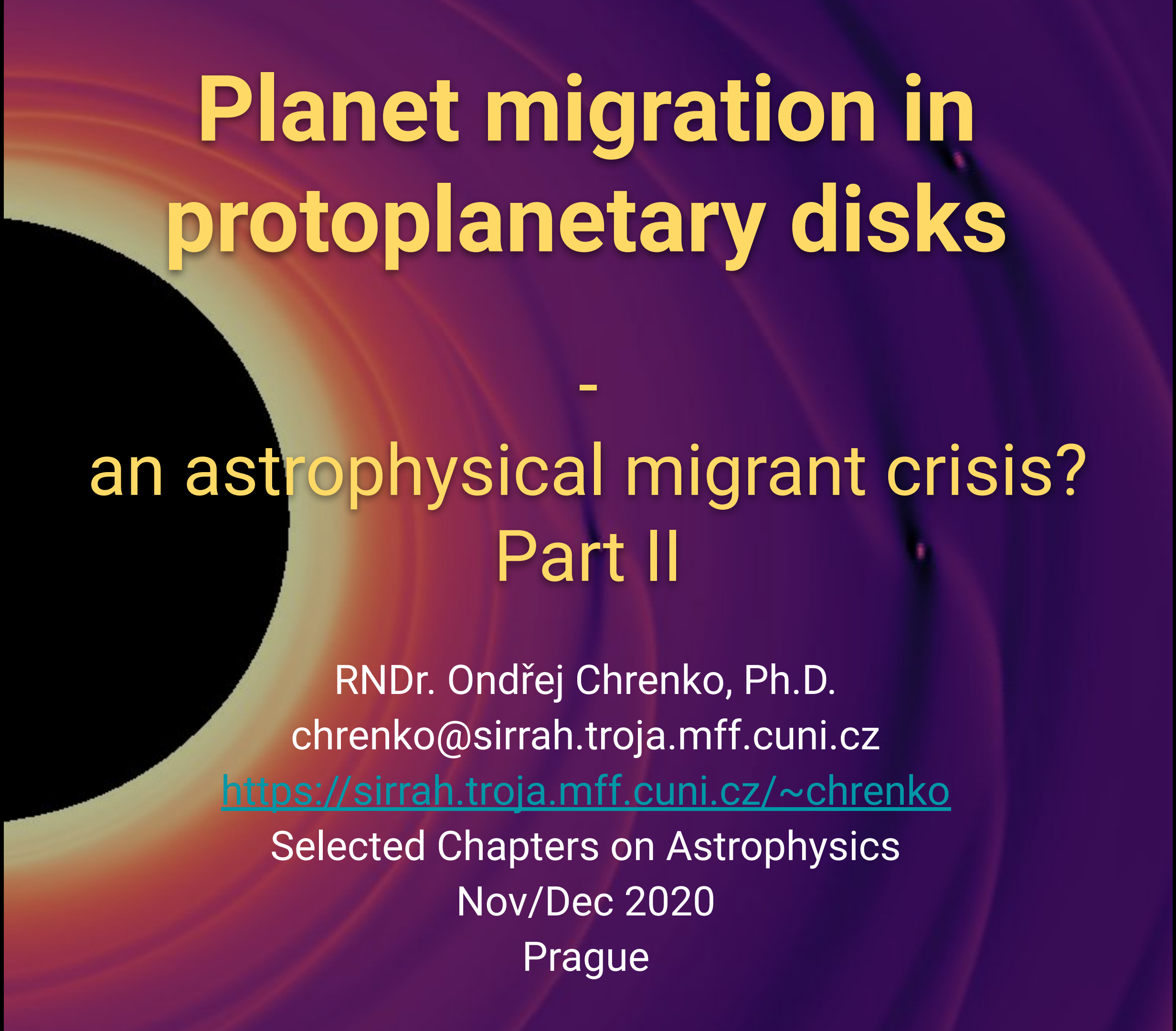

Planet migration in protoplanetary disks

-

an astrophysical migrant crisis?
Part II

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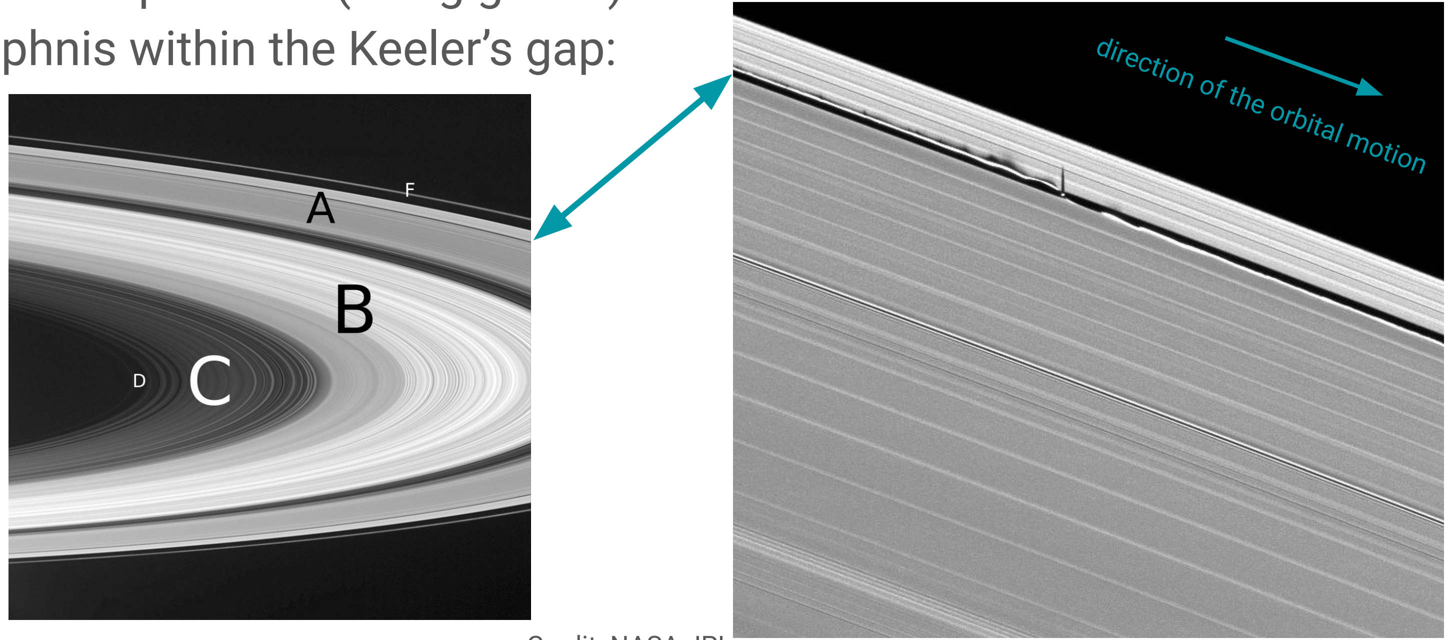
Selected Chapters on Astrophysics
Nov/Dec 2020
Prague

