Introduction to the dynamics of collisional stellar systems: the early life of star clusters

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Prague Dec. 2021- Selected Chapters on Astrophysics

My research interests focus on

- **Dark Matter Problem:** Alternative theories of gravity, Modeling of dwarf galaxies, Rotation Curves of spirals, Dark matter problems in galactic scales, …

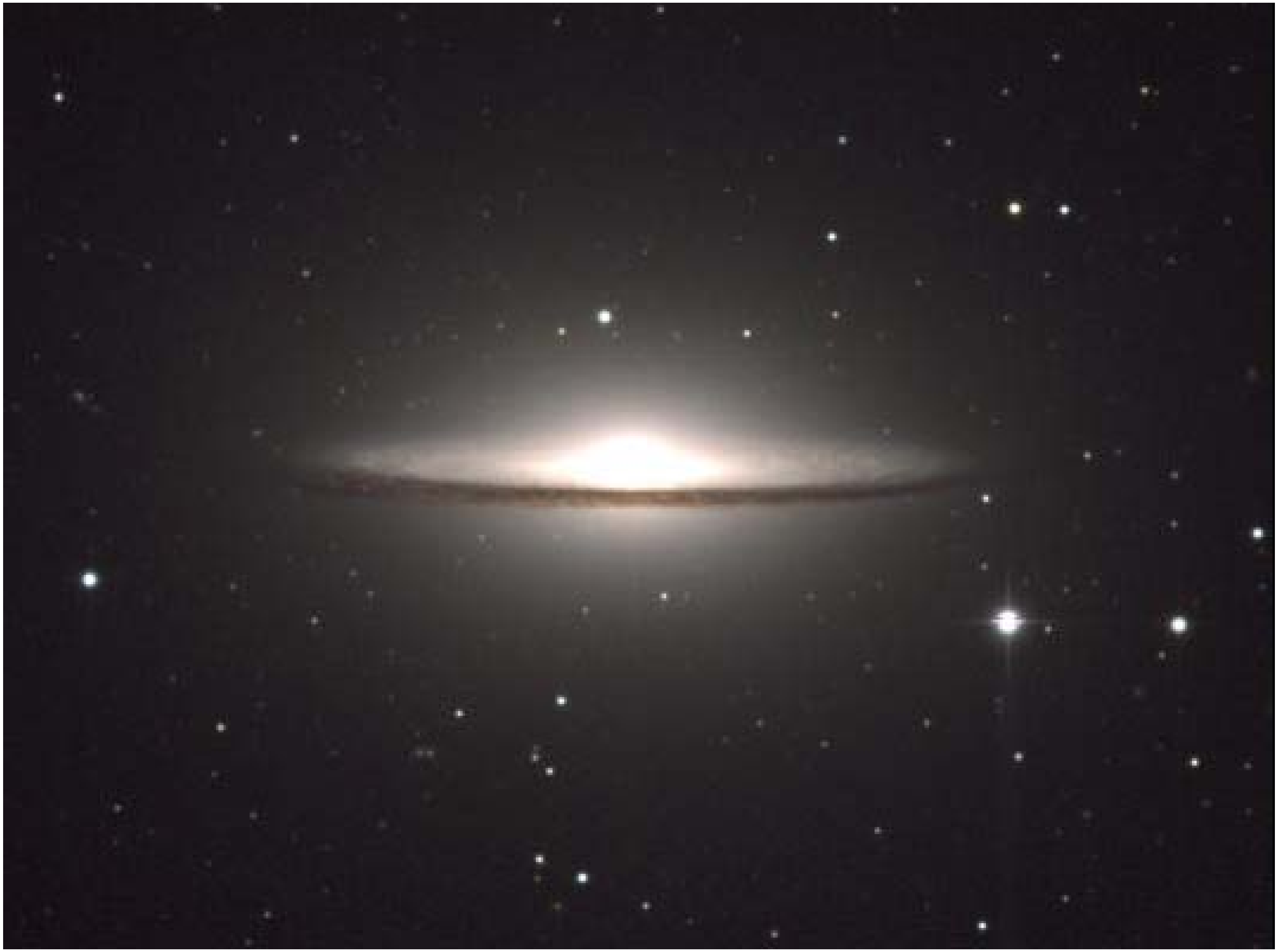
- **The Stellar Populations and Dynamics:**

  Nbody models of globular clusters, The birth and fate of star clusters

Outline

- Introduction to Globular Clusters (GCs) and evidences for dynamical evolution
- Collisional vs. Collisionless systems
- Two-body relaxation and its consequences
- Multiple stellar populations in GCs
- Specific topics in star cluster dynamics:
  - Outer halo GCs of the MW
  - Dissolution Rate of Star Clusters
  - The Metallicity- and Density-Dependent IMF in the Globular Clusters in M31

Introduction to star clusters



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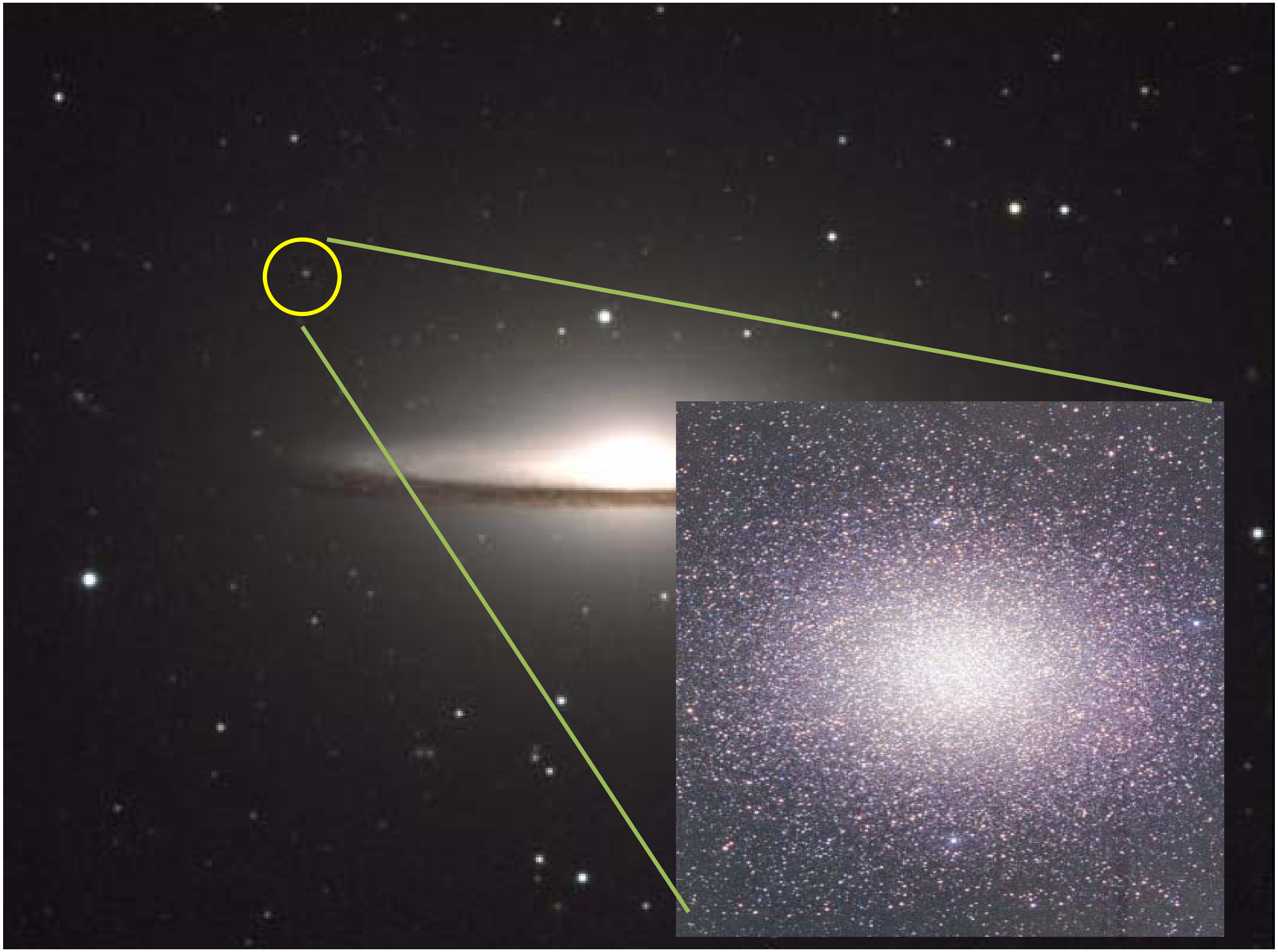
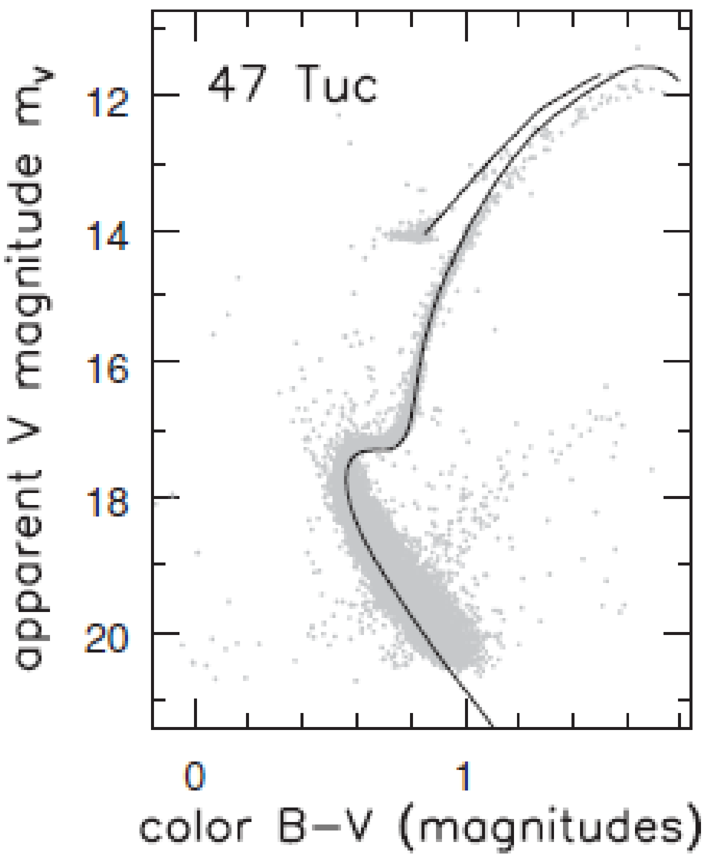
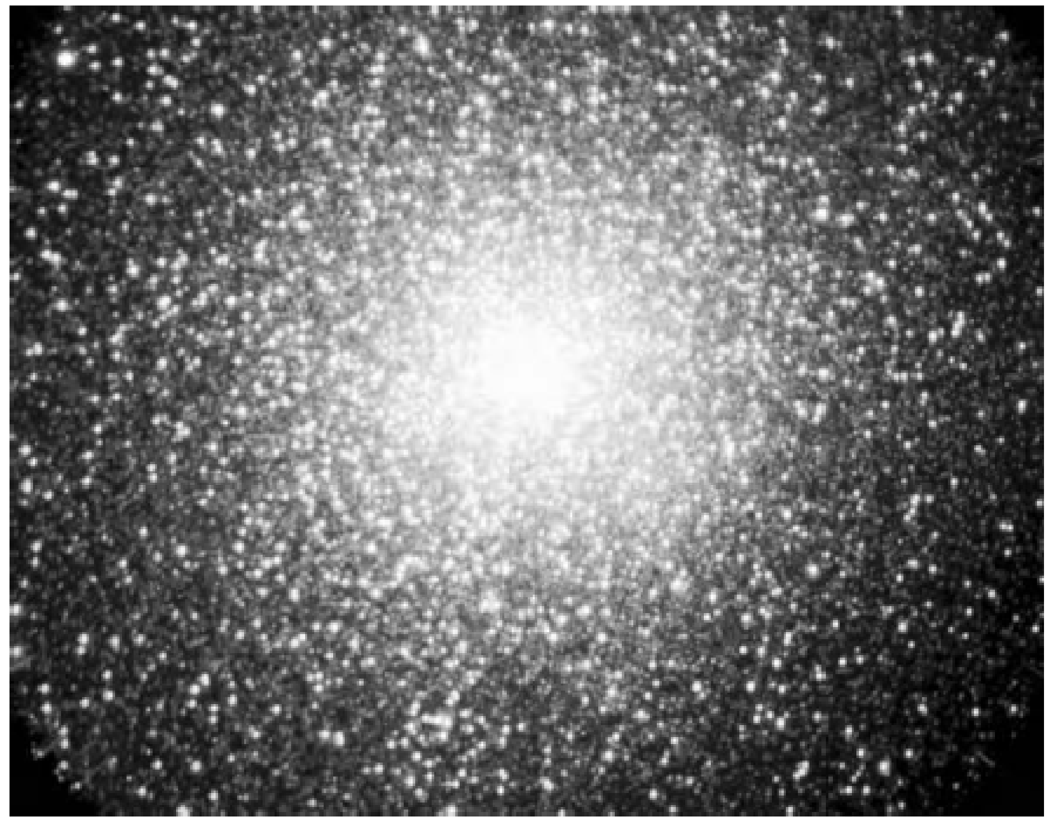
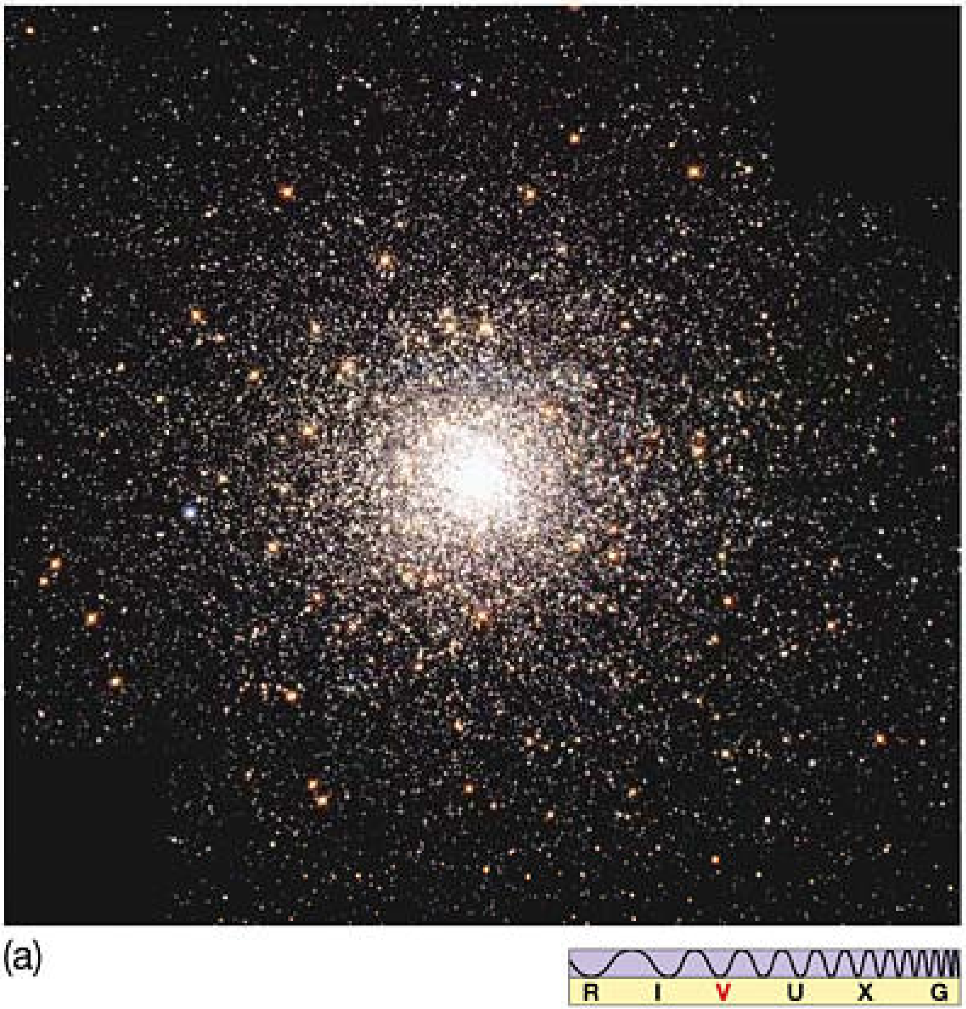


