Planet migration in protoplanetary disks

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an astrophysical migrant crisis? Part I

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Selected Chapters on Astrophysics
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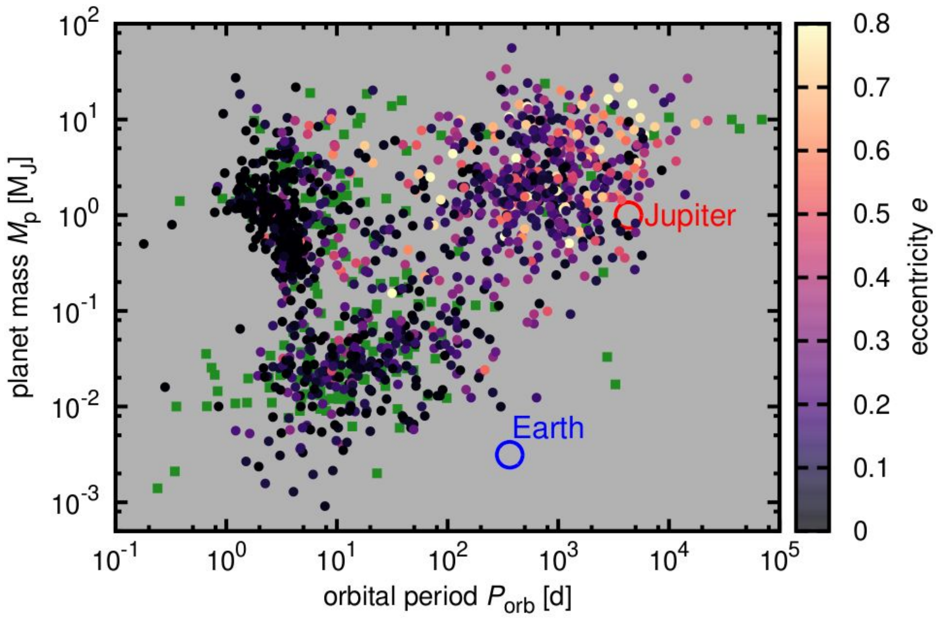
Outline

- Lecture 1
  - migration: what, when and why?
  - basic concepts of the angular momentum exchange (fluid as test particles)
- Lecture 2
  - Lindblad and corotation torques in gaseous disks; gap opening; Type I and Type II migration
  - introduction to the linear perturbation analysis of fluid equations
- Lecture 3
  - numerical methods (orbital advection; heating and cooling of disks)
  - latest breakthroughs in planet migration (~too many regimes of migration)
  - origin scenarios for exo- and solar-system planets

Motivation - exoplanets

- big planets have envelopes
- formation of super-Earths via solar-system scenarios is not easy
- close-in orbits and tightly packed systems are difficult to explain without some damping effects



Chrenko (2019); NASA Exoplanet Archive

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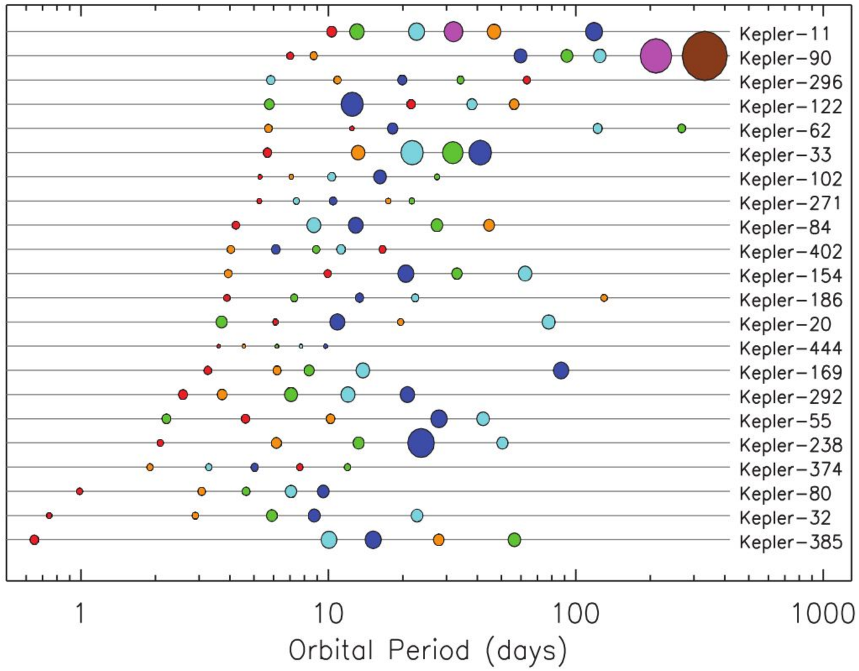

Borucki (2016)