Today's plan

- Finish torque estimates from Part I
    - gas as particles ~ weakly perturbed orbits (preparation for the Lindblad torque)
    - gas as particles ~ horseshoe orbits (preparation for the non-linear corotation torque)
- Discuss Type I migration
    - gas as a fluid
    - Lindblad torque
    - corotation torque
    - comparison to the impulse approximation; relation to the disk properties

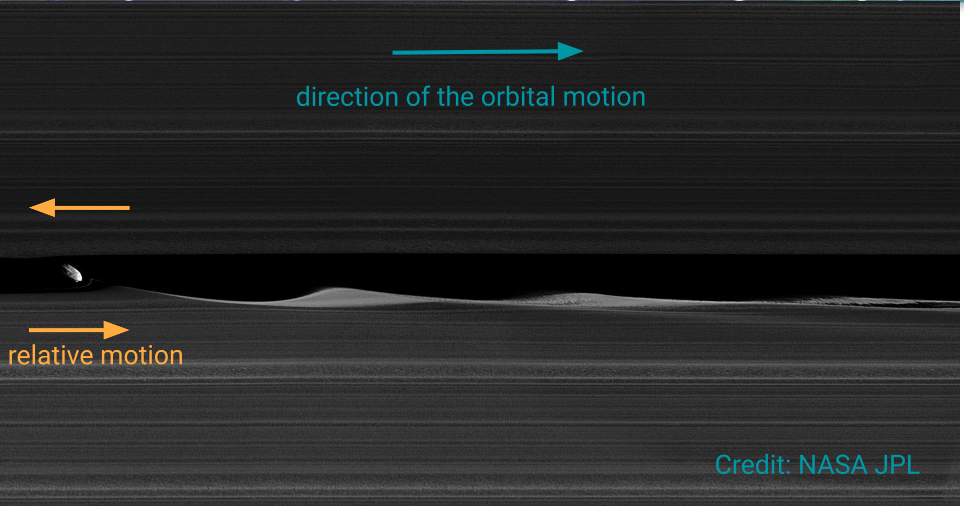
Perturbers in particle disks (<- related to Part I)

- Saturn’s rings as a laboratory for massive perturbers (~moons) interacting with test particles (~ring grains)
- Daphnis within the Keeler’s gap:



Credit: NASA JPL

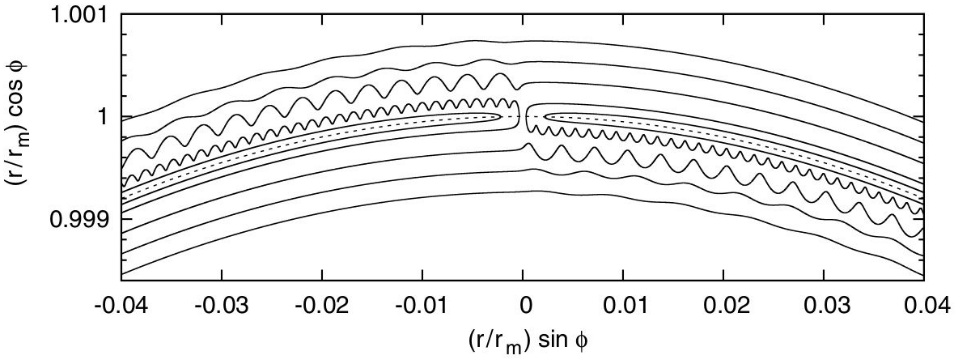
Perturbers in particle disks (<- related to Part I)