Globular clusters



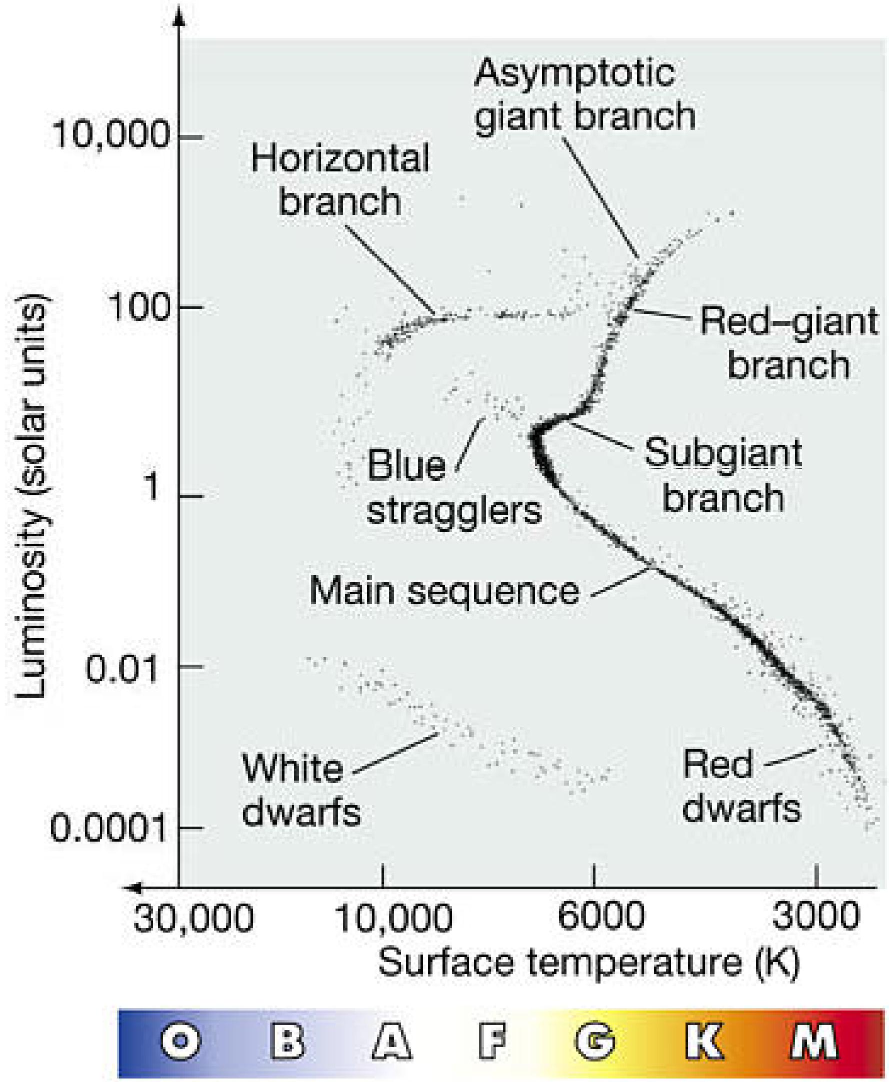


The luminous globular cluster 47 Tucanae – Southern African Large Telescope.



R I V U X G

(a)



Asymptotic giant branch
Horizontal branch
Red–giant branch
Blue stragglers
Subgiant branch
Main sequence
White dwarfs
Red dwarfs
Luminosity (solar units)
10,000
100
1
0.01
0.0001
30,000
10,000
6000
3000
Surface temperature (K)
O B A F G K M
Spectral classification

(b)

Galactic Globular clusters (GCs)

Median mass $\sim 3 \times 10^5 M_\odot$

Median Size: ~ 3 pc

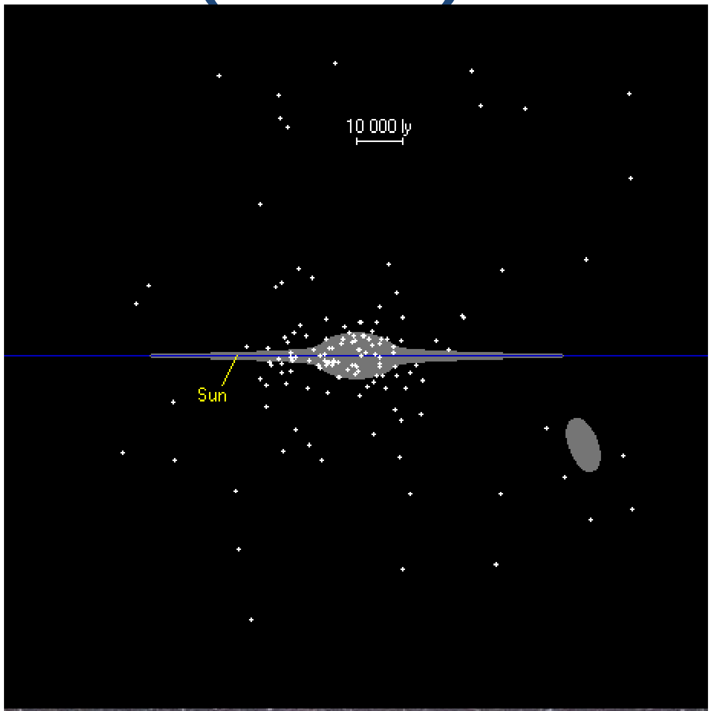
ages ~ 10 – 12 Gyr

✓About 160 GCs are discovered in the Milky Way

✓They are distributed out to more than 100 kpc

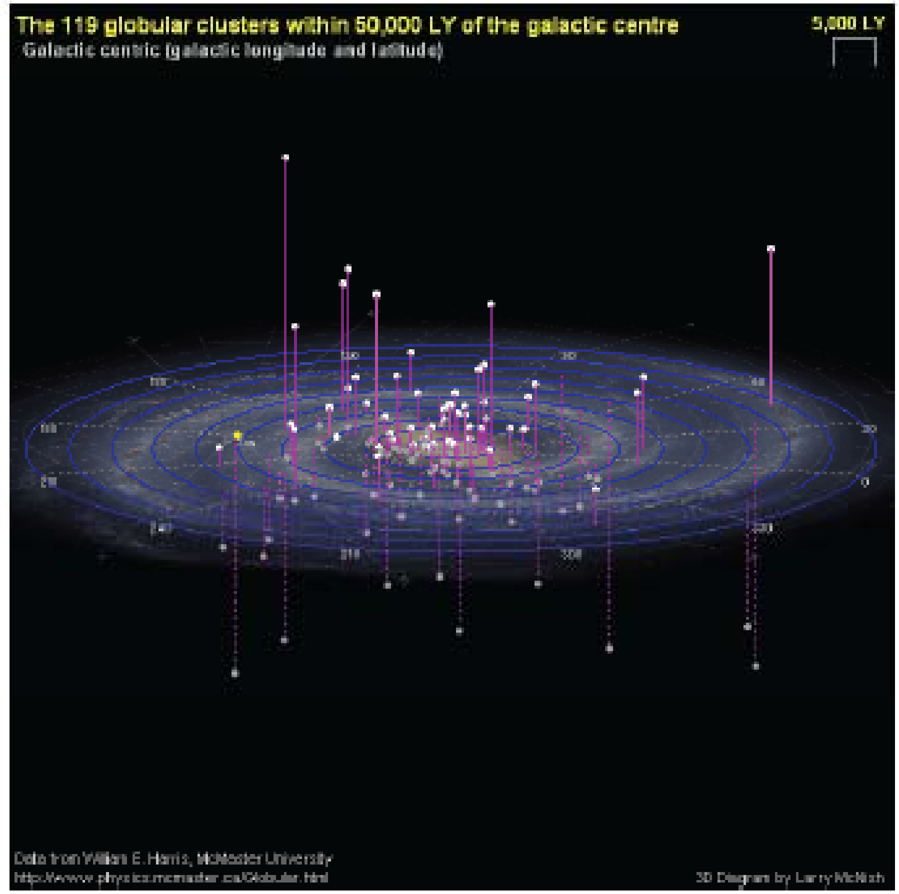
✓Multiple populations

✓Gas/dust free



Star Clusters Are Everywhere

- Spiral Galaxies
  - Halo Clusters
  - Disk Clusters
- Dwarf Galaxies
- Elliptical Galaxies



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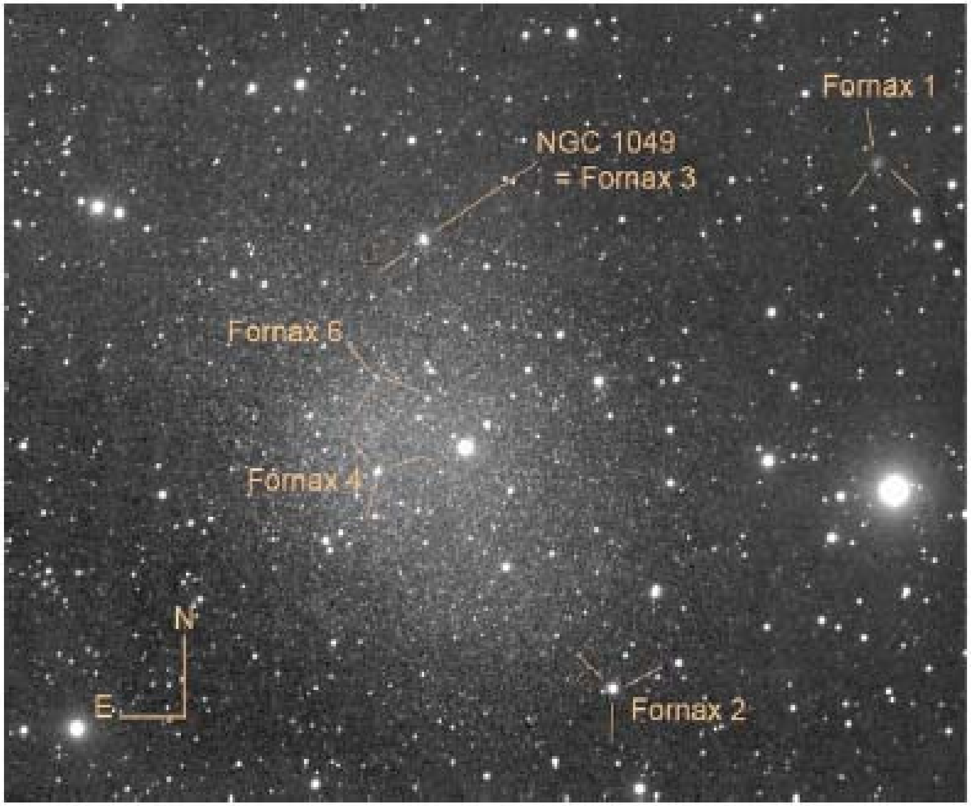

