Planet migration - homework assignment

Imagine that an Earth-mass protoplanet ($M_{\rm p} = 1\,M_\oplus$) is formed at an orbital distance $a_{\rm p} = 5.2\,$au in a protoplanetary disk similar to the Minimum Mass Solar Nebula (MMSN; Hayashi 1981) that has the gas surface density $\Sigma(r) = 1700(r/\mathrm{au})^{-3/2}\,\mathrm{g\,cm^{-2}}$ and uniform aspect ratio $h = H/r = 0.05$. The latter implies a radial temperature profile $T(r) \propto r^{-1}$. The mass of the central protostar is $M_\star = 1\,M_\odot$. The planet is not capable to open a gap and therefore it migrates in the Type I regime.

1. First, assume that the migration is driven solely by the Lindblad torque. Evaluate the migration rate $\mathrm{d}a_{\rm p}/\mathrm{d}t$ (in au/yr) and migration timescale $\tau_{\rm mig}$ (in Myr). Use the expression for the Lindblad torque from Paardekooper et al. (2010):

$$\gamma_{\rm eff}\Gamma_{\rm L} = -\left(2.5 - 0.1\alpha + 1.7\beta\right)\Gamma_0\,,$$

   and assume that the effective adiabatic index is $\gamma_{\rm eff} = 1.4$. Does the planet migrate inwards or outwards? Will the planet survive its migration if the typical lifetime of protoplanetary disks is $\simeq$ 3–10 Myr?

2. (a) Use a simple numerical integration (e.g. the Euler method) to calculate and plot how $a_{\rm p}$ evolves in time t. Is your $\tau_{\rm mig}$ from Task 1 an accurate measure of the evolution or not? Why?

   (b) Plot the evolution of $a_{\rm p}(t)$ for surface density profiles $\Sigma(r) = 1700(r/\mathrm{au})^{-\alpha}\,\mathrm{g\,cm^{-2}}$ with $\alpha = 0.5$ and $\alpha = 2$. Would it be safe to rely on $\tau_{\rm mig}$ in these cases or not? Why? Do steeper gas density profiles lead to faster or slower migration?

3. In addition to the Lindblad torque, consider that the non-linear corotation torque is also affecting the protoplanet. It can be estimated as (Paardekooper et al. 2011)

$$\gamma_{\rm eff}\Gamma_{\rm c} = \left(1.1\left(\frac{3}{2} - \alpha\right) + 7.9\frac{\xi}{\gamma}\right)\Gamma_0\,,$$

   where the entropy slope is $\xi = \beta - (\gamma - 1)\alpha$ and suppose that $\gamma = \gamma_{\rm eff} = 1.4$. Find the radial temperature profile in which the migration (driven by both $\Gamma_{\rm L}$ and $\Gamma_{\rm c}$) stops. Do steeper temperature profiles increase or decrease the magnitude of the non-linear corotation torque?

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4. For the corotation torque to remain active (unsaturated), a special ordering is required for the libration timescale τ_{lib}, U-turn timescale $\tau_{\mathrm{U-turn}}$ and viscous evolution timescale τ_{visc}. Assume in the following that the characteristic width of the horseshoe region is

$$x_{\mathrm{s}} = a_{\mathrm{p}} \frac{1.1}{\gamma_{\mathrm{eff}}^{1/4}} \sqrt{\frac{q}{h}} \, ,$$

and that the disk has the kinematic viscosity

$$\nu = \alpha_{\mathrm{visc}} H^2 \Omega \, ,$$

where α_{visc} is the dimensionless viscosity parameter of Shakura & Sunyaev (1973) and Ω is the Keplerian frequency.

   (a) Let $\alpha_{\mathrm{visc}} = 10^{-3}$. Calculate and plot τ_{lib}, $\tau_{\mathrm{U-turn}}$ and τ_{visc} as functions of the planet mass M_{p}. Use logarithmic axes, the mass range 0.1–30 $M_{\oplus}$, and the local orbital period as the time unit. Will the Earth-mass protoplanet from previous Tasks experience an unsaturated or saturated corotation torque? What is the range of M_{p} for which the corotation torque remains unsaturated?

   (b) Calculate and plot τ_{lib}, $\tau_{\mathrm{U-turn}}$ and τ_{visc} for the Earth-mass protoplanet as functions of the alpha viscosity α_{visc}. Use logarithmic axes, the alpha viscosity range 10^{-5}–10^{-2}, and the local orbital period as the time unit. What is the range of α_{visc} for which the corotation torque remains unsaturated?

   [Hint: Expressions for τ_{lib}, $\tau_{\mathrm{U-turn}}$ and τ_{visc} were not discussed during the lectures but all ingredients to put them together should be available. For τ_{lib}, a gas element needs to complete an entire horseshoe orbit (thus one can take it as the characteristic length) and its relative velocity is given by the Keplerian shear (which provides the characteristic velocity). For $\tau_{\mathrm{U-turn}}$, take $\tau_{\mathrm{U-turn}} \simeq h\tau_{\mathrm{lib}}$ for simplicity. For τ_{visc}, you can start by recalling the viscous accretion velocity $-(3\nu)/(2r)$.]