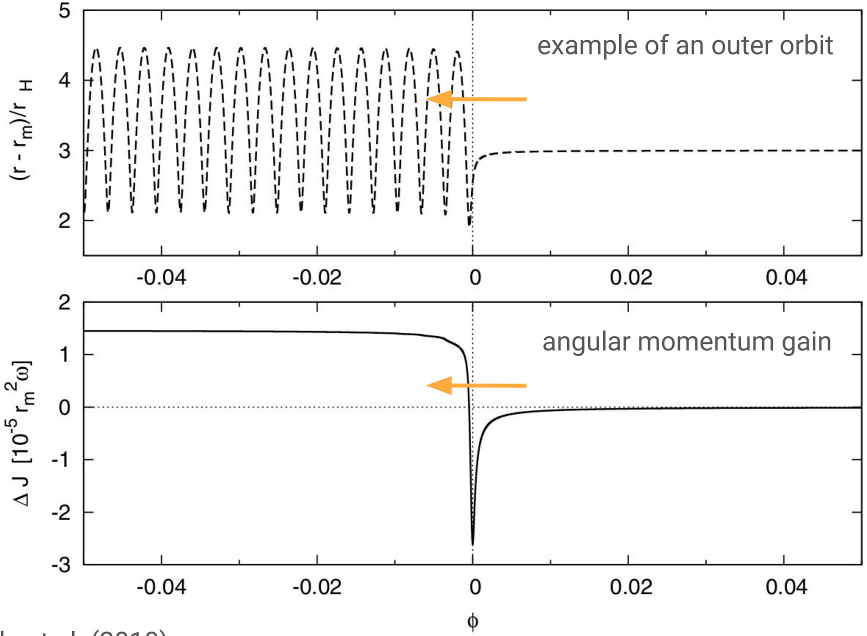


direction of the orbital motion

relative motion

Credit: NASA JPL





example of an outer orbit

angular momentum gain

Crida et al. (2010)

Gas as a fluid

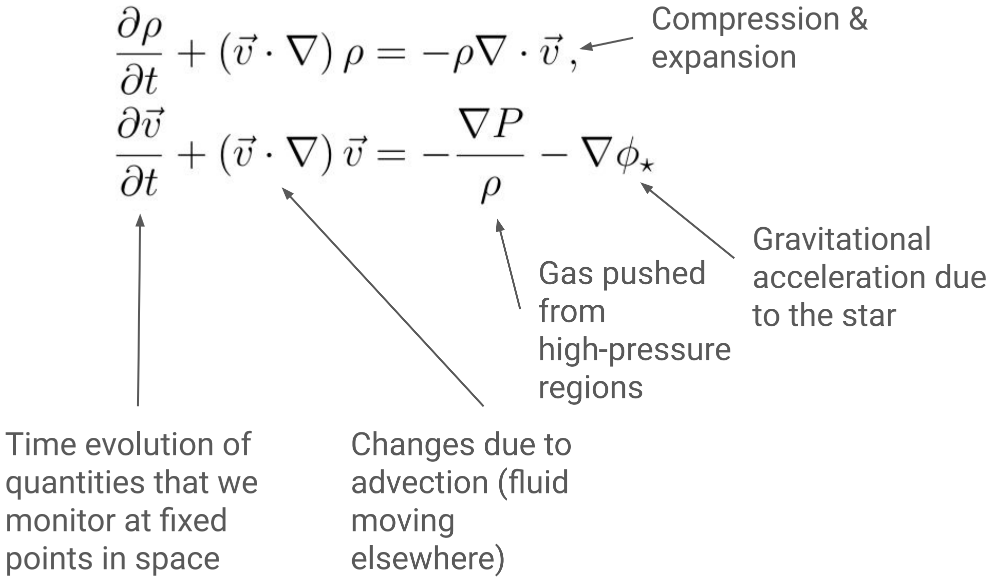
- minimal form for a gas disk around a star + pressure support:

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla)\, \rho = -\rho \nabla \cdot \vec{v}\,,$$
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} = -\frac{\nabla P}{\rho} - \nabla \phi_{\star}$$

- continuity equation
- Navier-Stokes equation

Gas as a fluid

- minimal form for a gas disk around a star + pressure support:



$$\frac{\partial\rho}{\partial t} + (\vec{v}\cdot\nabla)\,\rho = -\rho\nabla\cdot\vec{v}\,,$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v} = -\frac{\nabla P}{\rho} - \nabla\phi_\star$$

Compression & expansion

Gas pushed from high-pressure regions

Gravitational acceleration due to the star

Time evolution of quantities that we monitor at fixed points in space

Changes due to advection (fluid moving elsewhere)

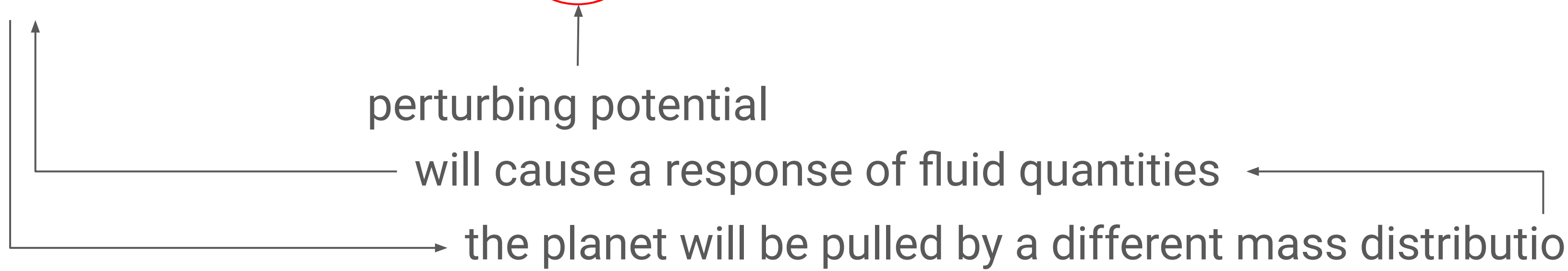
- continuity equation
- Navier-Stokes equation

Gas as a fluid

- a perturber (~planet) added:

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla)\,\rho = -\rho \nabla \cdot \vec{v}\,,$$
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\,\vec{v} = -\frac{\nabla P}{\rho} - \nabla \phi_\star \boxed{- \nabla \phi_{\mathrm{p}}}$$

- continuity equation
- Navier-Stokes equation



perturbing potential

will cause a response of fluid quantities

the planet will be pulled by a different mass distribution

Gas as a fluid

- adding relevant physics & closure relations:

$$\frac{\partial\rho}{\partial t} + (\vec{v}\cdot\nabla)\,\rho = -\rho\nabla\cdot\vec{v}\,,$$
$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v} = -\frac{\nabla P}{\rho} - \nabla\phi_\star - \nabla\phi_{\mathrm{p}} + \boxed{\frac{\nabla\cdot\mathbb{T}}{\rho}}\,,$$

- continuity equation
- Navier-Stokes equation

viscous stress term
(viscosity to drive the disk accretion)

$$\mathbb{T} = \rho\boxed{\nu}\left[\nabla\vec{v} + (\nabla\vec{v})^{\intercal} - \frac{2}{3}\,(\nabla\cdot\vec{v})\,\mathbb{1}\right]$$

Gas as a fluid

- adding relevant physics & closure relations:

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla)\,\rho = -\rho \nabla \cdot \vec{v}\,,$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\,\vec{v} = -\frac{\nabla P}{\rho} - \nabla \phi_\star - \nabla \phi_{\mathrm{p}} + \frac{\nabla \cdot \mathbb{T}}{\rho}\,,$$

- continuity equation
- Navier-Stokes equation

- pressure needs to be specified via the equation of state:

$$P = c_{\mathrm{s}}^2 \rho \wedge c_{\mathrm{s}}\,(r)\ \text{ is fixed \& prescribed}$$ —— locally isothermal approximation

Gas as a fluid

- adding relevant physics & closure relations:

$$
\begin{aligned}
\frac{\partial\rho}{\partial t} + (\vec{v}\cdot\nabla)\,\rho &= -\rho\nabla\cdot\vec{v}\,,\\
\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v} &= -\frac{\nabla P}{\rho} - \nabla\phi_{\star} - \nabla\phi_{\mathrm{p}} + \frac{\nabla\cdot\mathbb{T}}{\rho}\,,\\
\frac{\partial\epsilon}{\partial t} + (\vec{v}\cdot\nabla)\,\epsilon &= -\epsilon\nabla\cdot\vec{v}\boxed{-P\nabla\cdot\vec{v}}
\end{aligned}
$$

- continuity equation
- Navier-Stokes equation
- energy equation

- pressure needs to be specified via the equation of state:

$P = c_{\mathrm{s}}^2\rho \wedge c_{\mathrm{s}}(r)$ is fixed & prescribed — locally isothermal approximation

$P = (\gamma - 1)\,\epsilon = \rho\frac{\mathcal{R}}{\mu}T$ — adiabatic approx. (compressional heating only)

Gas as a fluid

- adding relevant physics & closure relations:

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla)\, \rho = -\rho \nabla \cdot \vec{v}\,,$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} = -\frac{\nabla P}{\rho} - \nabla \phi_\star - \nabla \phi_{\rm p} + \frac{\nabla \cdot \mathbb{T}}{\rho}\,,$$

$$\frac{\partial \epsilon}{\partial t} + (\vec{v} \cdot \nabla)\, \epsilon = -\epsilon \nabla \cdot \vec{v} - P \nabla \cdot \vec{v} + \boxed{Q_{\rm heat}\,.}$$

- continuity equation
- Navier-Stokes equation
- energy equation

- pressure needs to be specified via the equation of state:

$$P = c_{\rm s}^2 \rho \wedge c_{\rm s}\,(r) \text{ is fixed \& prescribed}$$

locally isothermal approximation

$$P = (\gamma - 1)\, \epsilon = \rho \frac{\mathcal{R}}{\mu} T$$

adiabatic approx. (compressional heating only)

non-isothermal fluid (any extra heating/cooling)

Gas as a fluid

- adding relevant physics & closure relations:

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla)\, \rho = -\rho \nabla \cdot \vec{v}\,,$$

- continuity equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} = -\frac{\nabla P}{\rho} - \nabla \phi_\star - \nabla \phi_{\rm p} + \frac{\nabla \cdot \mathbb{T}}{\rho}\,,$$

- Navier-Stokes equation

$$\frac{\partial \epsilon}{\partial t} + (\vec{v} \cdot \nabla)\, \epsilon = -\epsilon \nabla \cdot \vec{v} - P \nabla \cdot \vec{v} + \boxed{Q_{\rm heat}\,.}$$

- energy equation

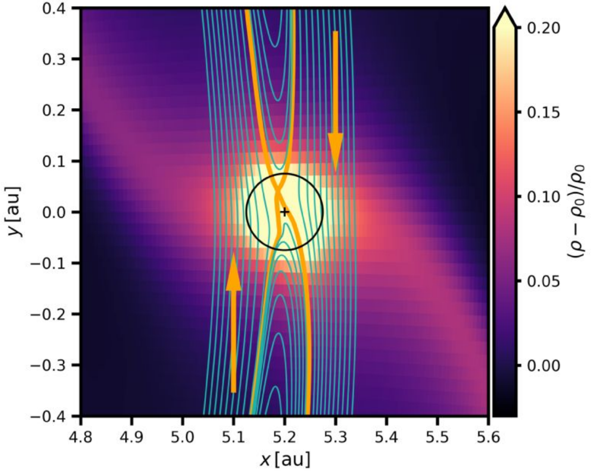
- the heat balance term can contain e.g.: viscous friction; stellar irradiation; radiative transfer and escape (might require a standalone equation for the energy of a field of photons)
- for practical reasons, 2D versions of the equations above are often used, e.g.:

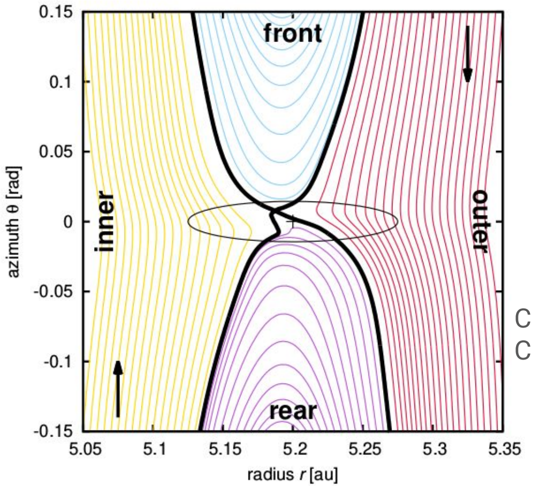
$$\frac{\partial \Sigma}{\partial t} + (\vec{v} \cdot \nabla)\, \Sigma = -\Sigma \nabla \cdot \vec{v}$$

- ...not mentioned: magnetic fields, self-gravity, etc...

Type I migration (no gap)

- usually dominated by resonant torques (Lindblad and corotation torques)
- gas orbits the star and receives kicks from the planet (e.g. outer circulating orbits gain *L*, inner circulating orbits loose *L*)
- the interaction is amplified if the frequency of the perturbation resonates with some natural dynamical frequency of the gas disk

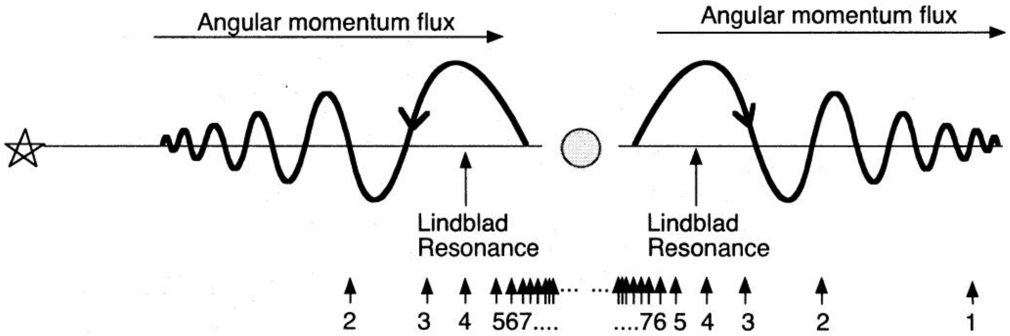




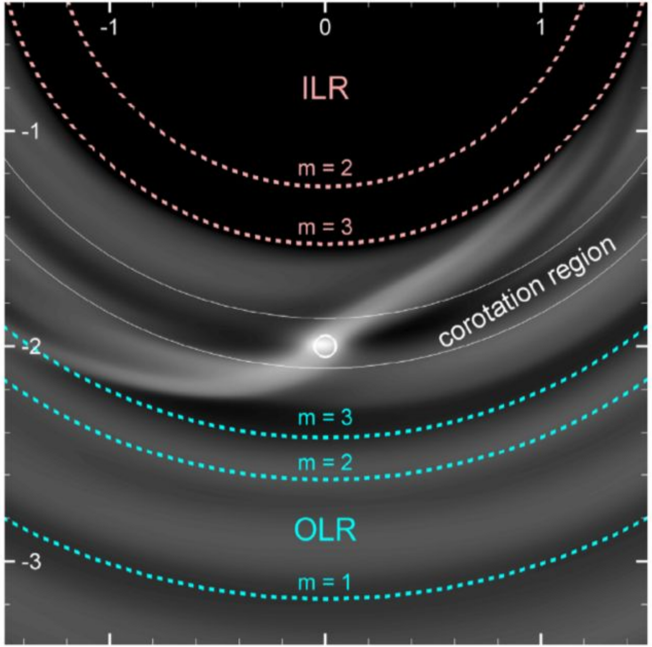
Chrenko (2019),
Chrenko & Lambrechts (2019)

Lindblad torque

- Lindblad resonance condition: $m\left[\Omega(r)-\Omega_{\rm p}\right]=\pm\kappa(r)\,,\;m\left[\Omega(r)-\Omega_{\rm p}\right]\simeq\pm\Omega\sqrt{1+\frac{m^2H^2}{r^2}}$



Takeuchi et al. (1996)



Romanova et al. (2019)

Lindblad torque

- Lindblad torque formulae from advanced fluid models:

  Tanaka et al. (2002) ~ linear pertur. theory, 3D disk with $\Sigma(r) \sim r^{-\alpha}$ and uniform temperature

  $$\Gamma_{\rm L} = -(2.34 - 0.1\alpha)\Gamma_0$$

  Paardekooper et al. (2010) ~ numerical study, 2D non-isothermal disk $\Sigma(r) \sim r^{-\alpha}$, $T(r) \sim r^{-\beta}$

  $$\gamma_{\rm eff}\Gamma_{\rm L} = -(2.5 - 0.1\alpha + 1.7\beta)\Gamma_0$$

  where $\Gamma_0 = \Sigma_{\rm p} a_{\rm p}^4 \Omega_{\rm p}^2 \left(\frac{q}{h_{\rm p}}\right)^2$, $q = M_{\rm p}/M_\star$, $h = H/r = c_{\rm s}/\Omega r$