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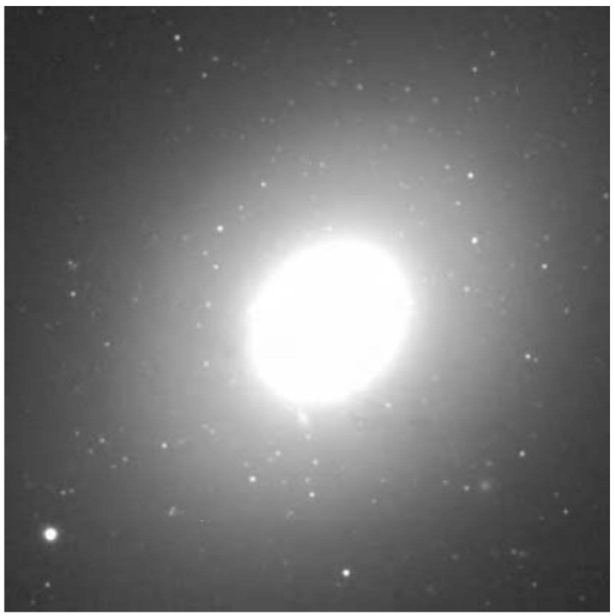
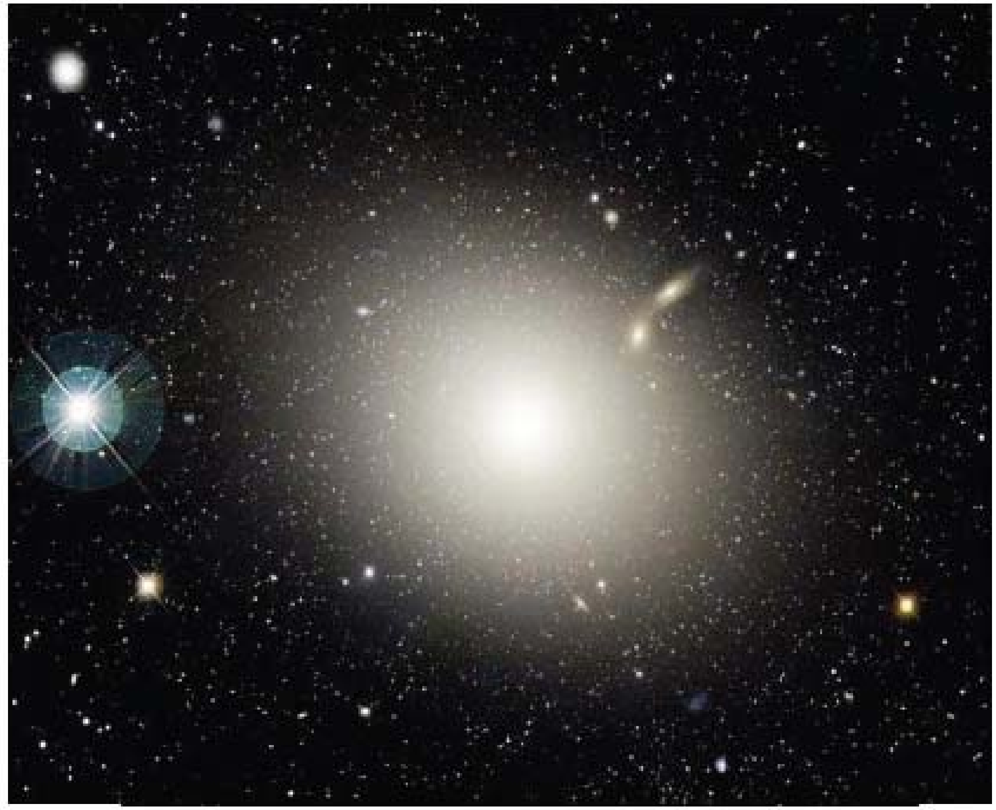
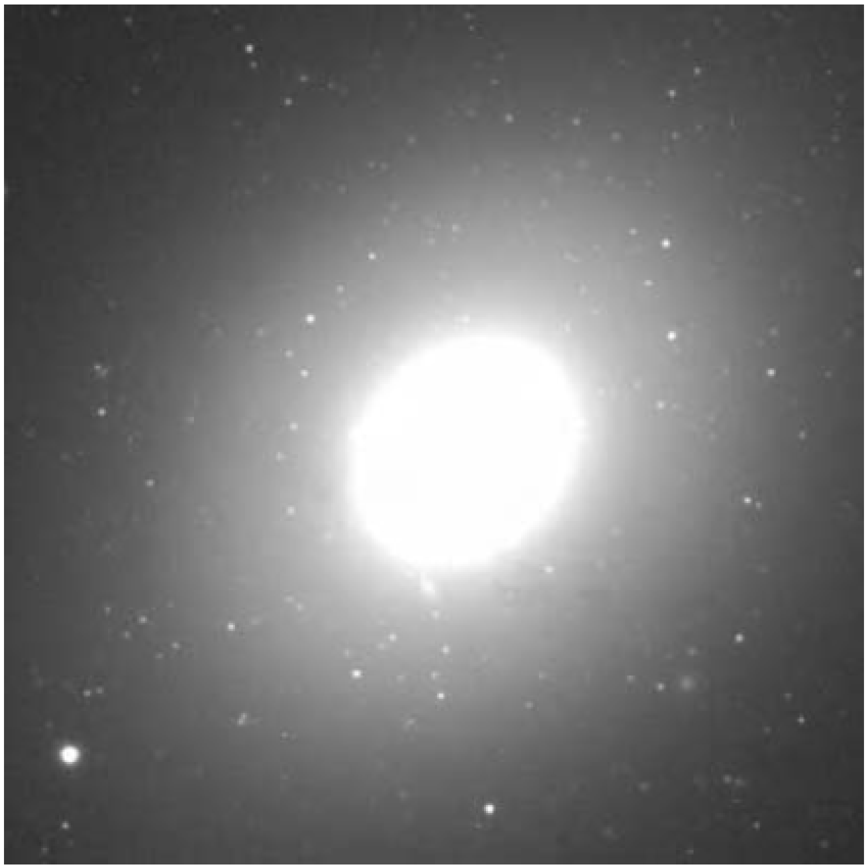




Dwarf vs. Giant Elliptical galaxies



Globular Clusters as tools: They have been used to ….

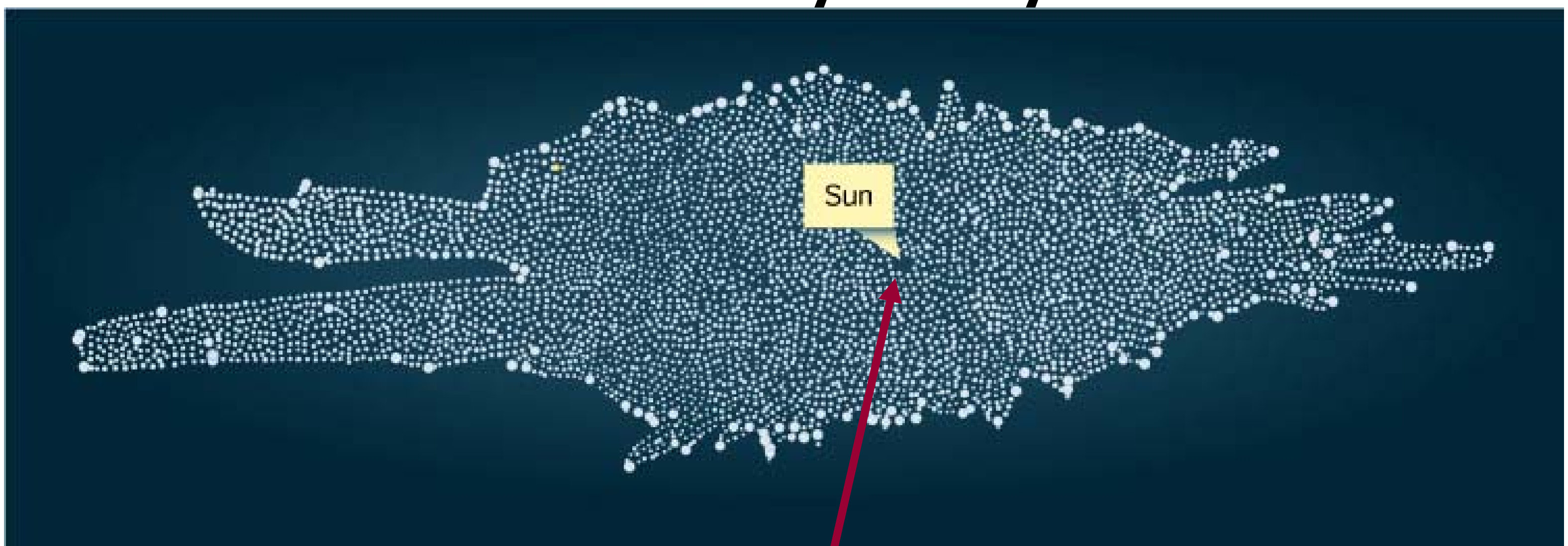
- Determine the structure of the MW (Shapely 1918)

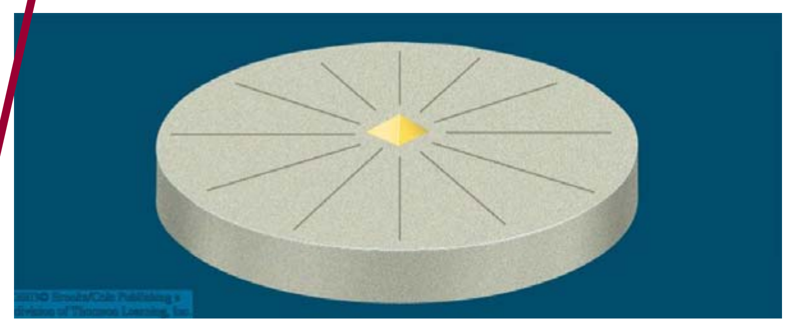
- Understand the star formation and evolution and stellar populations (Eddingtone 1926)

- Constrain the formation and evolution of the MW (Searle & Zinn 1978) as well as other nearby galaxies (Brodie & Strader 2006): GC formation intimately linked with galaxy formation

- A perfect laboratory to explore the effects of 2-body encounters

Historical example: First Studies to Explore the Structure of Our Milky Way



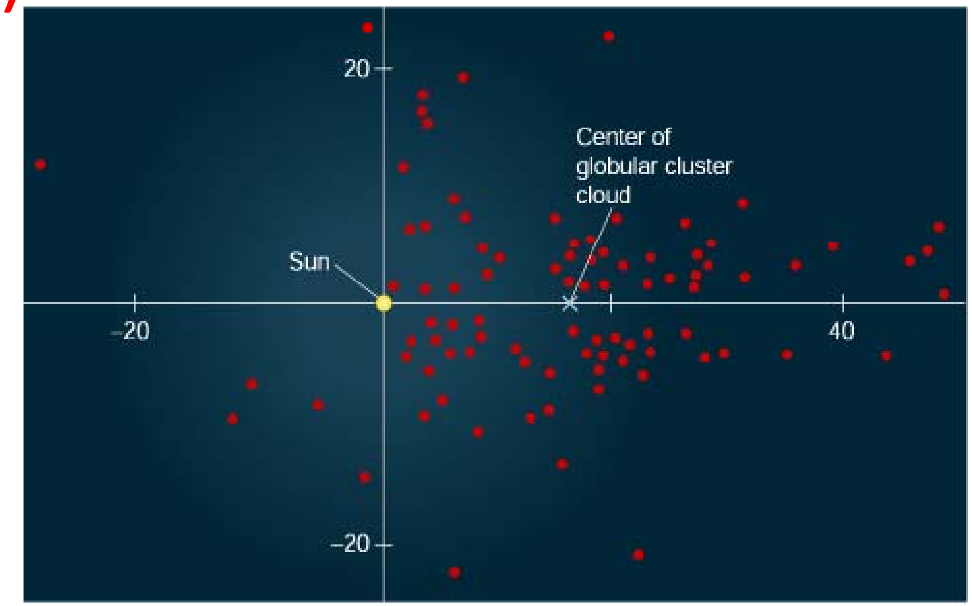


First attempt to unveil the structure of our Galaxy by William Herschel **(1785),** based on optical observations

Two errors:
1. assumed all stars identical so that brightness gave distance
2. did not know about dark clouds and interstellar extinction…

The shape of the Milky Way was believed to resemble a grindstone, with the sun close to the center

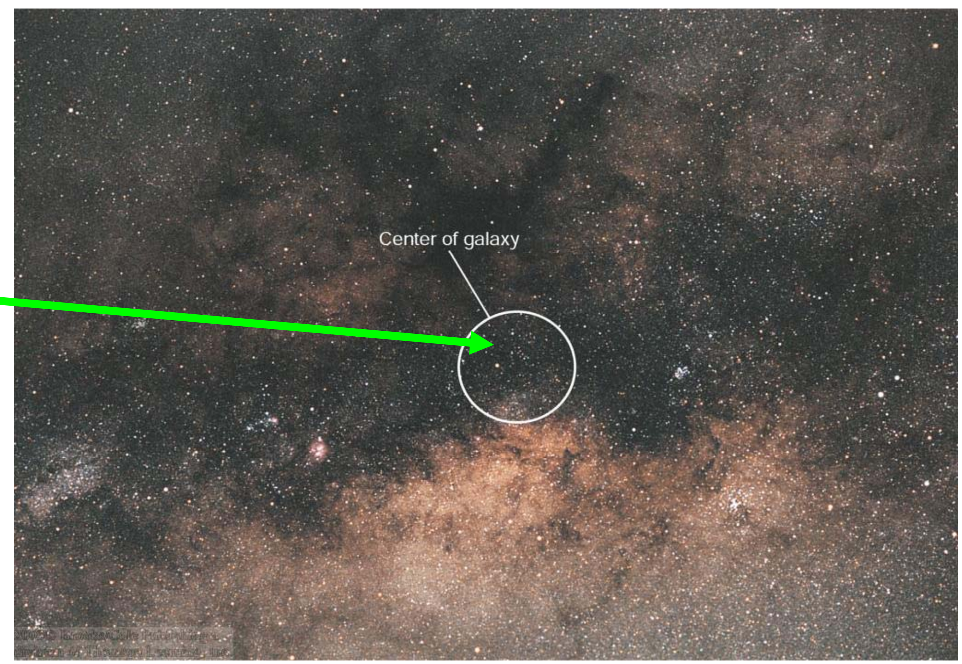
Using GCs to find the location of the Center of the Milky Way (Shapely 1918)



Distribution of globular clusters is ***not*** centered on the sun…

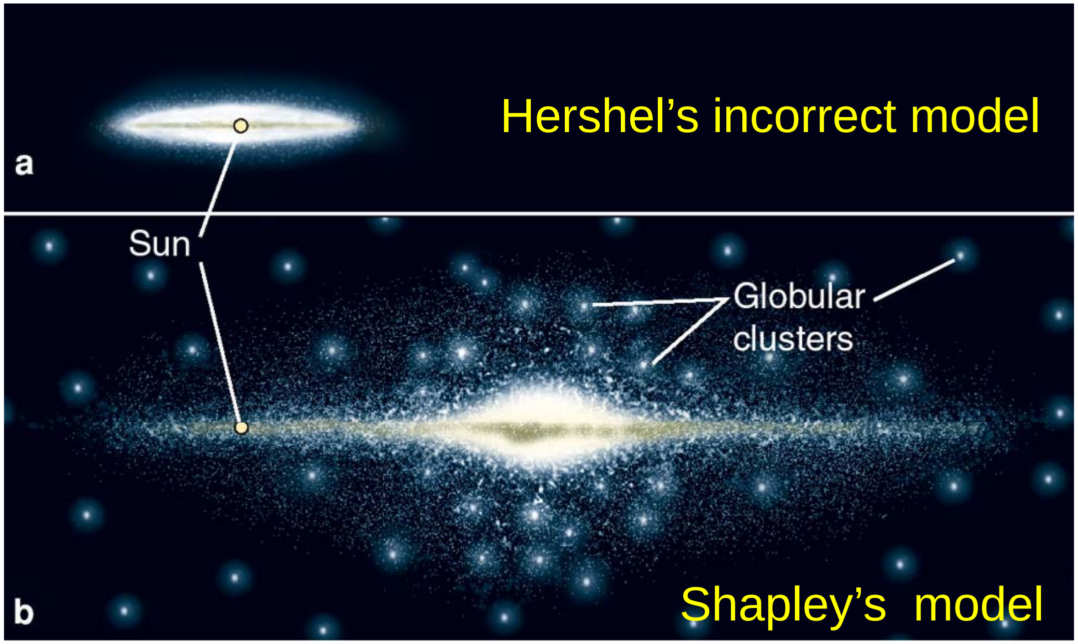
Transition from Heliocentrism to Galactocentrism

…but on a location which is heavily obscured from direct (visual) observation



Shapely-Curtis Debate: Great Debate 26 April 1920

Harlow Shapley's Realization… (1920s)



globular cluster

(has lots of Cepheid variable stars in it!)