Motivation - exoplanets

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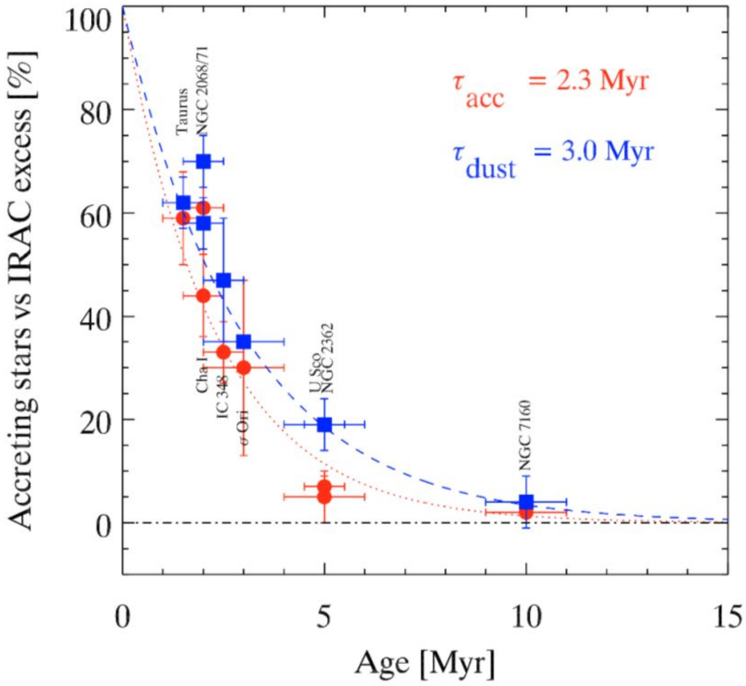
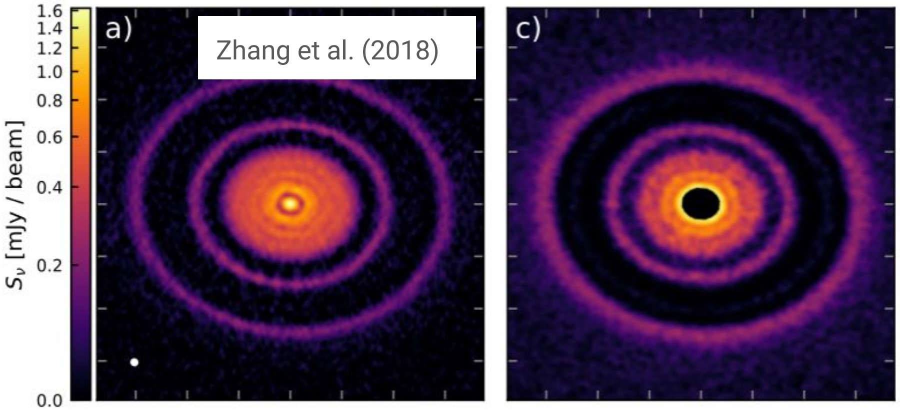
-> important evolutionary phase within protoplanetary disks (~Myr time scales)

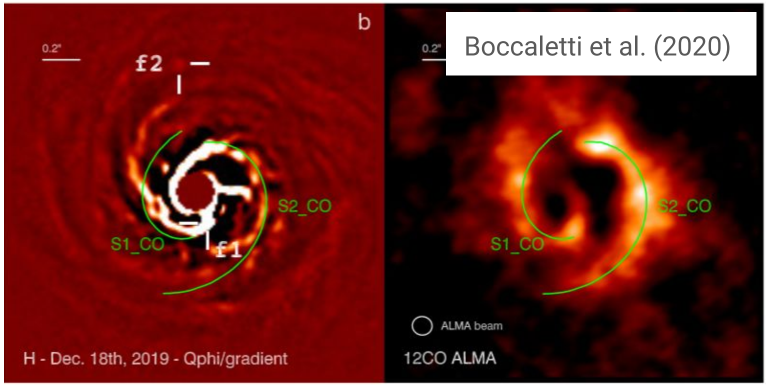


Fedele et al. (2010)

Motivation - disk imaging

- ubiquitous substructures = imprints of perturbations by unseen planets?
- rings = dust accumulation in pressure maxima?
- spiral arms = gravity-induced waves?





