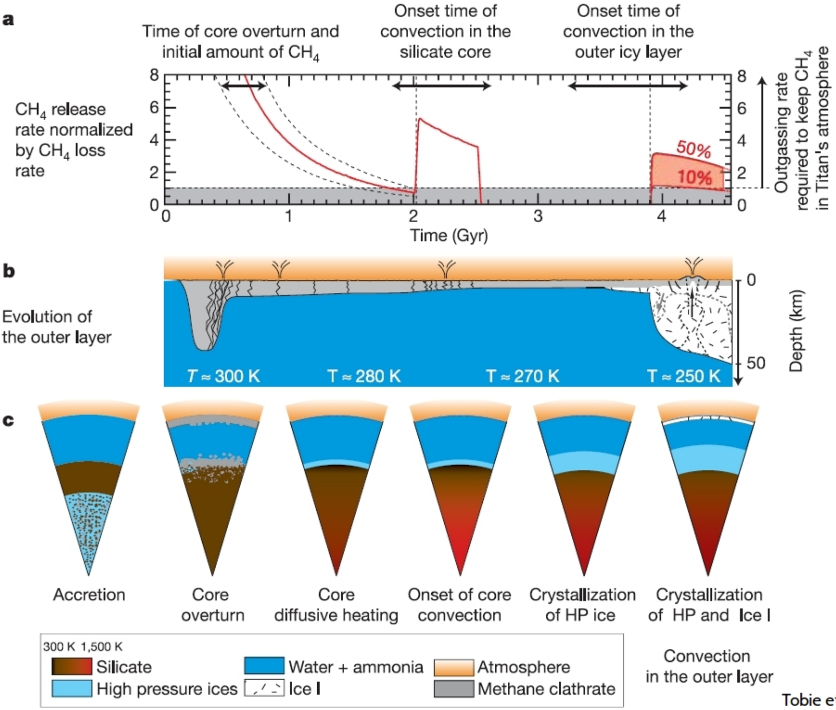

Tobie et al. (2006)

Implications for the long-term stability of subsurface oceans

(*Nimmo & Pappalardo*, 2016)

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e.g. $\Delta q \sim 1$ mW m^{-2} $\rightarrow \Gamma \sim 100$ km Gyr^{-1}

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- accretion heat: modest even for largest moons
- radioactive decay (mainly K, U and Th)
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heat transfer:

- convection (thick, mobile crusts) vs conduction (thin, rigid crusts)

→ more information/data on the interior structure are needed

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