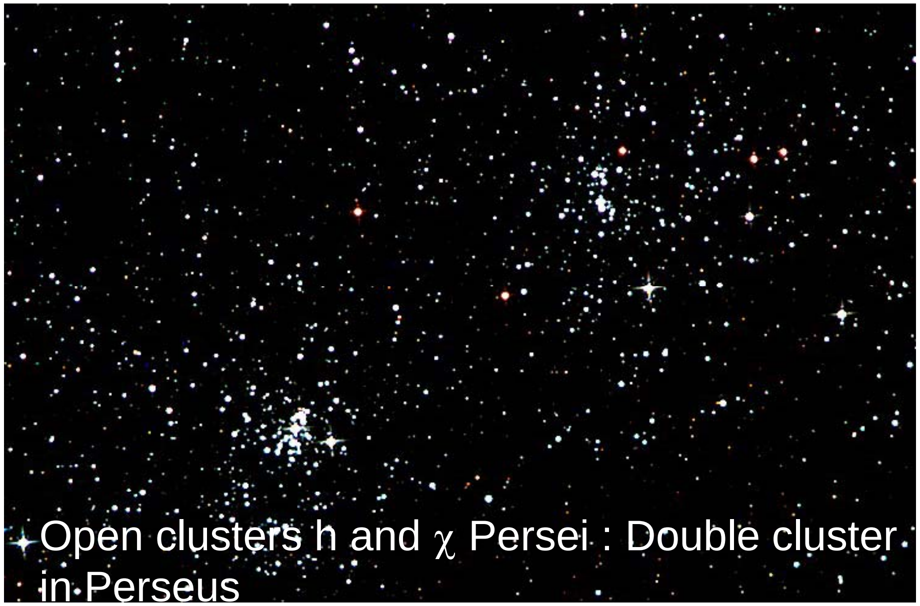
Introduction to Star Clusters

Introduction to Star Clusters: **Two types of star clusters**

Open Clusters:

young clusters of recently formed stars;

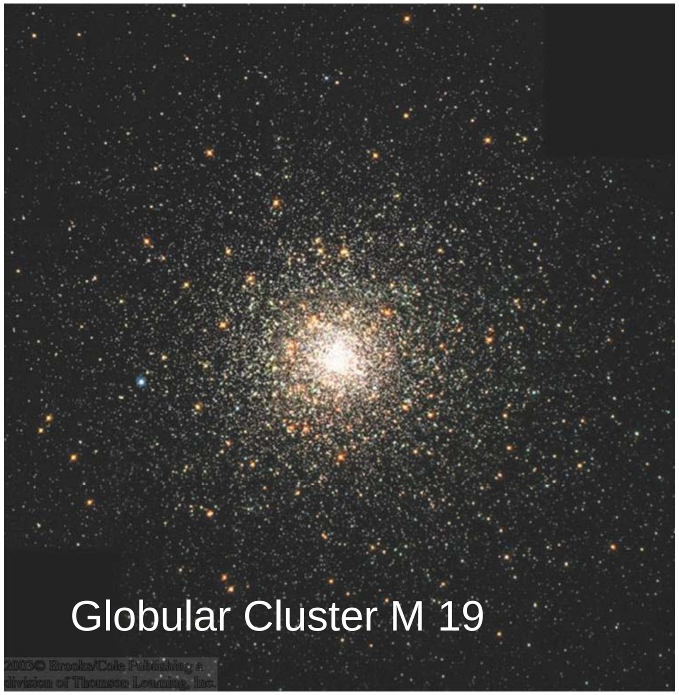
within the disk of the Galaxy



Open clusters h and χ Persei : Double cluster in Perseus

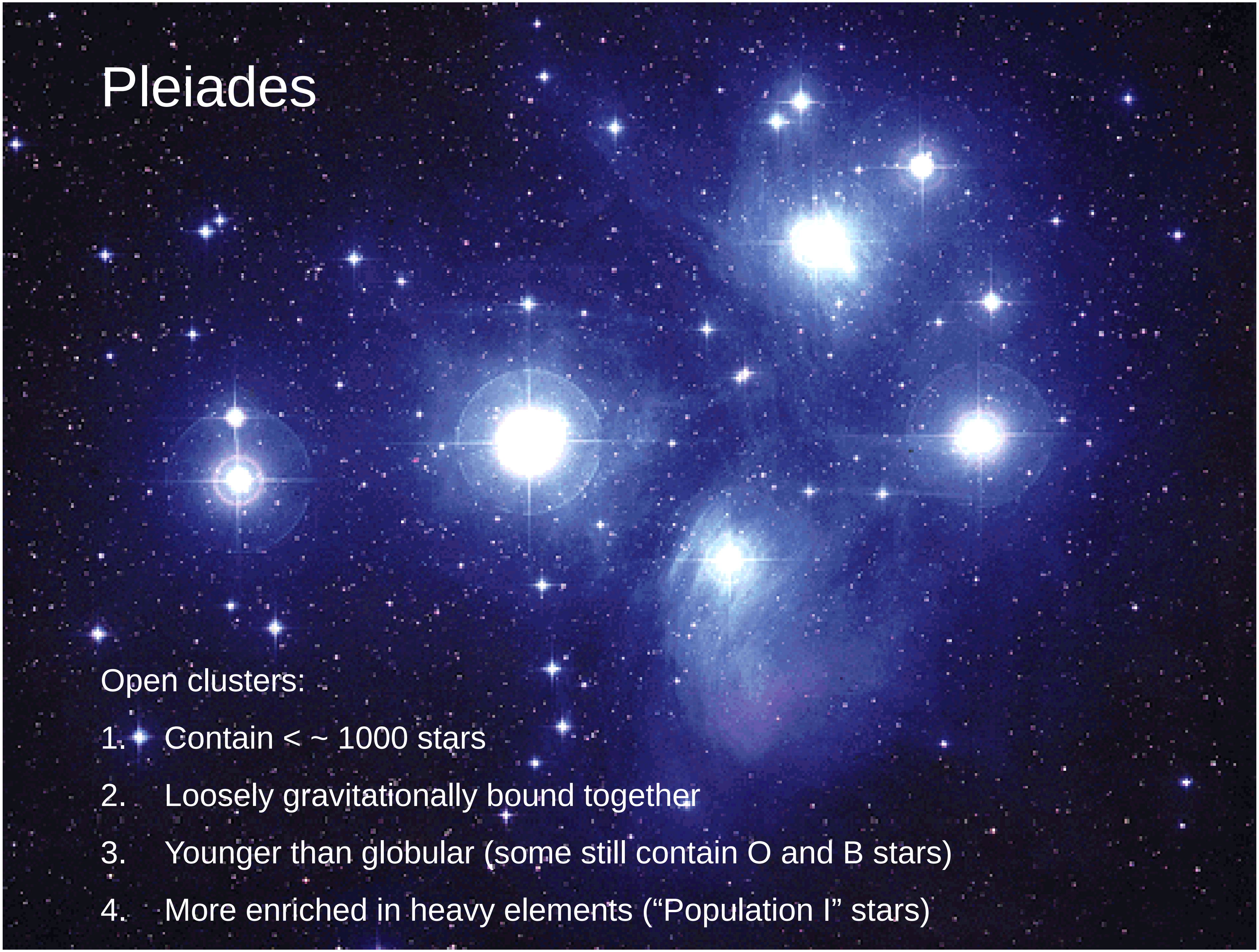
Globular clusters:

old, centrally concentrated clusters of stars; mostly in a halo around the Galaxy



Globular Cluster M 19

Pleiades



Open clusters:

1. Contain < ~ 1000 stars
2. Loosely gravitationally bound together
3. Younger than globular (some still contain O and B stars)
4. More enriched in heavy elements (“Population I” stars)


globular clusters:
1. Contains ~1000-10^6 stars
2. Extremely old: billions of years
3. Population II (low in heavy elements)

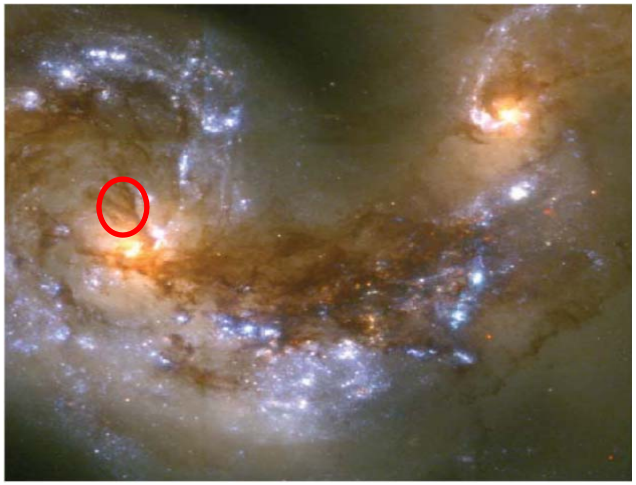
Clusters historically viewed as simple stellar populations

- All stars have the same age (very small spread < 1-2 Myr)
- All stars have the same abundances: “simple stellar populations”.
- Range of stellar masses
- GCs could only form in the special conditions of the early Universe.

Surprise: Young Massive Clusters (YMCs) as the third types of star clusters ?? !!!!!

Omega Cen in the MW:
Mass=few million Msun,
~ 12 Gyr

Antennae Colliding galaxies



Westerlund 1 in the MW

NGC 3603, in the MW

Introduction to Stellar Dynamics

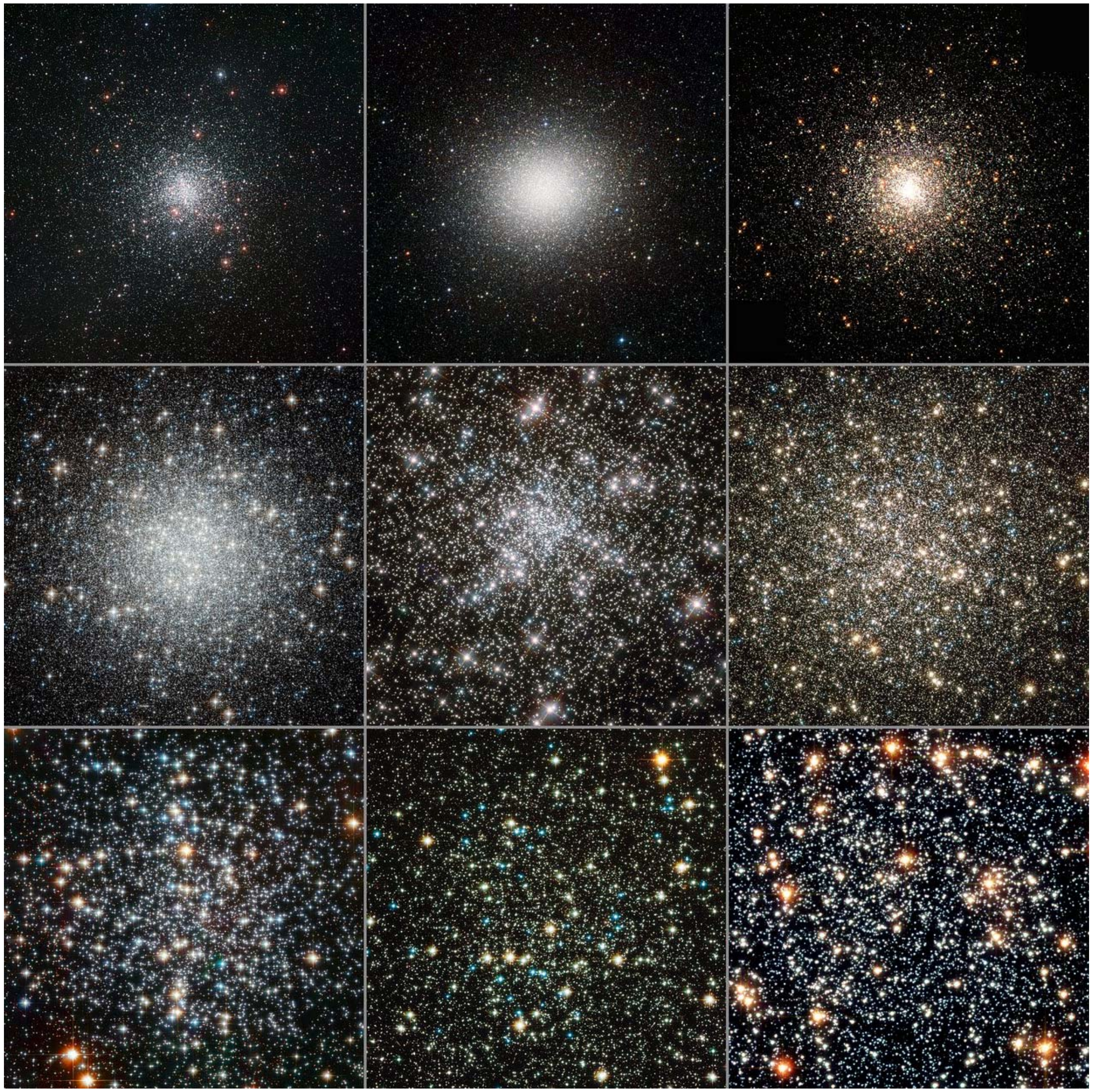
References

1. **Galactic Dynamics**, James Binney and Scott Tremaine, Princeton University Press, 1988, 755 pp.