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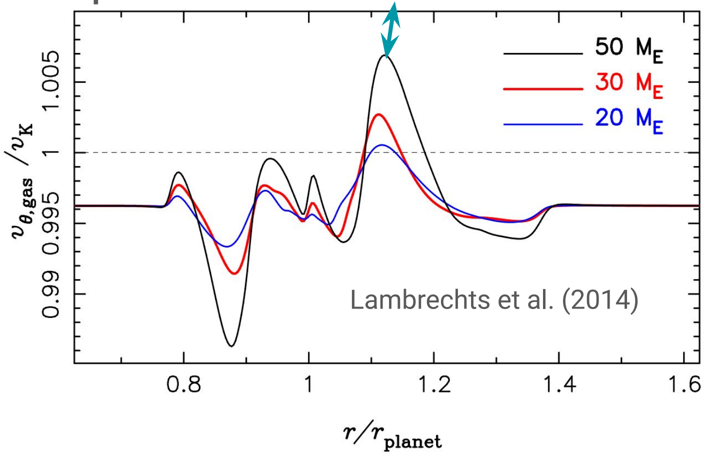
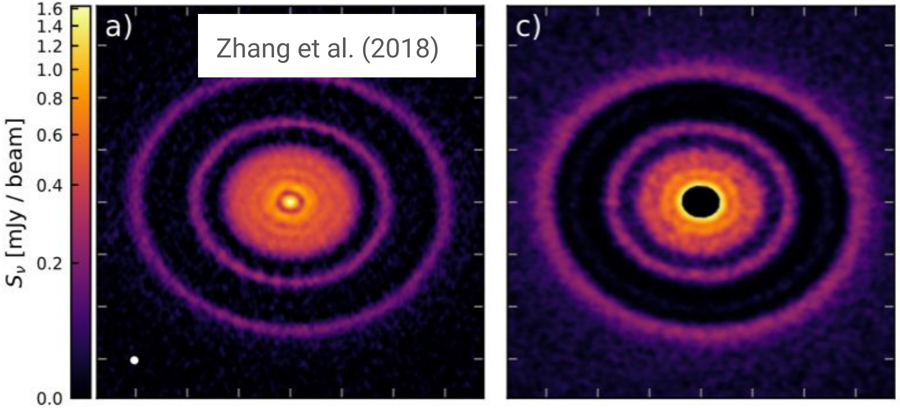
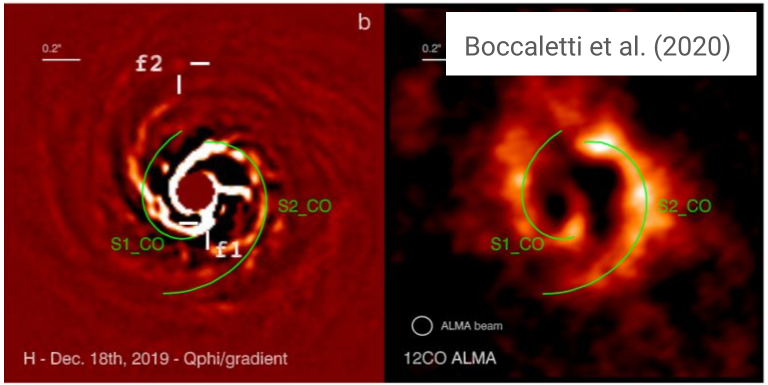
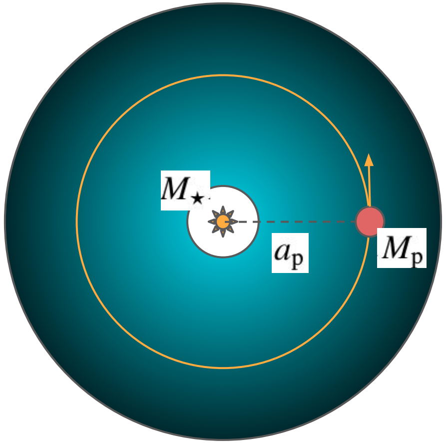






Why planets migrate



- for a planet on a circular orbit:

$$G\frac{M_\star M_p}{a_p^2} = \frac{M_p v_p^2}{a_p}, \qquad v_p = \sqrt{\frac{GM_\star}{a_p}} \equiv v_K, \qquad \Omega_p = \frac{v_p}{a_p} = \sqrt{\frac{GM_\star}{a_p^3}} \equiv \Omega_K,$$

- the orbital angular momentum:

$$\boldsymbol{L} = M_p \boldsymbol{r_p} \times \boldsymbol{v_p} \overset{\text{circ.}}{\Longrightarrow} |\boldsymbol{L}| = L = M_p a_p v_p = M_p a_p^2 \Omega_p = M_p \sqrt{GM_\star a_p},$$

- **angular momentum change -> migration**; it can only be related to a **non-zero torque**

$$\frac{dL}{dt} \Longrightarrow \frac{da_p}{dt},$$

$$\frac{dL}{dt} = \Gamma \Longrightarrow \Gamma = M_p \sqrt{GM_\star} \frac{1}{2\sqrt{a_p}} \frac{da_p}{dt} = \frac{M_p a_p \Omega_p}{2} \frac{da_p}{dt} \Longrightarrow \frac{da_p}{dt} = \frac{2\Gamma}{M_p a_p \Omega_p},$$

