Selected chapters on astrophysics

Icy moons in the Solar System

homework: reconstruction of interior structure of synthetic moons

contact: klara.kalousova@mff.cuni.cz

Assumptions:

- moons consist of j_{max} layers of different materials
- each layer has a constant density ρ_j and radius R_j (note that $R_{j_{max}}$ is the total radius R and $R_0 = 0$ is the center of the moon)
- total mass:

$$M = \int dm = \int \rho(\vec{r}) dV = \int_0^{2\pi} \int_0^{\pi} \int_0^{R} \rho(r) r^2 \sin\theta dr d\theta d\phi = 4\pi \int_0^R \rho(r) r^2 dr$$
$$= 4\pi \sum_{j=1}^{j_{max}} \int_{R_{j-1}}^{R_j} \rho_j r^2 dr = 4\pi \sum_{j=1}^{j_{max}} \rho_j \int_{R_{j-1}}^{R_j} r^2 dr = \frac{4}{3}\pi \sum_{j=1}^{j_{max}} \rho_j (R_j^3 - R_{j-1}^3)$$

- moment of inertia with respect to rotational axis:

$$C = \int (x^2 + y^2) dm = \int r^2 \sin^2\theta \rho(\vec{r}) dV = \int_0^{2\pi} \int_0^{\pi} \int_0^{R} \rho(r) r^4 \sin^3\theta dr d\theta d\phi = \frac{8}{3}\pi \int_0^R \rho(r) r^4 dr$$
$$= \frac{8}{3}\pi \sum_{j=1}^{j_{max}} \int_{R_{j-1}}^{R_j} \rho_j r^4 dr = \frac{8}{3}\pi \sum_{j=1}^{j_{max}} \rho_j \int_{R_{j-1}}^{R_j} r^4 dr = \frac{8}{15}\pi \sum_{j=1}^{j_{max}} \rho_j (R_j^5 - R_{j-1}^5)$$

- reduced moment of inertia:

$$\text{MoI} = \frac{C}{MR^2}$$

Task:

- using the provided data (cf. next 2 pages), find the missing information on interior structure (densities and/or radii)

Method (suggestion) - systematic search of parameters space:

- for each moon, use two nested cycles for a reasonable range of the two unknown quantities
- reasonable range for ρ_r is between 2000 and 4000 kg/m^3
- compute M' and MoI$'$ for all combinations of the two unknowns
- compare the computed M and MoI with the provided values
- solution: values of unknowns that lead to the smallest difference between the provided (M, MoI) and computed (M', MoI$'$) values

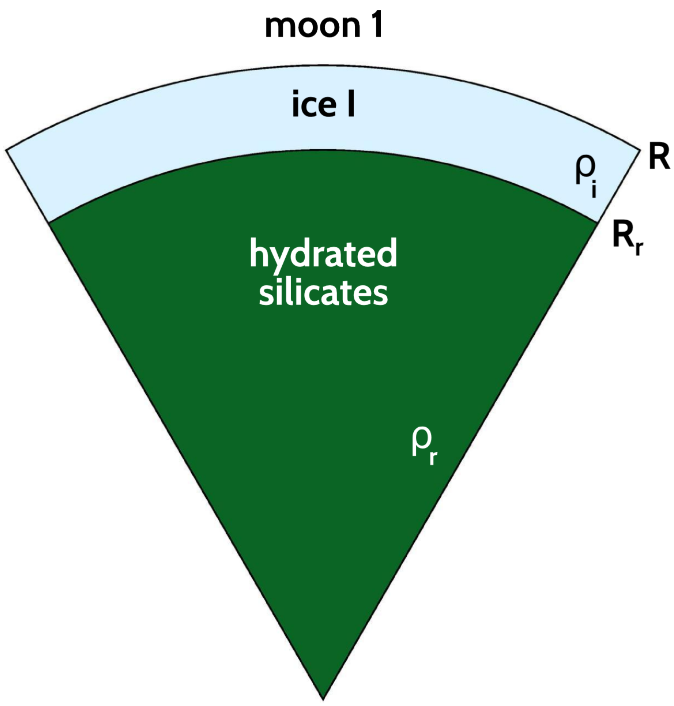
moon 1: 2-layered

- hydrated rock (silicates) mantle: R_r, ρ_r
- ice I crust: $R_i = R$, ρ_i

Inversion data

- $R = 1500$ km
- $M = 2.80{\times}10^{22}$ kg
- MoI $= 0.3464$
- $\rho_i = 920$ kg/m^3

Find the rock mantle radius R_r and its density ρ_r.