2. **Dynamical Evolution of Globular Clusters,** Lyman J. Spitzer, Princeton UP, 1988, 196 pp.

3. **The Gravitational Million Body Problem**, Douglas Heggie, Piet Hut; Cambridge UP, 2003

Star Clusters Evolve



Frerra et al 2014

Some examples of Galactic GCs with different dynamical ages.

The length scales of a globular cluster
Core radius

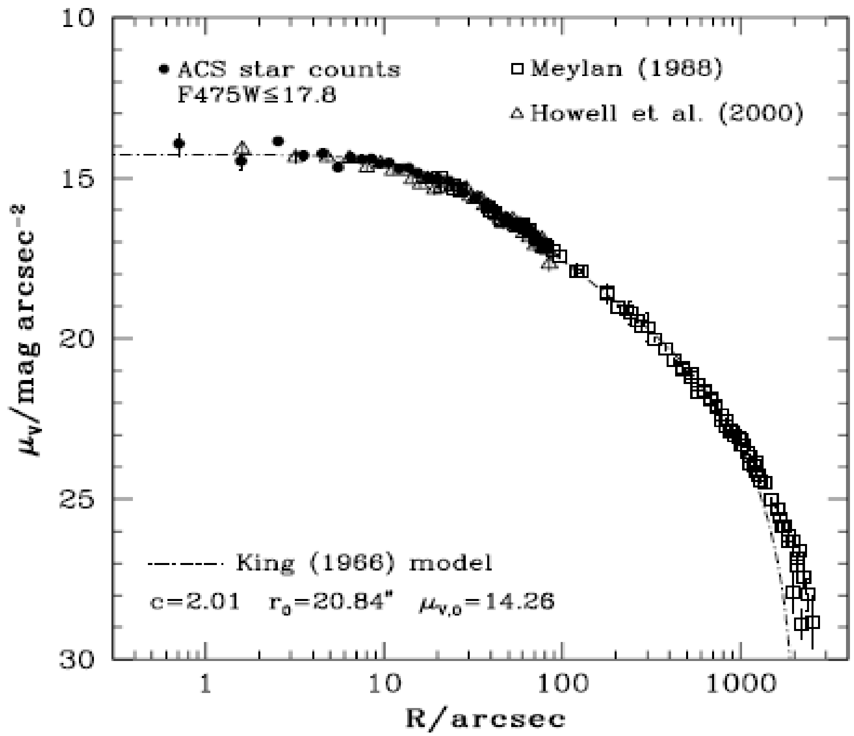
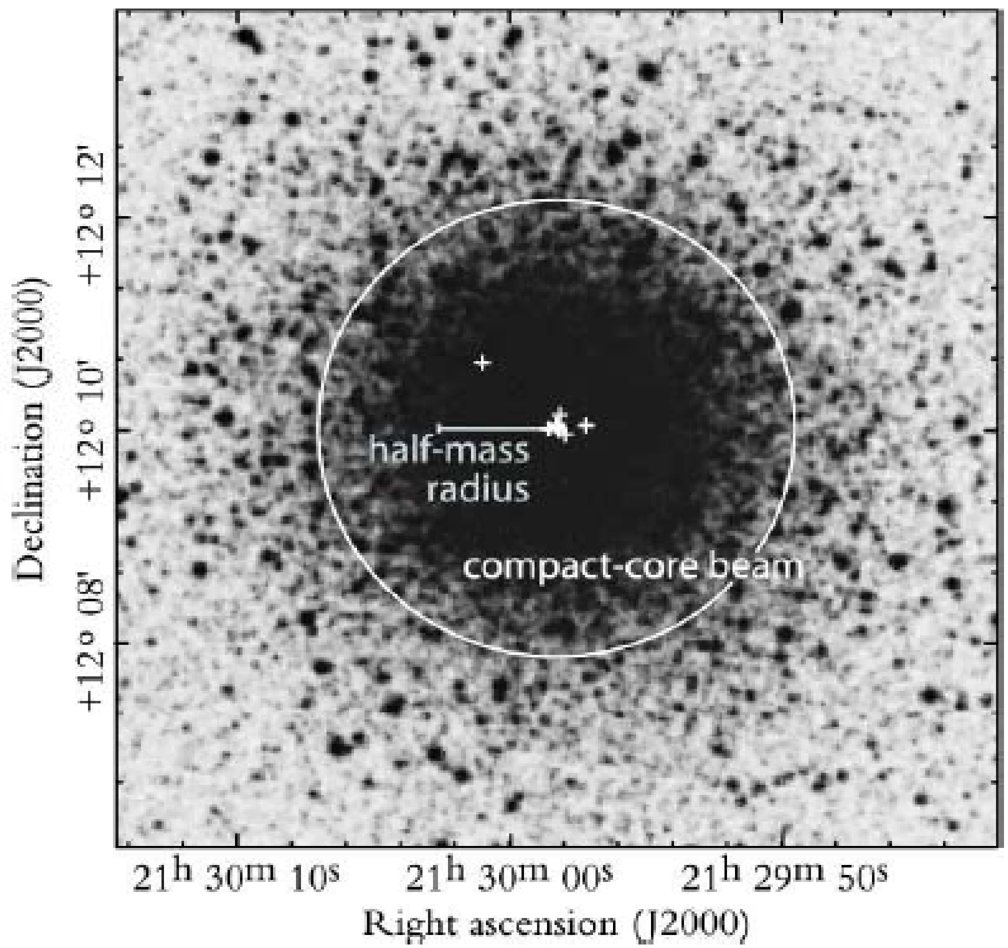


Figure: Surface brightness profile of 47 Tuc (McLaughlin et al, 2006)

- Core radius $r_c = 22''$
- Half-mass radius $r_h = 190''$ (half-light radius)
- Tidal radius $r_t = 2500''$

The half-mass radius

Definition: The radius of a sphere, centred at the centre of the cluster, which encloses half of the mass of the cluster.



Example: M15 (van Leeuwen & Stappers, 2010, A&A, 509, A7)

Dynamic Evolutionary modeling of GCs

- Until the late 1970s, GCs were thought of to be relatively static stellar systems: fitted with equilibrium models like King (1966) profiles. This view has changed significantly over the last thirty years:

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On the observational side:

Strong indications for the ongoing dynamical evolution:

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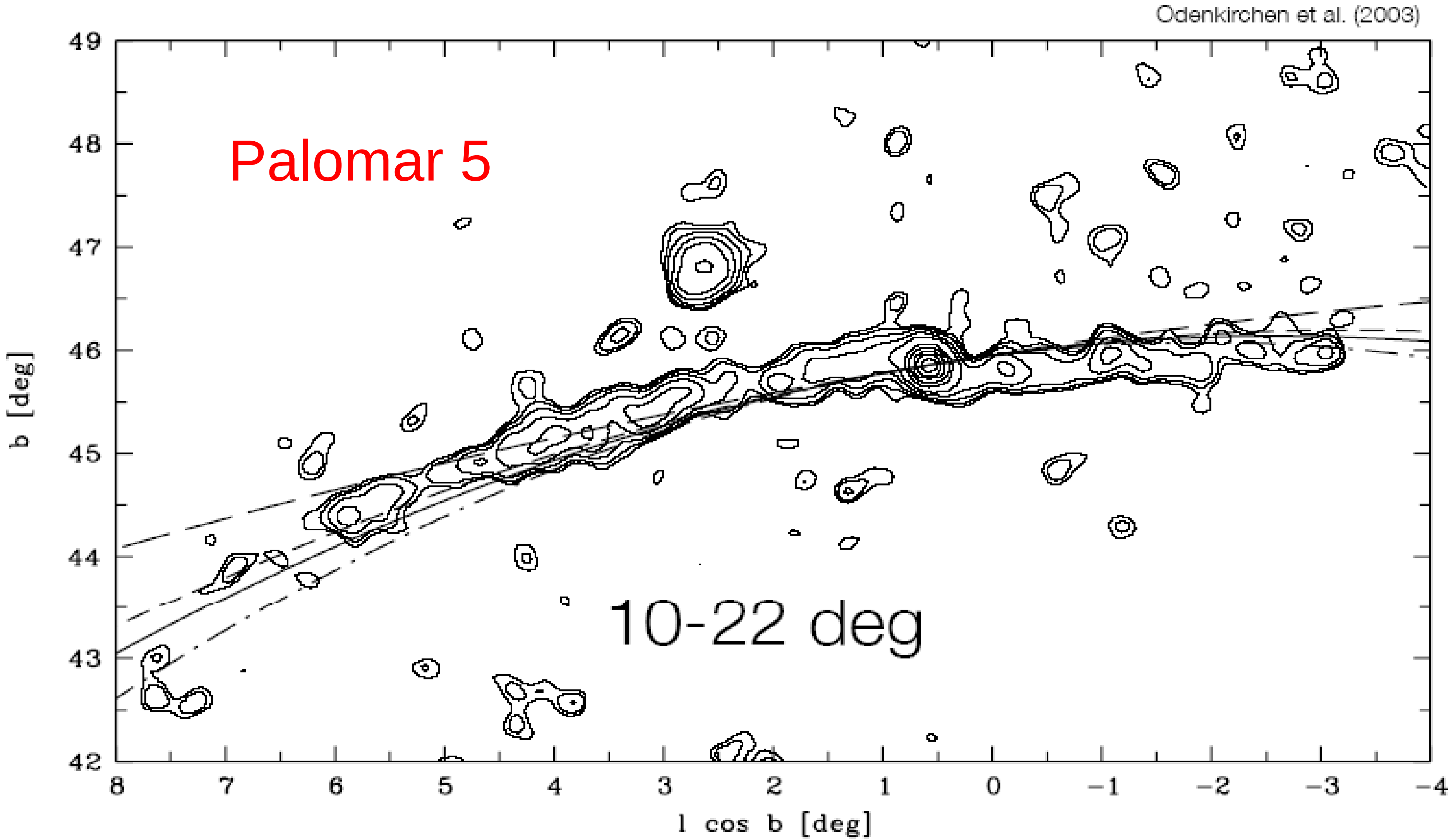
Dynamic Evolutionary modelling of GCs



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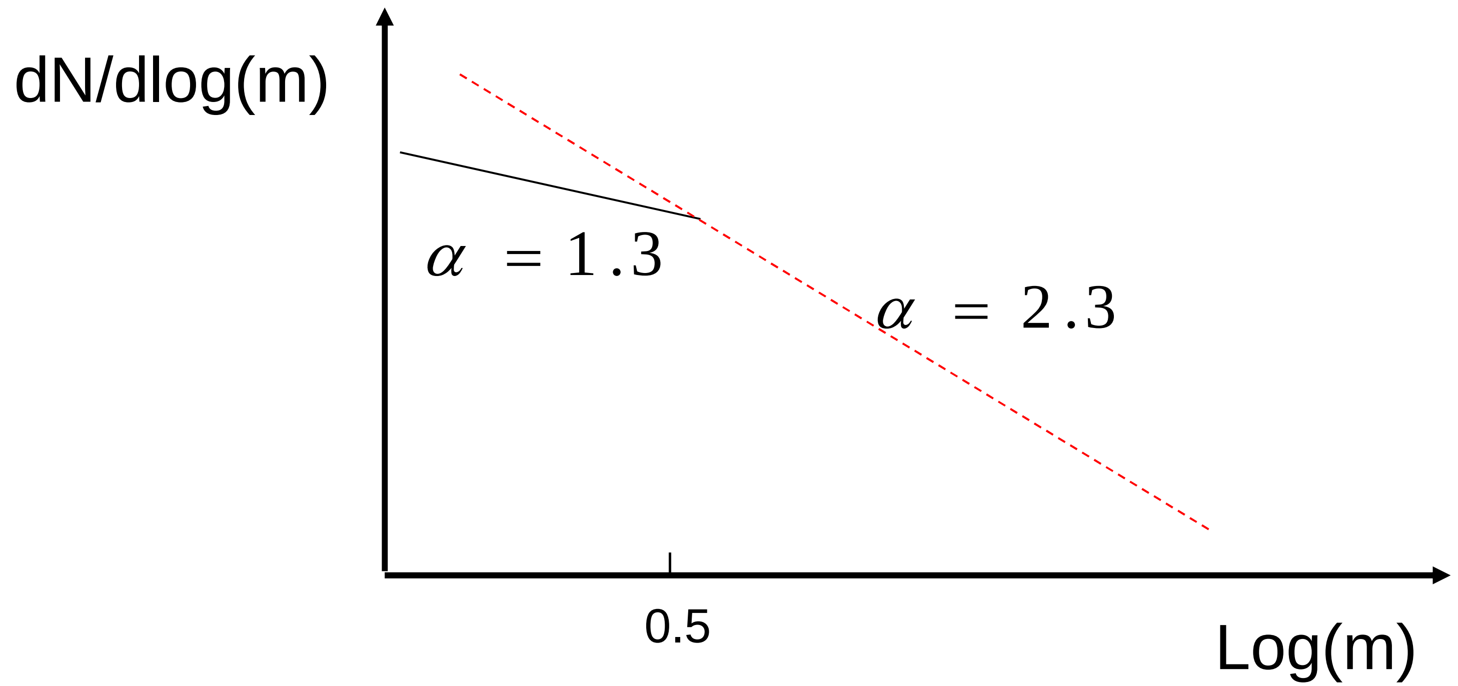
2- The differences in the stellar **mass-functions** of globular clusters (Piotto, Cool & King 1997, de Marchi et al. 1999).