Why planets migrate

- alternatively, use the Gauss perturbation equation for the semi-major axis

$$\frac{\mathrm{d}a_\mathrm{p}}{\mathrm{d}t} \stackrel{\text{circ.}}{=} \frac{2\mathcal{T}}{\Omega_\mathrm{p}},$$ and compare to the previous expression $$\frac{\mathrm{d}a_\mathrm{p}}{\mathrm{d}t} = \frac{2\Gamma}{M_\mathrm{p} a_\mathrm{p} \Omega_\mathrm{p}},$$

outward migration (semi-major axis grows) when there is a
  - **positive torque**
  - **acceleration in the direction of the orbital velocity**

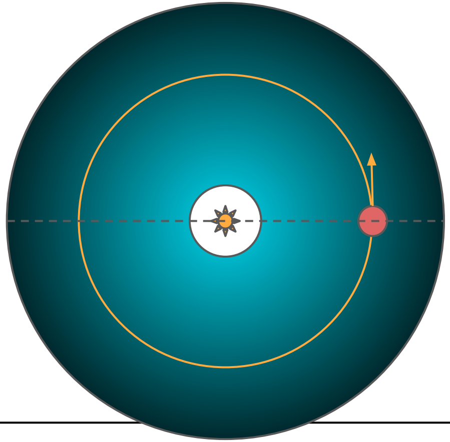
inward migration (semi-major axis shrinks) when there is a
  - **negative torque**
  - **acceleration against the orbital motion**

- one is often interested in the migration time scale

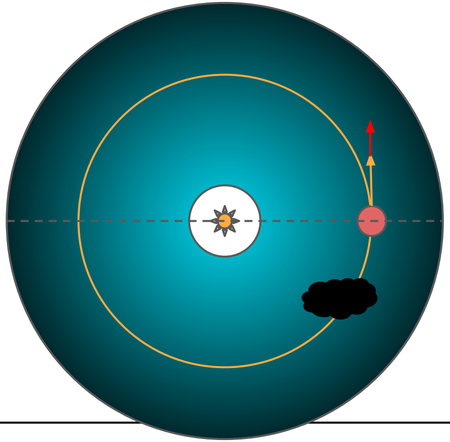
$$\frac{\Delta a_\mathrm{p}}{\tau_\mathrm{mig}} = \frac{-a_\mathrm{p}}{\tau_\mathrm{mig}} = \frac{2\Gamma}{M_\mathrm{p} a_\mathrm{p} \Omega_\mathrm{p}} \implies \tau_\mathrm{mig} = -\frac{M_\mathrm{p} a_\mathrm{p}^2 \Omega_\mathrm{p}}{2\Gamma} = -\frac{L}{2\Gamma},$$

Torques from a disk

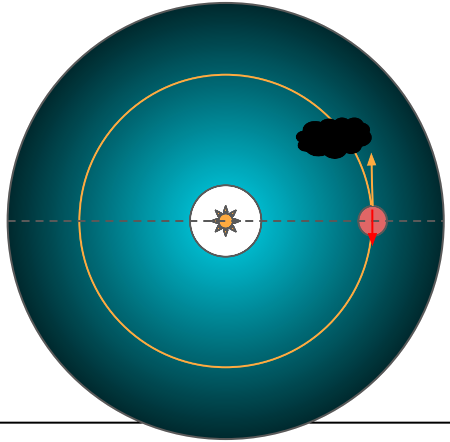
- the theory of planet-disk interactions aims to find the disk-driven torque responsible for the orbital migration
- basic considerations (dark = less dense; black blob = cavity in the disk):



axisymmetric disk -> no azimuthal acceleration -> no torque



more material ahead of the planet -> acceleration in the direction of the orbital velocity



more material behind the planet -> acceleration against the orbital motion

Disk properties (vague introduction)

- torques depend on how the disk mass is distributed -> understanding the disk structure is crucial
- disks are **mixtures of gas and dust**:
  - Gas is more abundant (~100:1) -> we will mostly focus on the **migration in gas disks**
  - Gas is subject to the sub-Keplerian orbital motion, accretion onto the protostar, and turbulence

$$\frac{v_{\rm gas}^2}{r} = \frac{GM_\star}{r^2} + \frac{1}{\rho}\frac{{\rm d}P}{{\rm d}r}$$

radial pressure support (disks get hotter and denser towards their centre)

$$v_{\rm gas} = (1-\eta)\, v_{\rm K} \approx (1-0.001)\, v_{\rm K}$$

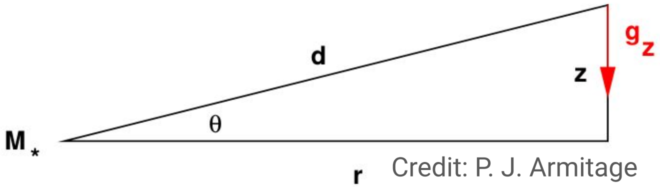
$$v_{\rm K} \propto r^{-1/2}, \quad \Omega_{\rm K} \propto r^{-3/2}, \quad l = r^2 \Omega_{\rm K} \propto r^{1/2}$$

radially increasing specific angular momentum

  - To drive the gas accretion (inward transport of gas), there has to be a **physical mechanism capable of redistributing the angular momentum** (e.g. disk winds; turbulence)