Lindblad torque

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- compare to the impulse approximation:

$$\Gamma_{\rm out} = -C_{\rm out}\Sigma_{\rm p} a_{\rm p}^4 \Omega_{\rm p}^2 \frac{q^2}{\left(\frac{\Delta r}{a_{\rm p}}\right)^3}\,,$$

$$\Gamma_{\rm in} = C_{\rm in}\Sigma_{\rm p} a_{\rm p}^4 \Omega_{\rm p}^2 \frac{q^2}{\left(\frac{\Delta r}{a_{\rm p}}\right)^3}\,,$$

→

taking $\frac{\Delta r}{a_{\rm p}} \sim \frac{H}{a_{\rm p}}$,

together with the char. scaling of the differential Lindblad torque (Ward 1997): $\Gamma_{\rm in/out} \sim h^{-3}, \Gamma_{\rm in} + \Gamma_{\rm out} \sim h^{-2}$,

→ $\Gamma_{\rm in} + \Gamma_{\rm out} \sim \Gamma_0$

Lindblad torque

- Lindblad torque formulae from advanced fluid models:
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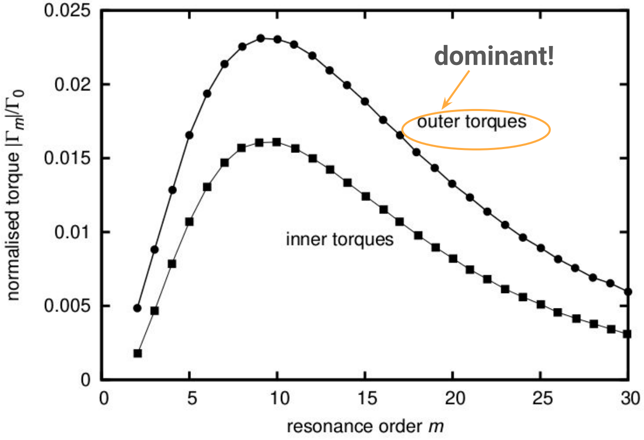
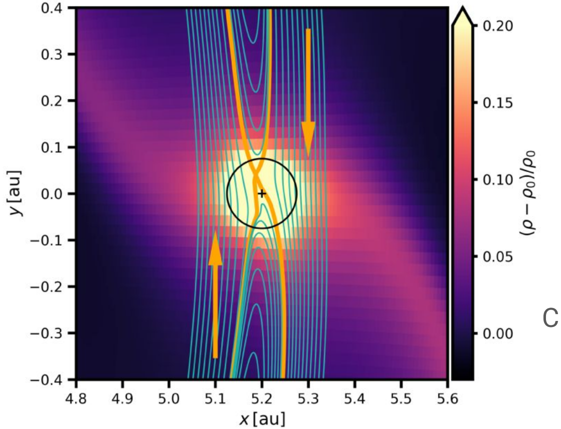
  $$\gamma_{\rm eff}\Gamma_{\rm L} = -(2.5 - 0.1\alpha + 1.7\beta)\Gamma_0$$
  where $\Gamma_0 = \Sigma_{\rm p} a_{\rm p}^4 \Omega_{\rm p}^2 \left(\frac{q}{h_{\rm p}}\right)^2$, $q = M_{\rm p}/M_\star,\ h = H/r = c_{\rm s}/\Omega r$
- usually leads to an inward migration
- resulting migration rates are relatively fast (migra. time scale for an Earth in the MMSN disk ~0.25 Myr!)

$$\frac{{\rm d}a_{\rm p}}{{\rm d}t} = \frac{2\Gamma}{M_{\rm p} a_{\rm p} \Omega_{\rm p}} \sim \frac{-\Gamma_0}{M_{\rm p} a_{\rm p} \Omega_{\rm p}}$$

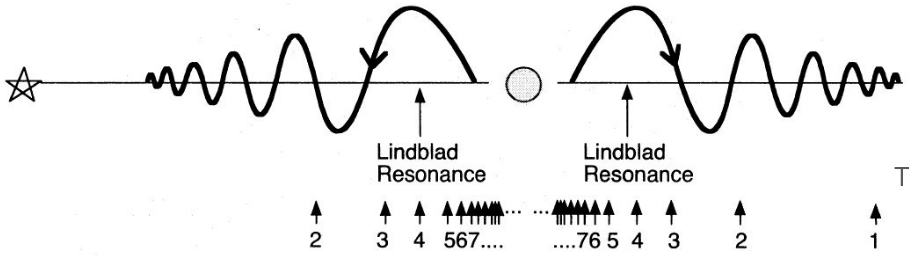
Lindblad torque

- strongest torque contribution generated close to the launching point of spiral arms:





Chrenko (2019)