Initial mass function (IMF)

IMF: The initial mass distribution of stars

(Salpeter 1955, Kroupa 2001, 2012)



$$dN = \xi(m)dm$$

IMF: The initial mass distribution of stars

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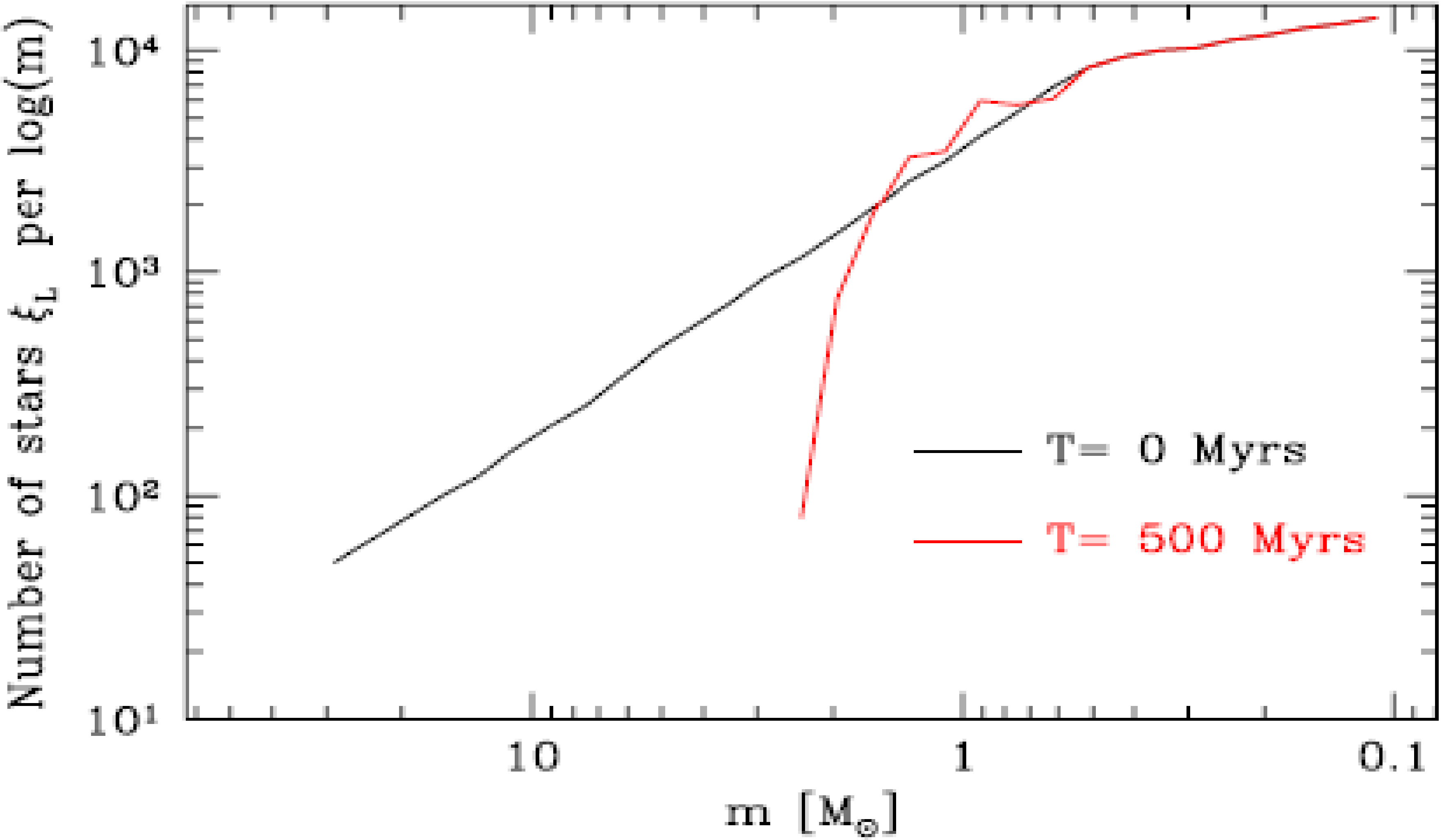
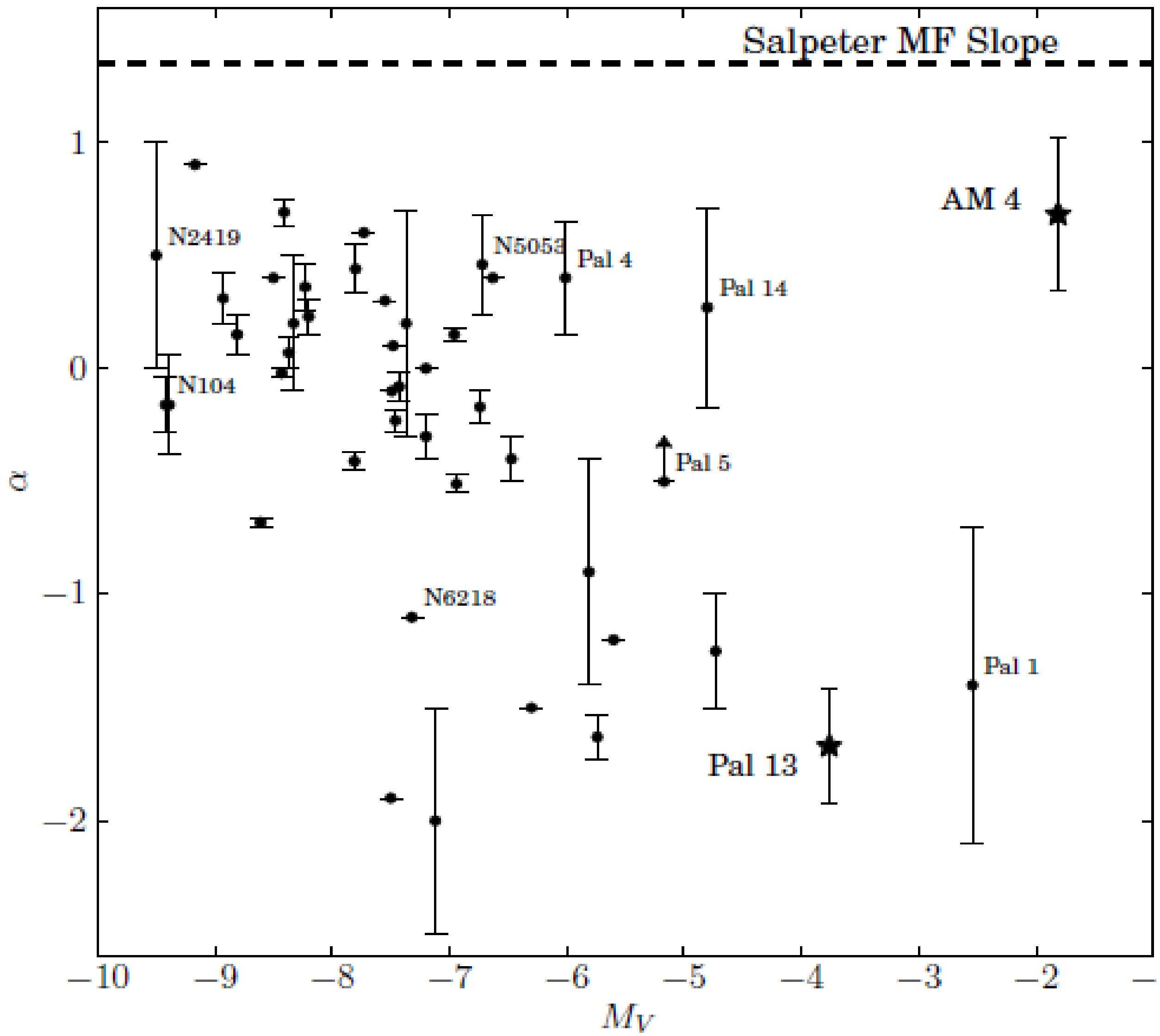


MF changes through long-term evolution
MF-slope is a tracer of mass loss



Hamren et al., 2013, ApJ

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On the theoretical side:

N-body simulations of star cluster evolution :

1- Progresses in simulation **techniques** (e.g. Mikkola & Aarseth 1993, Aarseth 1999) .

2- Development of the **hardware** (GRAPE: Makino et al. 2003, GPUs) which allows to simulate the evolution of star clusters with increasingly larger particle numbers.

Dissolution of star clusters

Mass Loss From Star Clusters

Stellar & Dynamical Evolution

Characteristic parameters of star clusters change with time at early stages and also during the cluster long-term evolution

Vesperini & Heggie 1997; Giersz & Heggie 1996, **Baumgardt & Makino 2003**, Zonoozi et al. 2011, 2014, 2017, Haghi et al. 2015, Bianchini et al. 2017, webb et al 2017 .....,
MC method : Giersz et al. , Rasio et al., ...

Internal and External Mechanisms:

- Stellar evolution

- Two body (collisional) relaxation: energy equipartition and mass segregation, binary heating, 3 and 4-body encounters, core evolution

- Violent relaxation: Tidal interactions, dynamical friction, bulge/disk shocking, tidal stripping

Two-body relaxation and its consequences:

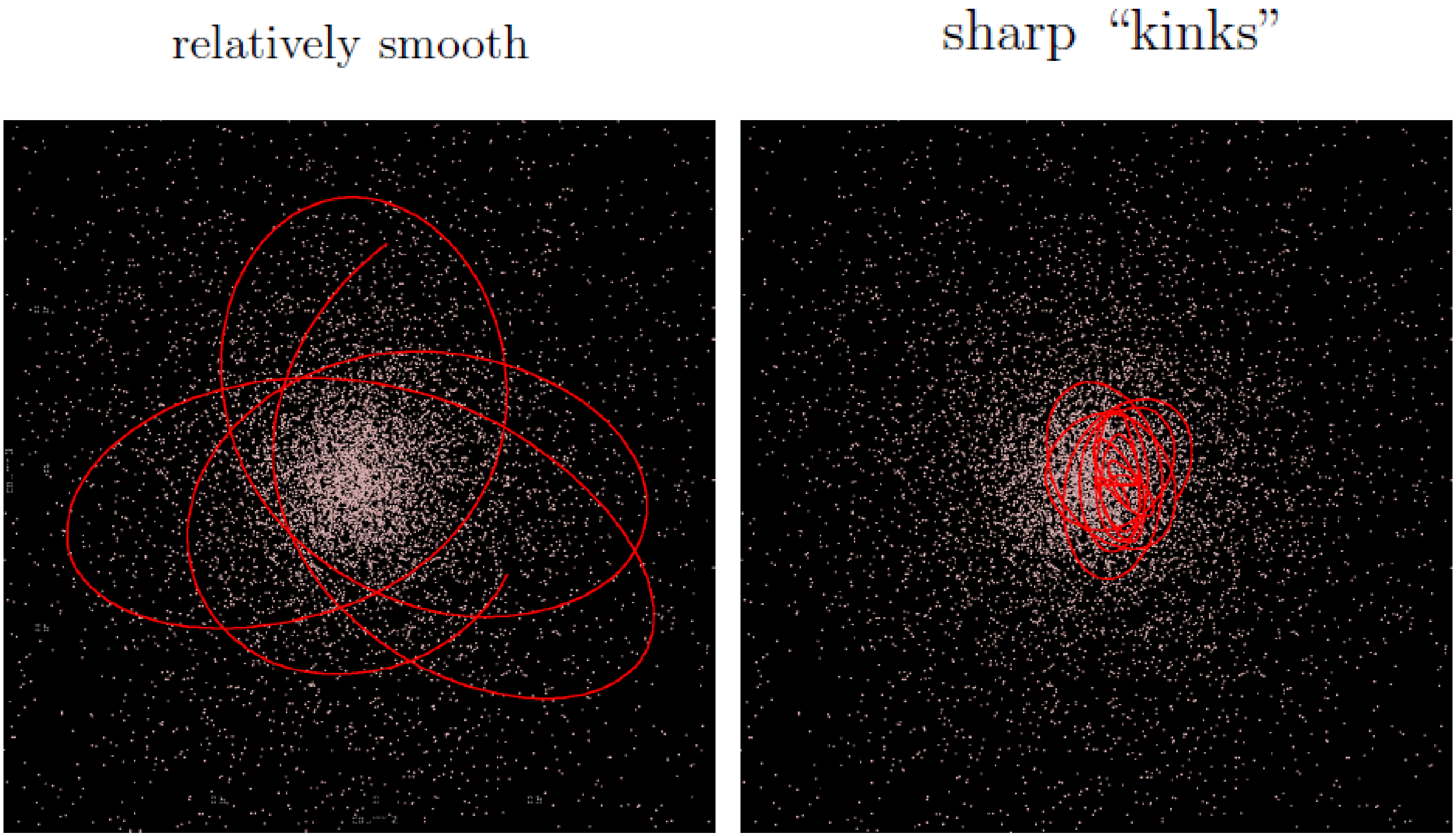
- Escape and Evaporation
- Mass segregation, energy equipartion
- Core collapse and Binary formation
- Gravothermal instability: Gravothermal oscillations
- External tidal perturbations

Additional astrophysical processes

- Primordial gas loss
- Stellar & Binary evolution

2-body Relaxation

- In a star cluster, thermal evolution is driven by two-body relaxation.