Disk properties (vague introduction)

- torques depend on how the disk mass is distributed -> understanding the disk structure is crucial
- disks are **mixtures of gas and dust**:
    - Gas disks tend to be flat



Credit: P. J. Armitage

$$-\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}z} = \frac{GM_\star}{d^2}\sin\theta = \frac{GM_\star}{d^2}\frac{z}{d} \simeq \frac{GM_\star}{r^3}z = \Omega_\mathrm{K}^2 z$$

Assuming the locally-isothermal EOS $P = \rho c_\mathrm{s}^2$

one obtains

$$\rho = \frac{\Sigma}{\sqrt{2\pi}H}\exp\left(-\frac{z^2}{2H^2}\right)$$

where

$$H = \frac{c_\mathrm{s}}{\Omega_\mathrm{K}}$$

is the **pressure scale height** ~ a measure of the disk thickness & a typical scale of disk perturbations

Disk properties (vague introduction)

- torques depend on how the disk mass is distributed -> understanding the disk structure is crucial
- disks are **mixtures of gas and dust**:
  - Gas is often modelled as a **viscous fluid** (to mimic the turbulence)

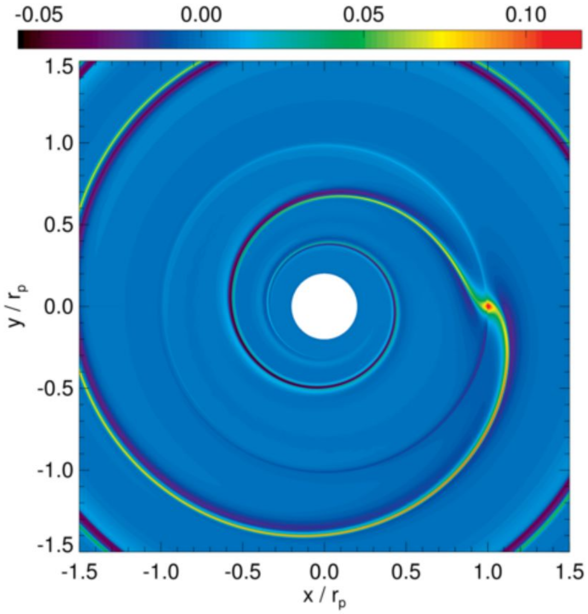
    α-viscosity parametrization $\nu = \alpha_{\rm visc} c_s H$, results in the mass accretion rate $\dot{M}_{\rm acc} = 3\pi\nu\Sigma$,

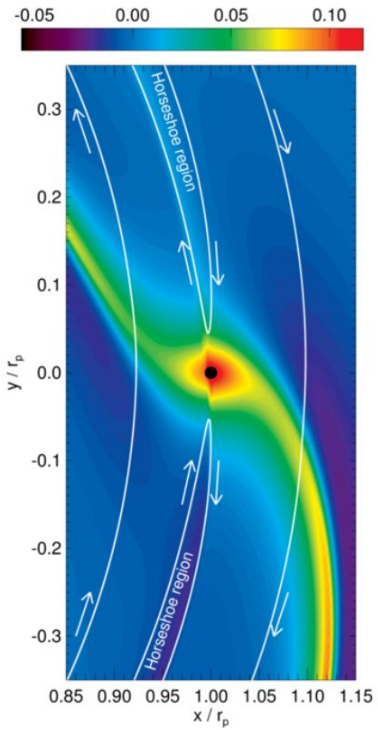
    and viscous accretion velocity $v_{r,\rm visc} = -\frac{3\nu}{2r}$
  - Characteristic quantities are typically outward-decreasing power laws $\Sigma \propto r^{-\alpha}, T \propto r^{-\beta}$
  - Dust is important for the radiative transfer (opacity agent) and provides thermoregulation
  - Dust can be distributed differently than gas

Migration in gas disks (basic regimes)

- perturbations leading the orbital motion ~ positive torque, outward migration
- perturbations trailing the orbital motion ~ negative torque, inward migration
- mathematically:

$$\Gamma = \int_{\text{disk}} \Sigma\left(\boldsymbol{r}_{\mathbf{p}} \times \boldsymbol{a}_{\mathbf{g}}\right) \mathrm{d}S = \int_{\text{disk}} \Sigma\left(\boldsymbol{r}_{\mathbf{p}} \times \nabla\Phi_{\mathrm{p}}\right) \mathrm{d}S = \int_{\text{disk}} \Sigma \frac{\partial \Phi_{\mathrm{p}}}{\partial \theta} \mathrm{d}S$$