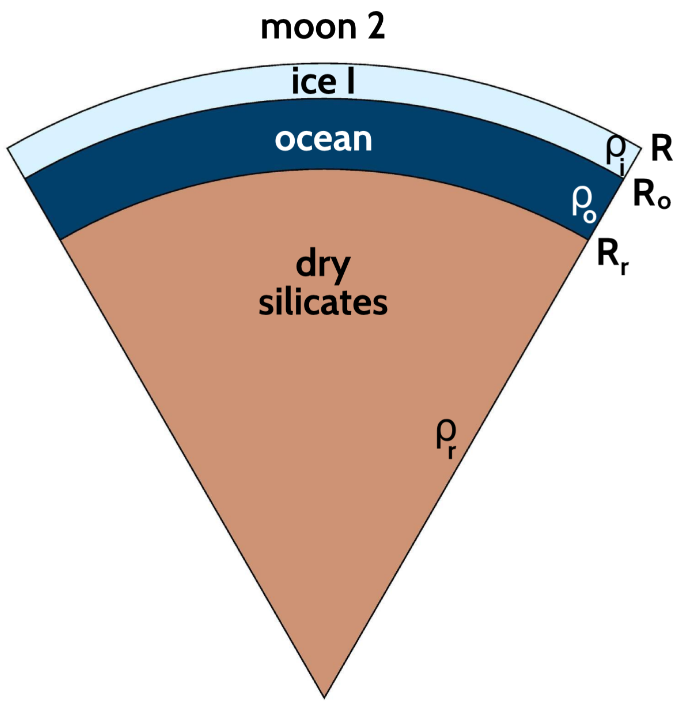
moon 2: 3-layered

- dry rock (silicates) mantle: R_r, ρ_r
- ocean: R_o, ρ_o
- ice I crust: $R_i = R$, ρ_i

Inversion data

- $R = 1800$ km
- $M = 5.90{\times}10^{22}$ kg
- MoI $= 0.3273$
- $\rho_i = 920$ kg/m^3
- $\rho_o = 1100$ kg/m^3
- $R_o = 1700$ km

Find the rock mantle radius R_r and its density ρ_r.



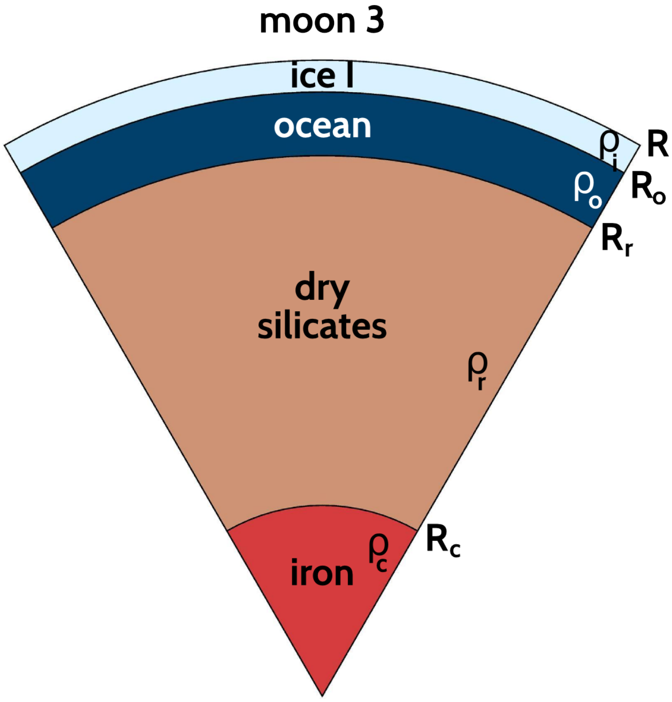
moon 3: 4-layered

- iron core: R_c, ρ_c
- dry rock (silicates) mantle: R_r, ρ_r
- ocean: R_o, ρ_o
- ice I crust: $R_i = R$, ρ_i

Inversion data

- $R = 2000$ km
- $M = 8.84{\times}10^{22}$ kg
- MoI $= 0.3182$
- $\rho_i = 920$ kg/m^3
- $\rho_o = 1100$ kg/m^3
- $\rho_r = 3500$ kg/m^3
- $\rho_c = 8000$ kg/m^3
- $R_o = 1900$ km

Find the core radius R_c and the rock mantle radius R_r.



moon 4: 4-layered

- hydrated rock (silicates) mantle: R_r, ρ_r
- high-pressure (HP) ice layer: R_{hp}, ρ_{hp}
- ocean: R_o, ρ_o
- ice I crust: $R_i = R$, ρ_i

Inversion data

- $R = 3000$ km
- $M = 1.68{\times}10^{23}$ kg
- MoI $= 0.3374$
- $\rho_i = 920$ kg/m^3
- $\rho_o = 1100$ kg/m^3
- $\rho_{hp} = 1350$ kg/m^3
- $\rho_r = 2600$ kg/m^3
- $R_o = 2850$ km

Find the rock mantle radius R_r and the high-pressure ice layer radius R_{hp}.