2-body Relaxation

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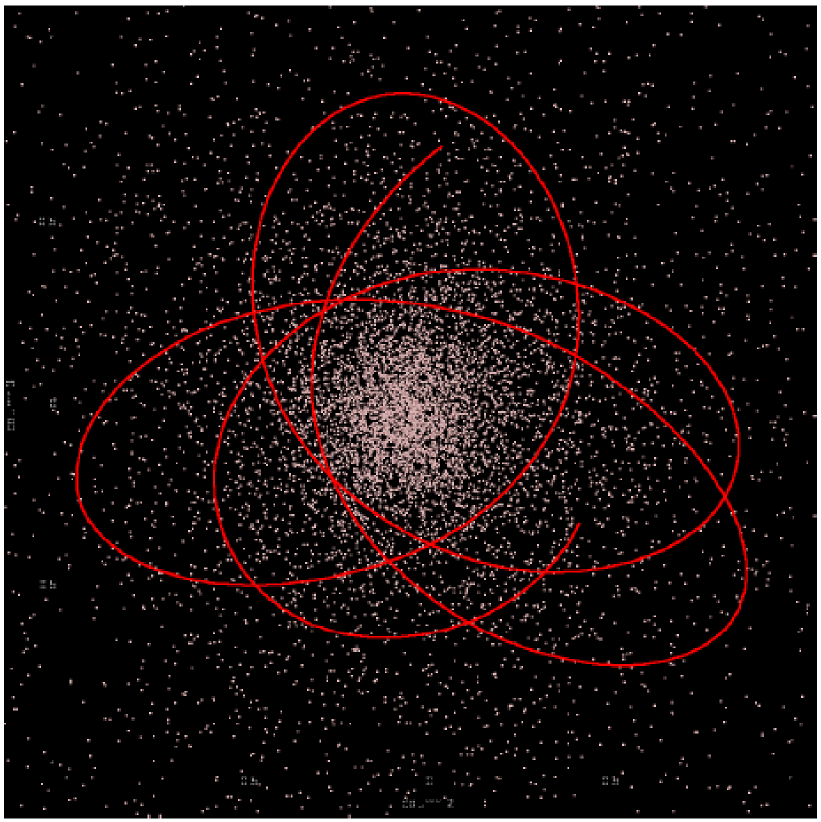
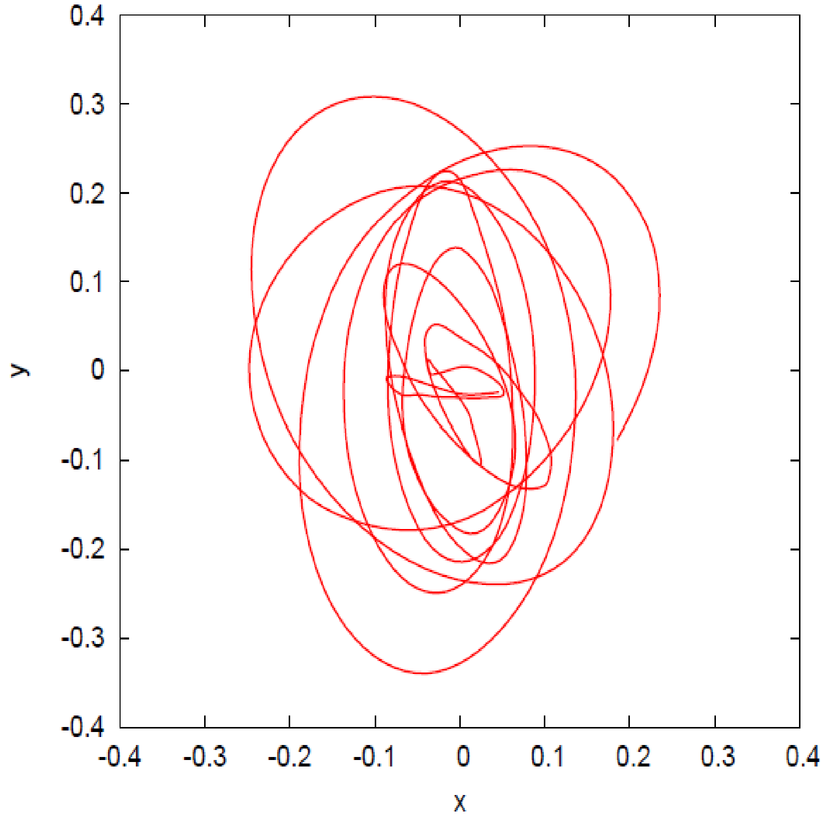
relatively smooth



sharp "kinks"



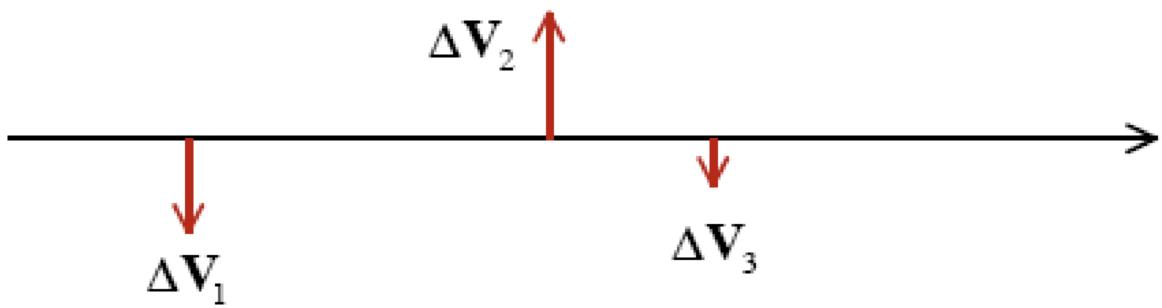
Strong encounters

$$\frac{Gm^2}{r} \gtrsim \frac{mV^2}{2}, \quad \text{which means } r \lesssim r_s \equiv \frac{2Gm}{V^2}$$

$$t_s = \frac{V^3}{4\pi G^2 m^2 n} \approx 4 \times 10^{12}\,\text{yr} \left(\frac{V}{10\,\text{km s}^{-1}}\right)^3 \left(\frac{m}{\mathcal{M}_\odot}\right)^{-2} \left(\frac{n}{1\,\text{pc}^{-3}}\right)^{-1}$$

$n \approx 0.1\,\text{pc}^{-3}$ for stars near the Sun; so $t_s \sim 10^{15}$ years

Relaxation of Stellar Systems : Weak encounters



If the star receives many independent deflections, each with a random direction, expected value of the perpendicular velocity after time t is obtained by summing the *squares* of the individual velocity kicks:

$$\left\langle \Delta V_{\perp}^{2} \right\rangle = \Delta V_{1}^{2} + \Delta V_{2}^{2} + \Delta V_{3}^{2} + \ldots$$

Writing this as an integral (i.e. assuming that there are very many kicks):

$$\left\langle \Delta V_{\perp}^{2} \right\rangle = \int_{b_{\min}}^{b_{\max}} \left(\frac{2Gm}{bV} \right)^{2} dN$$

Where dN is the expected number of encounters that occur in time t between impact parameter b and b + db.

Relaxation of Stellar Systems : Weak encounters

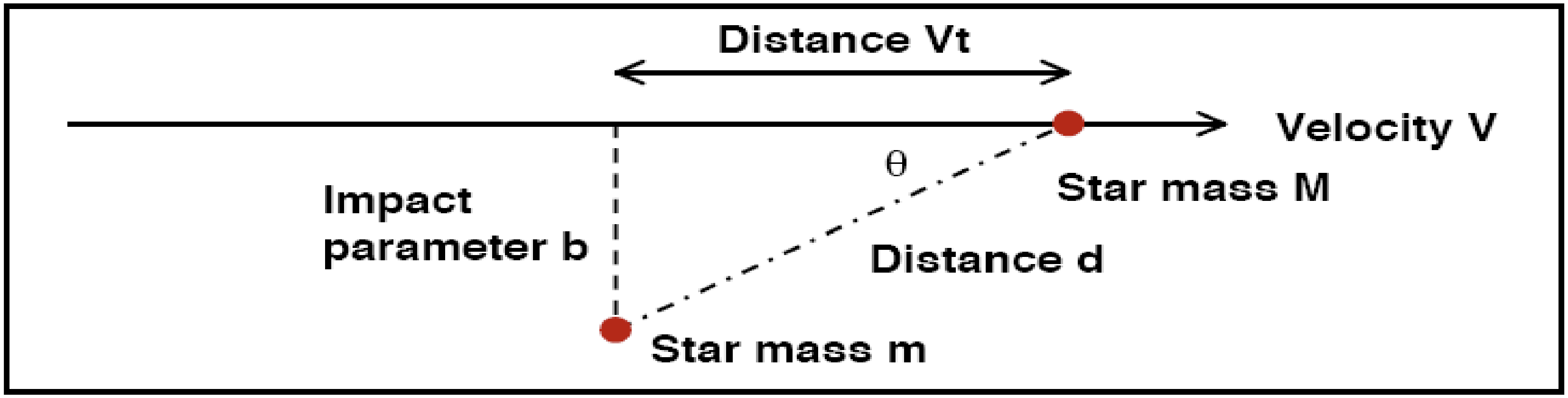
$$t_{\rm relax} \approx \frac{2\times 10^9\ {\rm yr}}{\ln\Lambda}\left(\frac{V}{10\,{\rm km\,s^{-1}}}\right)^3\left(\frac{m}{\mathcal{M}_\odot}\right)^{-2}\left(\frac{n}{10^3\,{\rm pc^{-3}}}\right)^{-1}$$

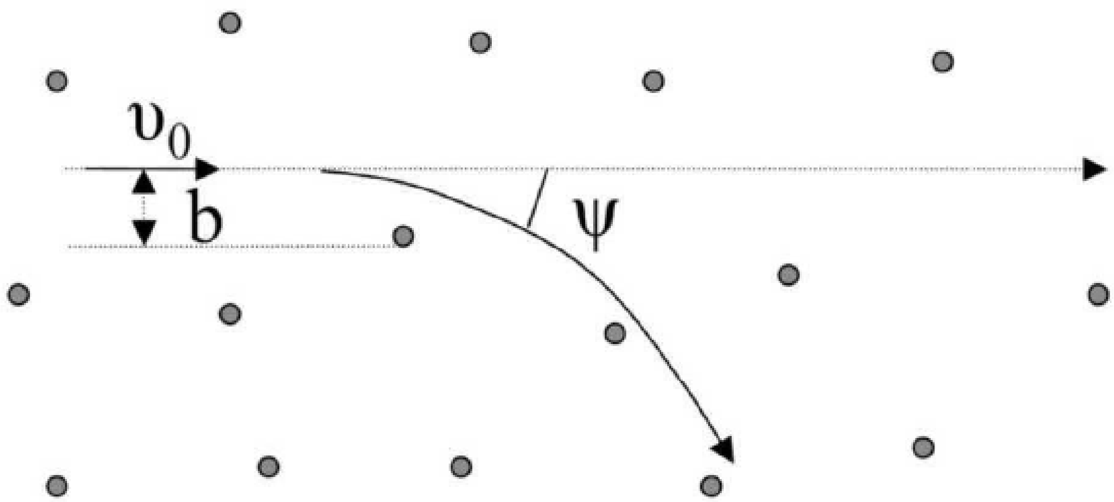
$$\frac{1}{2}NmV^2 \sim \frac{G(Nm)^2}{2R}, \qquad \frac{t_{\rm relax}}{t_{\rm cross}} \sim \frac{V^4R^2}{6NG^2m^2\ln\Lambda} \sim \frac{N}{6\ln(N/2)}$$

Examples:

	N	R	v	τ_{cross}	τ_{relax}	age/τ_{relax}
open cluster	100	2 pc	0.5 km/s	$4 \cdot 10^6$ yrs	10^7 yrs	≥ 1
globular cluster	10^5	4 pc	10 km/s	$4 \cdot 10^5$ yrs	$4 \cdot 10^8$ yrs	≥ 10
ellipt. galaxy	10^{12}	10 kpc	600 km/s	$2 \cdot 10^7$ yrs	10^{17} yrs	10^{-7}
dwarf galaxy	10^9	1 kpc	50 km/s	$2 \cdot 10^7$ yrs	10^{14} yrs	10^{-4}
galaxy cluster	1000	1 Mpc	1000 km/s	10^9 yrs	$2 \cdot 10^{10}$ yrs	10^{-1}

Relaxation of Stellar Systems : Weak encounters





$$t_{\rm relax} \approx \frac{2\times 10^9\ {\rm yr}}{\ln \Lambda}\left(\frac{V}{10\ {\rm km\,s^{-1}}}\right)^3\left(\frac{m}{\mathcal{M}_\odot}\right)^{-2}\left(\frac{n}{10^3\ {\rm pc^{-3}}}\right)^{-1}$$