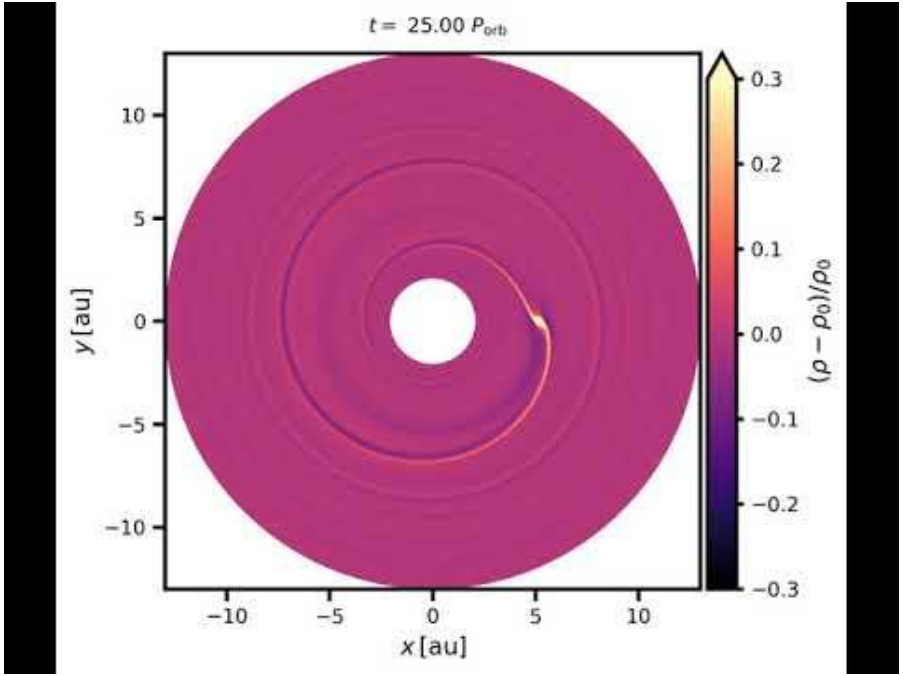


Baruteau et al. (2014)

Migration in gas disks (basic regimes)

- generally, the torque arises from
  - spiral arms = the **Lindblad torque**
  - the corotation region = the **corotation torque**
- low-mass planets undergo **Type I migration** (Lindblad + corotation equally important)
- planets massive enough to open a gap in the gas disk undergo **Type II migration** (Lindblad usually dominant + coupling to the disk flow can be important)

Type I (example with 10 Earth masses)



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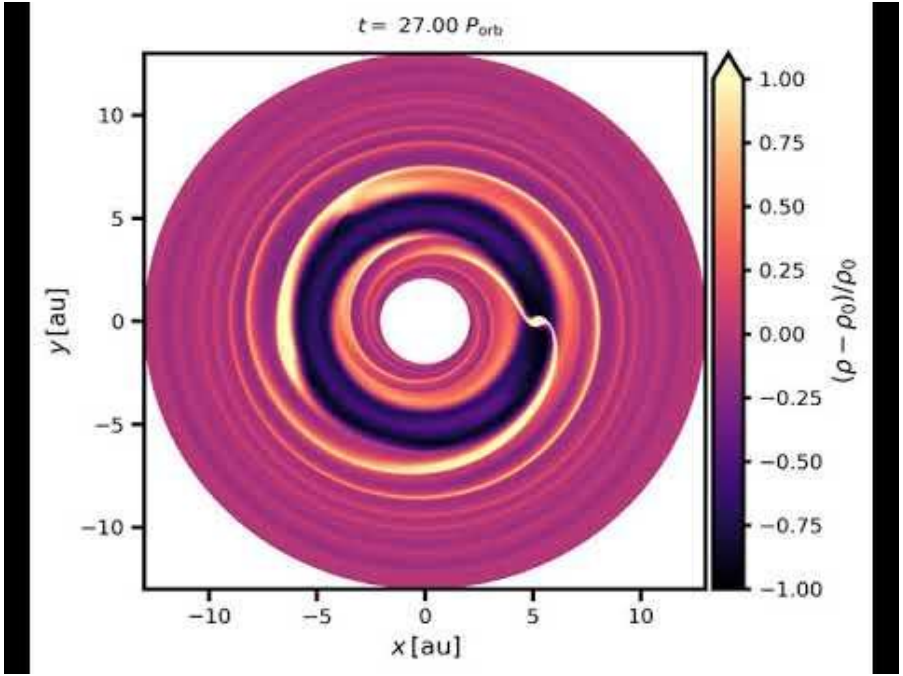
!!! now the colorbar is scaled differently !!!