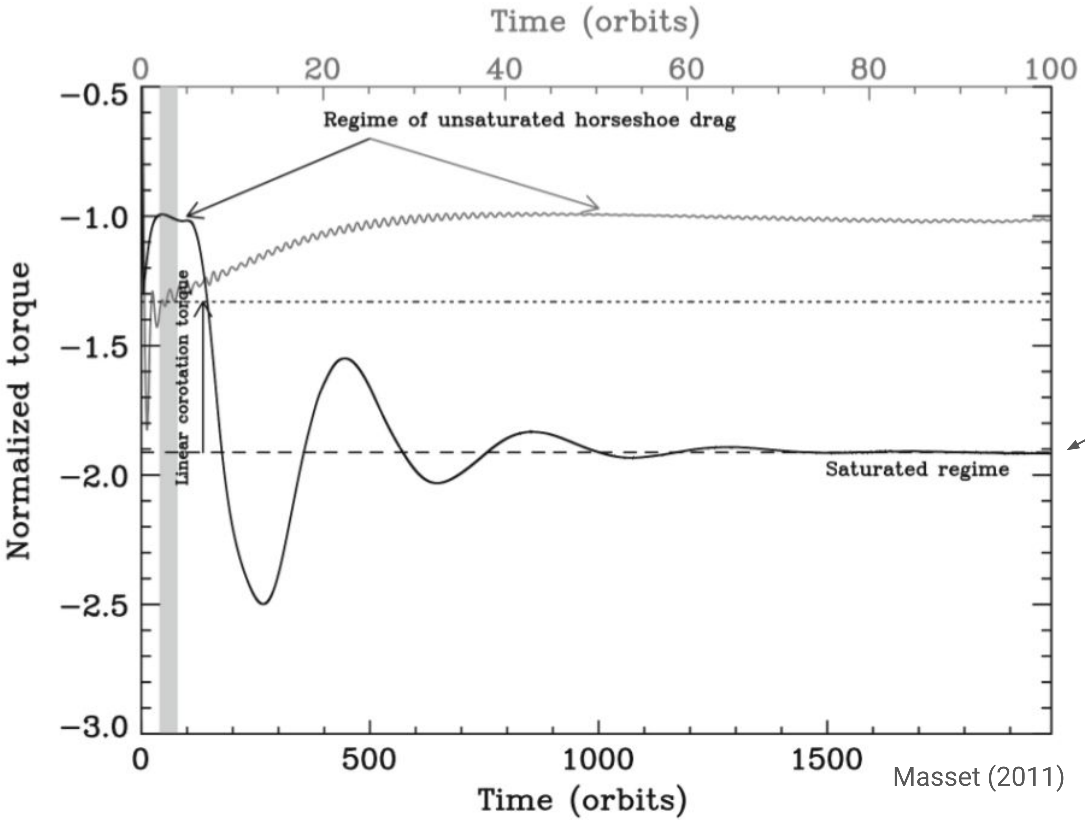
Takeuchi et al. (1996)

Corotation torque

- corotation resonance condition: $\left[\Omega\left(r\right)-\Omega_{\mathrm{p}}\right]=0$
- rich behaviour: can be positive/negative, dominant/negligible w.r.t. the Lindblad torque
- regimes:
  - linear
  - non-linear (a.k.a. horseshoe drag <- this is related to our calculations in the impulse approx.)
    - unsaturated
    - saturated
- components:
  - driven by the vortensity gradient across corotation
  - driven by the thermal gradient across corotation

Corotation torque - regimes

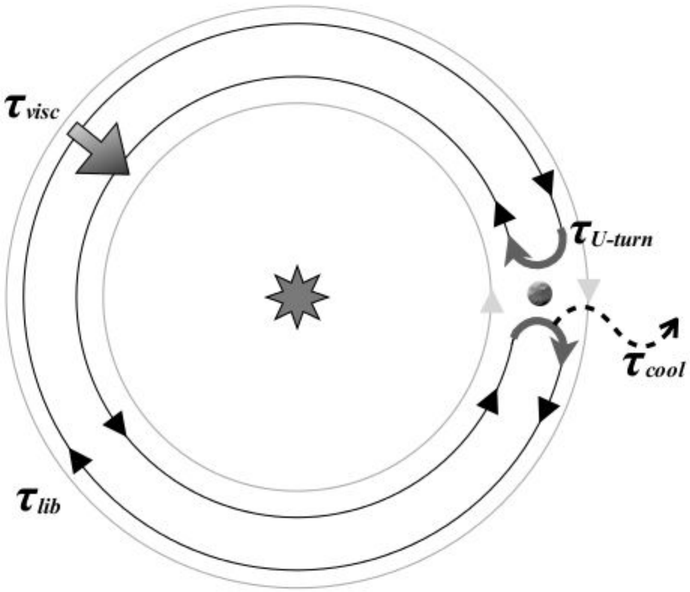
Normalized to Γ_0



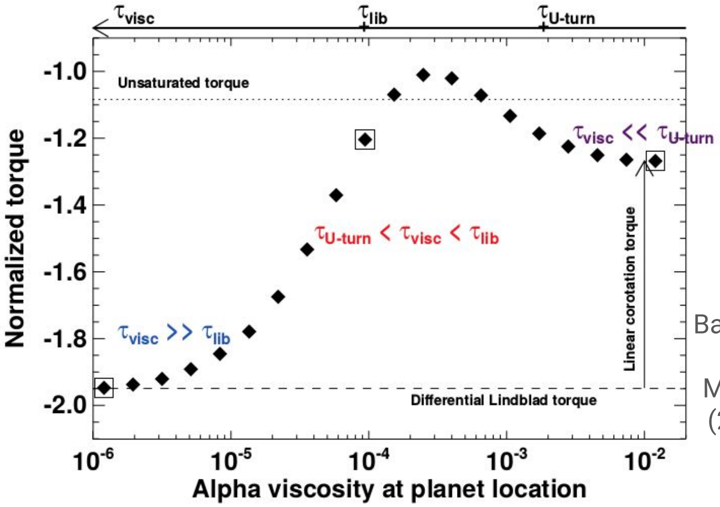
Masset (2011)