Type II (example with 1 Jupiter mass)



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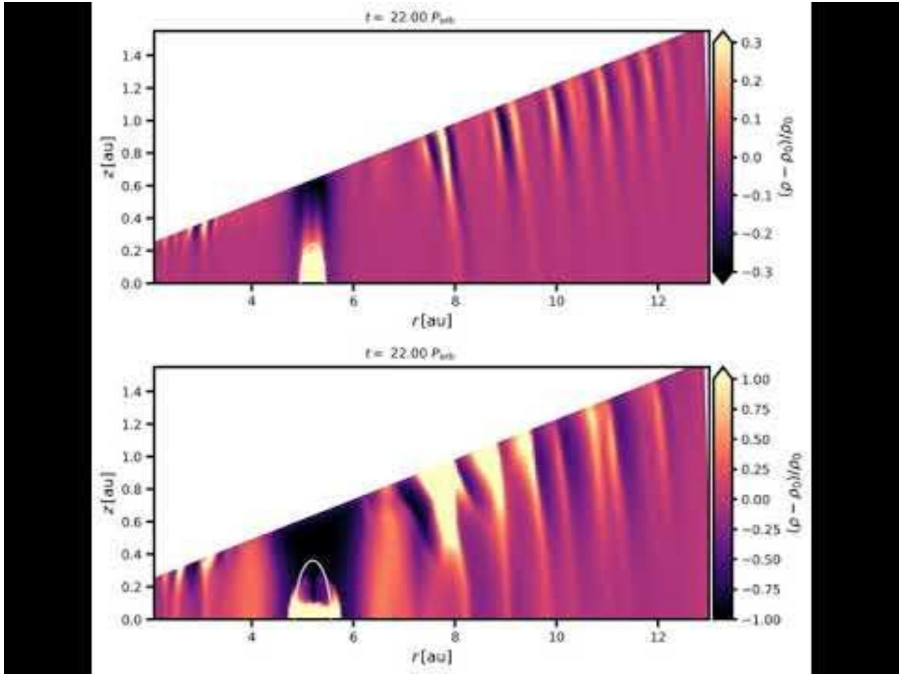
top: 10 Earth masses, bottom: 1 Jupiter mass

Vertical plane



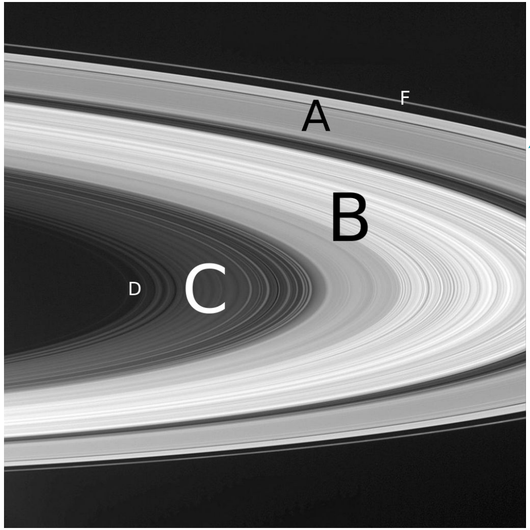
How to analyze the torque

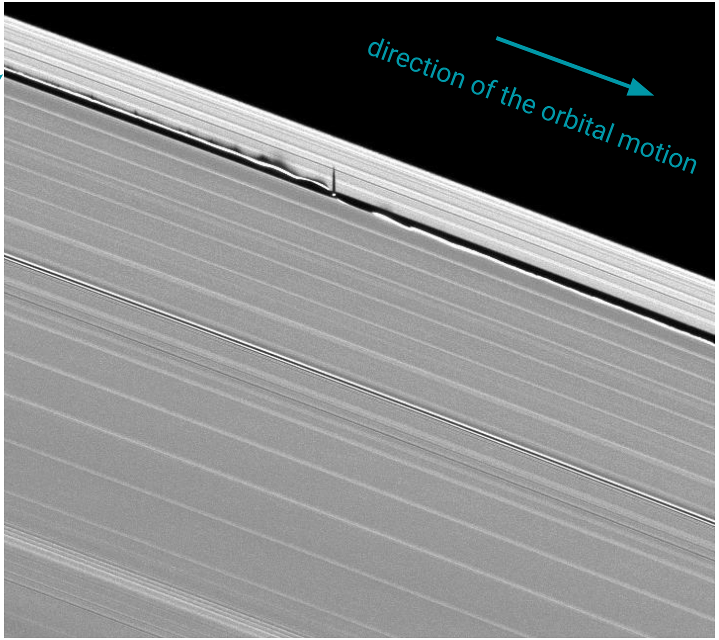
- analytically - calculate the rate of the angular momentum exchange at planet-induced perturbations
  - fluid approximated with non-interacting particles - today's lecture
  - linear perturbation analysis of hydrodynamic equations - 2nd lecture
- numerically - simulate the disk with an embedded planet and sum up the torque from the final distribution of gas

- **Today's plan:**
  - investigate the angular momentum exchange between a massive body and a disk of test particles (we can benefit from the results of the celestial mechanics)
  - the method leads to **correct functional dependencies but wrong scaling** (since we neglect the pressure, temperature, viscosity, …)