Lindblad torque remains

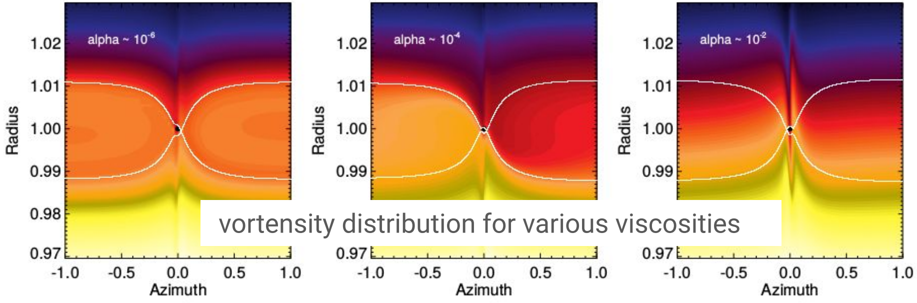
Corotation torque - question of saturation



Mordasini et al. (2011)



Baruteau & Masset (2012)



vortensity distribution for various viscosities

Corotation torque - components

- full formula for the corotation torque includes linear and non-linear regimes, their blending, and saturation of the non-linear regime
- for simplicity, let us assume that the torque operates in the non-linear unsaturated regime (i.e. the maximum possible corotation torque):

$$\gamma_{\mathrm{eff}}\Gamma_{\mathrm{c}} = \left[1.1\left(\frac{3}{2} - \alpha\right) + 7.9\frac{\xi}{\gamma_{\mathrm{eff}}}\right]\Gamma_0$$

Paardekooper et al. (2011)

Vortensity-related component

Entropy-related component

Corotation torque - components

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- for simplicity, let us assume that the torque operates in the non-linear unsaturated regime (i.e. the maximum possible corotation torque):

$$\gamma_{\rm eff}\Gamma_{\rm c} = \left[\underline{1.1\left(\frac{3}{2}-\alpha\right)} + 7.9\frac{\xi}{\gamma_{\rm eff}}\right]\Gamma_0$$

Paardekooper et al. (2011)

- compare to the impulse approximation:

$$\Gamma_{\rm hs} = \frac{3}{4}\left(\frac{3}{2}-\alpha\right)\Sigma_{\rm p}\boxed{x_{\rm s}^4}\Omega_p^2$$

half-width of the horseshoe region has to be determined accurately, Paardekooper & Papaloizou (2009) suggest: $x_{\rm s} = a_{\rm p}\frac{1.1}{\gamma_{\rm eff}^{1/4}}\sqrt{\frac{q}{h}}$

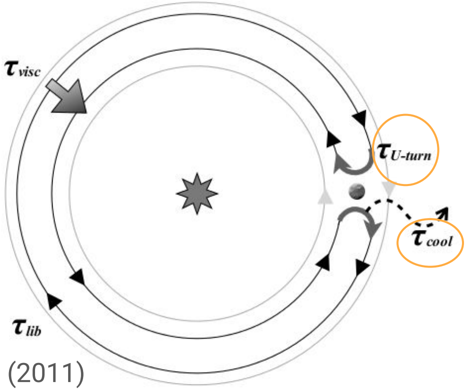
but then: $$\Gamma_{\rm hs} \simeq 1.1\left(\frac{3}{2}-\alpha\right)\Sigma_{\rm p}a_{\rm p}^4\Omega_{\rm p}^2\frac{q^2}{h^2\gamma_{\rm eff}} = \underline{1.1\left(\frac{3}{2}-\alpha\right)\frac{\Gamma_0}{\gamma_{\rm eff}}}$$

Corotation torque - components

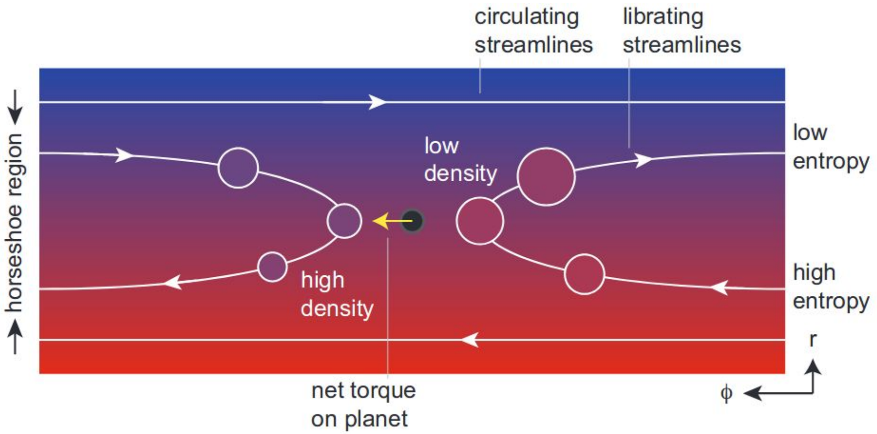
$$\gamma_{\mathrm{eff}}\Gamma_{\mathrm{c}} = \left[1.1\left(\frac{3}{2}-\alpha\right) + \underline{7.9\frac{\xi}{\gamma_{\mathrm{eff}}}}\right]\Gamma_0$$

Paardekooper et al. (2011)

- reasoning for the entropy-related component:
    - we consider a measure of the specific entropy $\mathcal{S} = \frac{P}{\Sigma^\gamma} \propto \frac{\Sigma T}{\Sigma^\gamma} \propto r^{-\xi}$ with the slope $\xi = \beta - (\gamma - 1)\alpha$
    - after a U-turn, a blob of gas finds itself surrounded by material with different entropy and can produce density variations
    - again, question of timescales:



Mordasini et al. (2011)



Credit: P.J. Armitage