Perturbers in particle disks

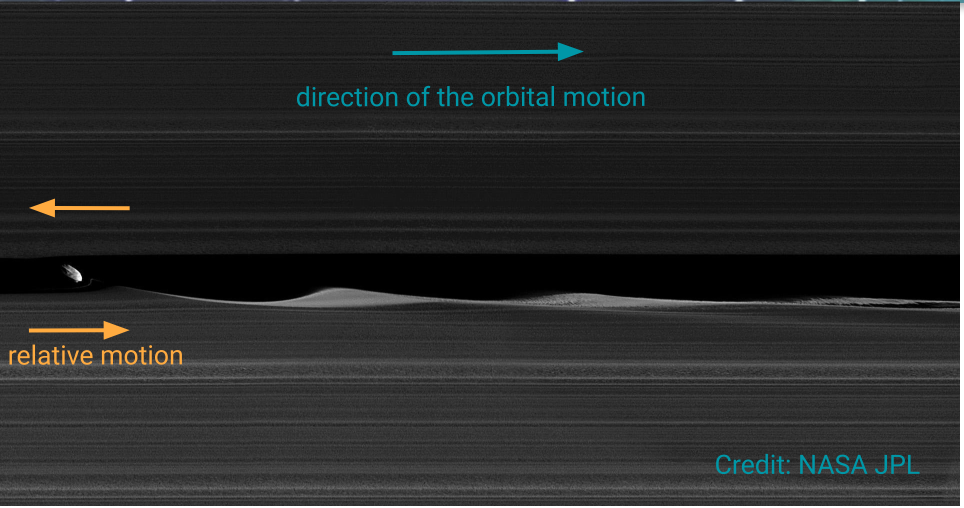
- Saturn's rings as a laboratory for massive perturbers (~moons) interacting with test particles (~ring grains)
- Daphnis within the Keeler's gap:

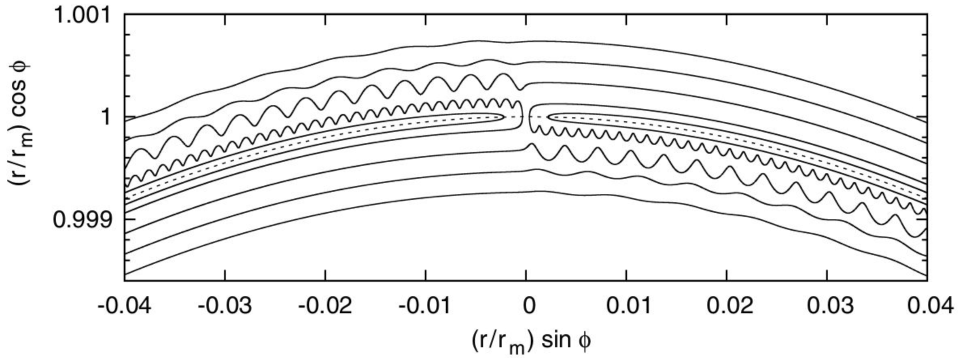


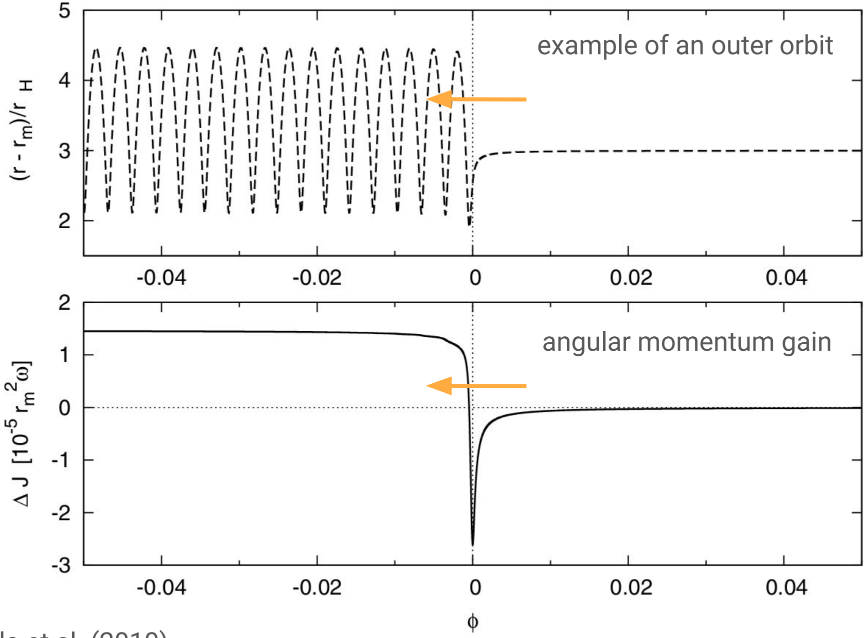


Credit: NASA JPL

Perturbers in particle disks







Crida et al. (2010)