GCs are collisional systems

2-body interactions of stars are important in driving the dynamical evolution

Galaxies that are collisionless

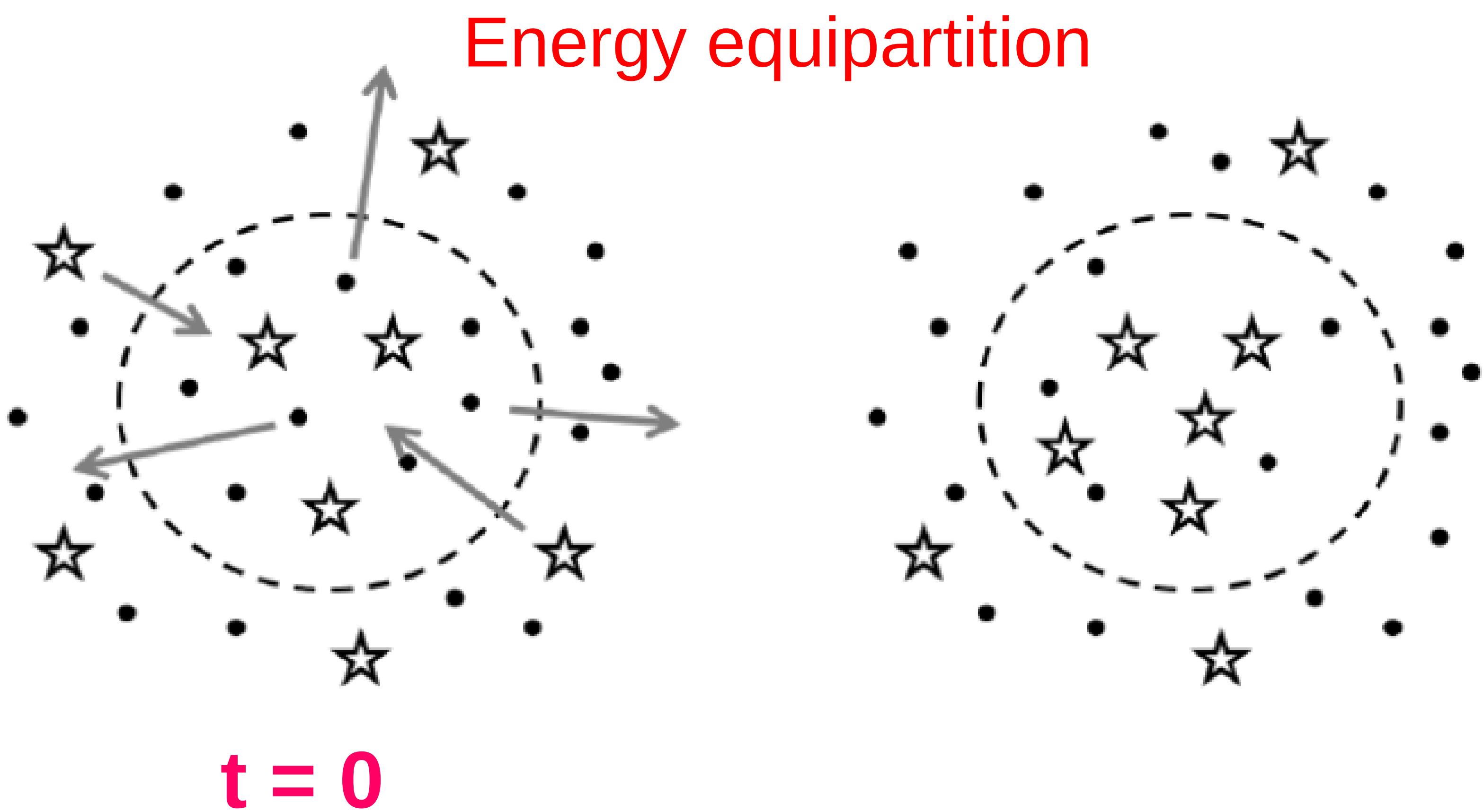
stars are mainly moving in the collective gravitational field

Significance of the relaxation time scale (T_r)

$$\mathbf{T_r \sim T_{cross} \,.\, N/\log(N)}$$

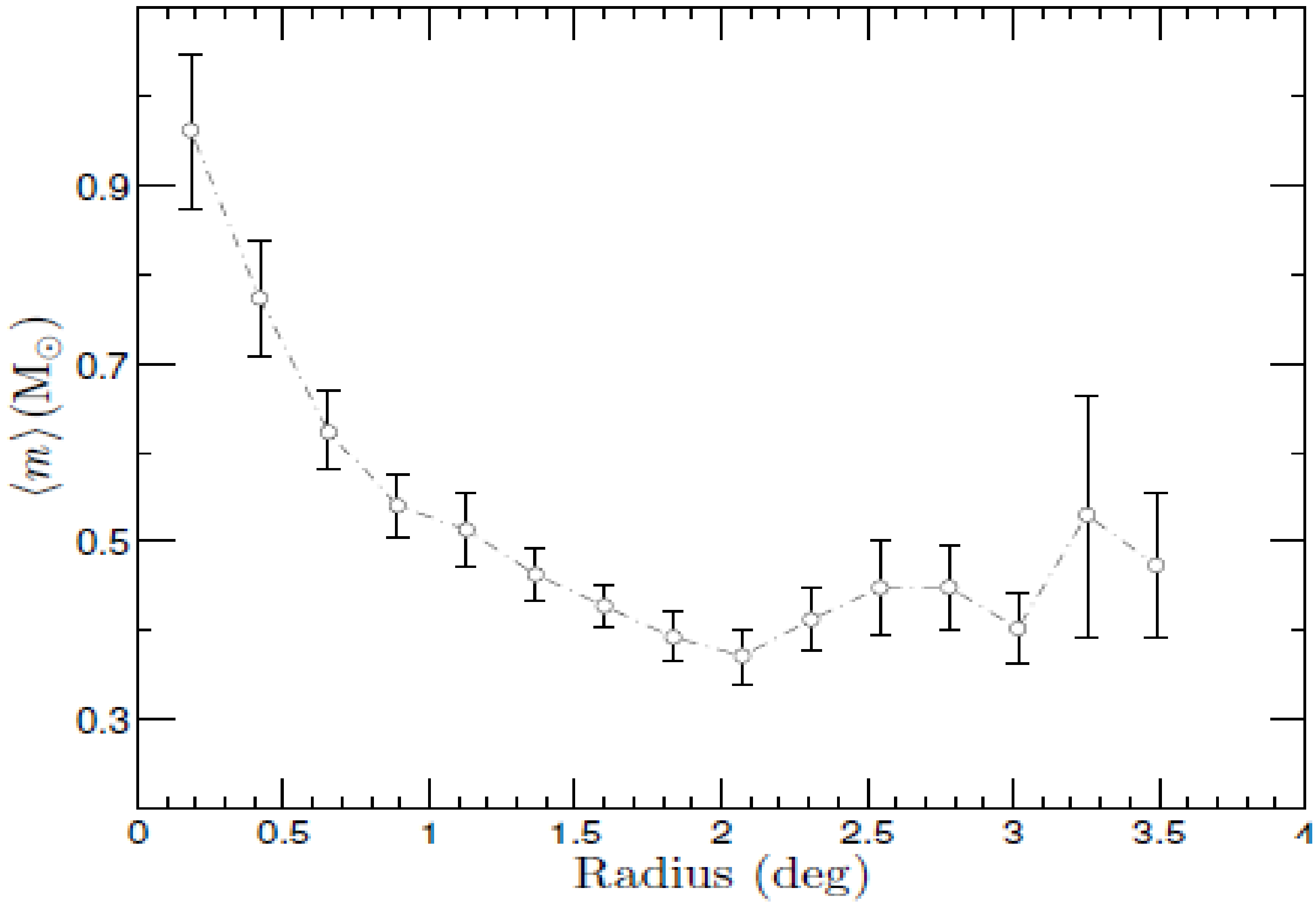
- Time scale of evolution of a virialised cluster
- Time scale of core collapse (~ T_r)
- Time scale of mass segregation (several T_r),
- Time scale of escape /evaporation (~100 T_r)

Consequences of 2-body relaxation: Dynamical Mass segregation

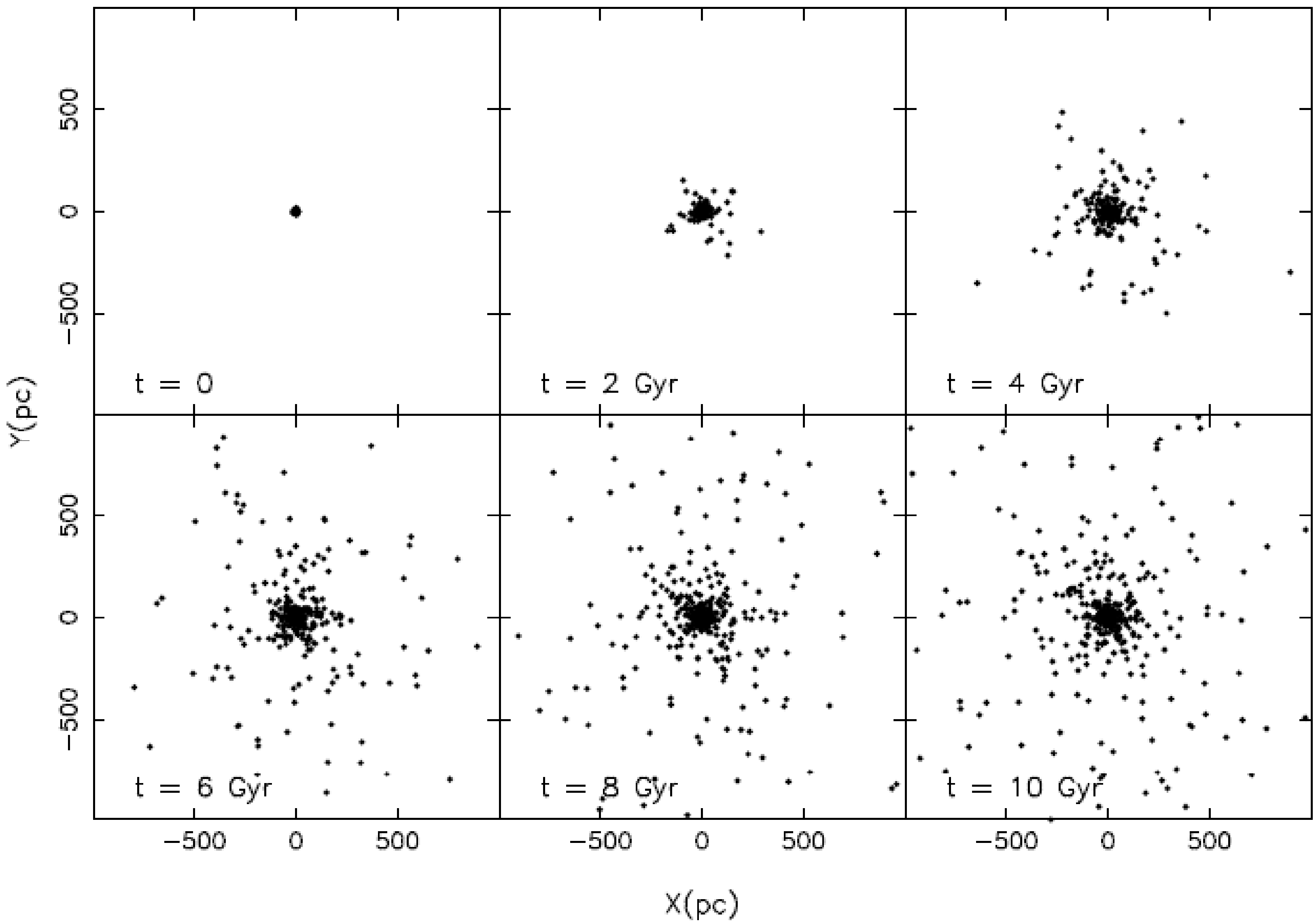


As a result of energy equipartition, the heavier stars tend to move toward the cluster centre while lighter stars tend to move further away from the cluster centre.

Consequences of 2-body relaxation: Dynamical Mass segregation



Consequences of 2-body relaxation: **Core collapse**



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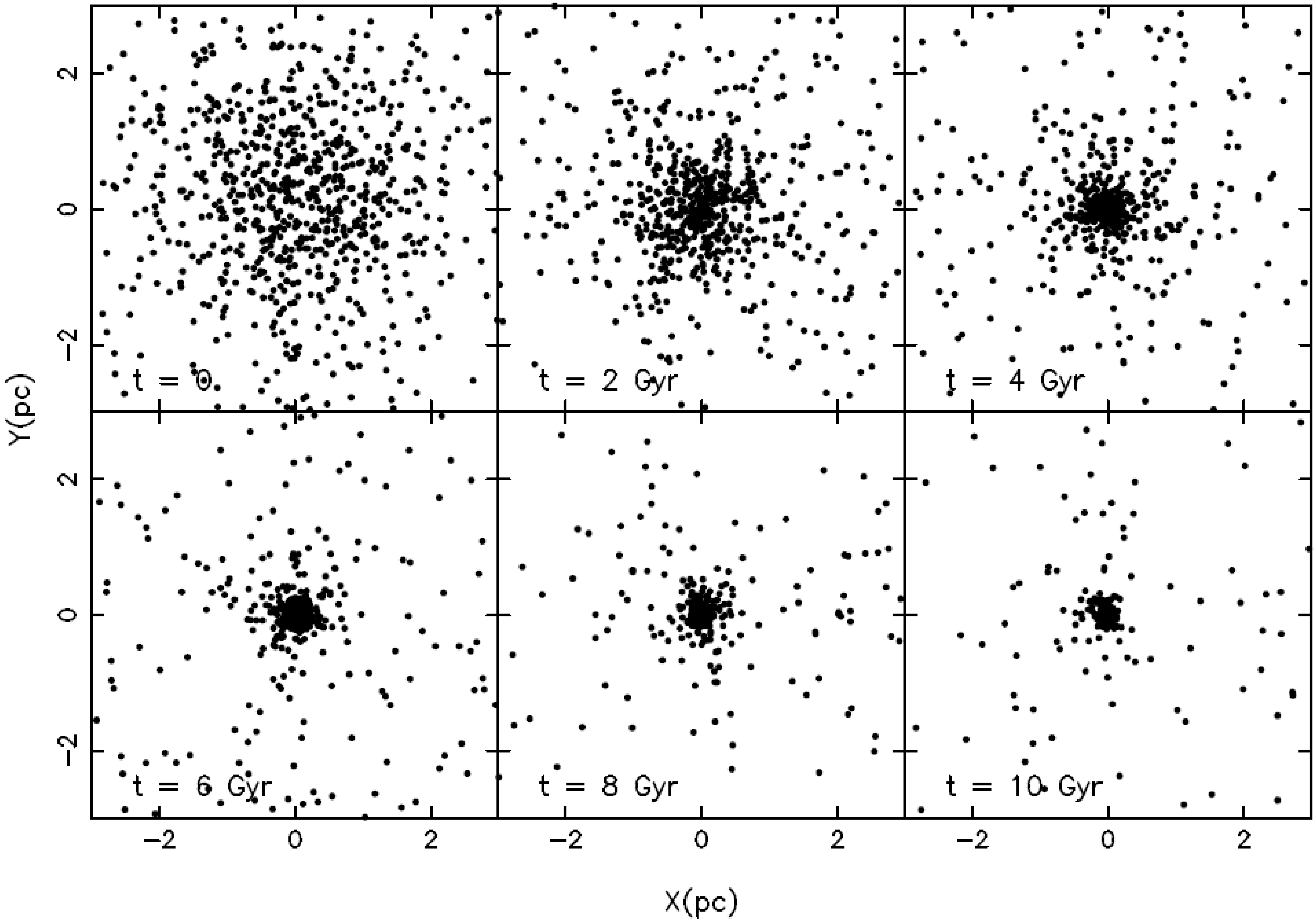
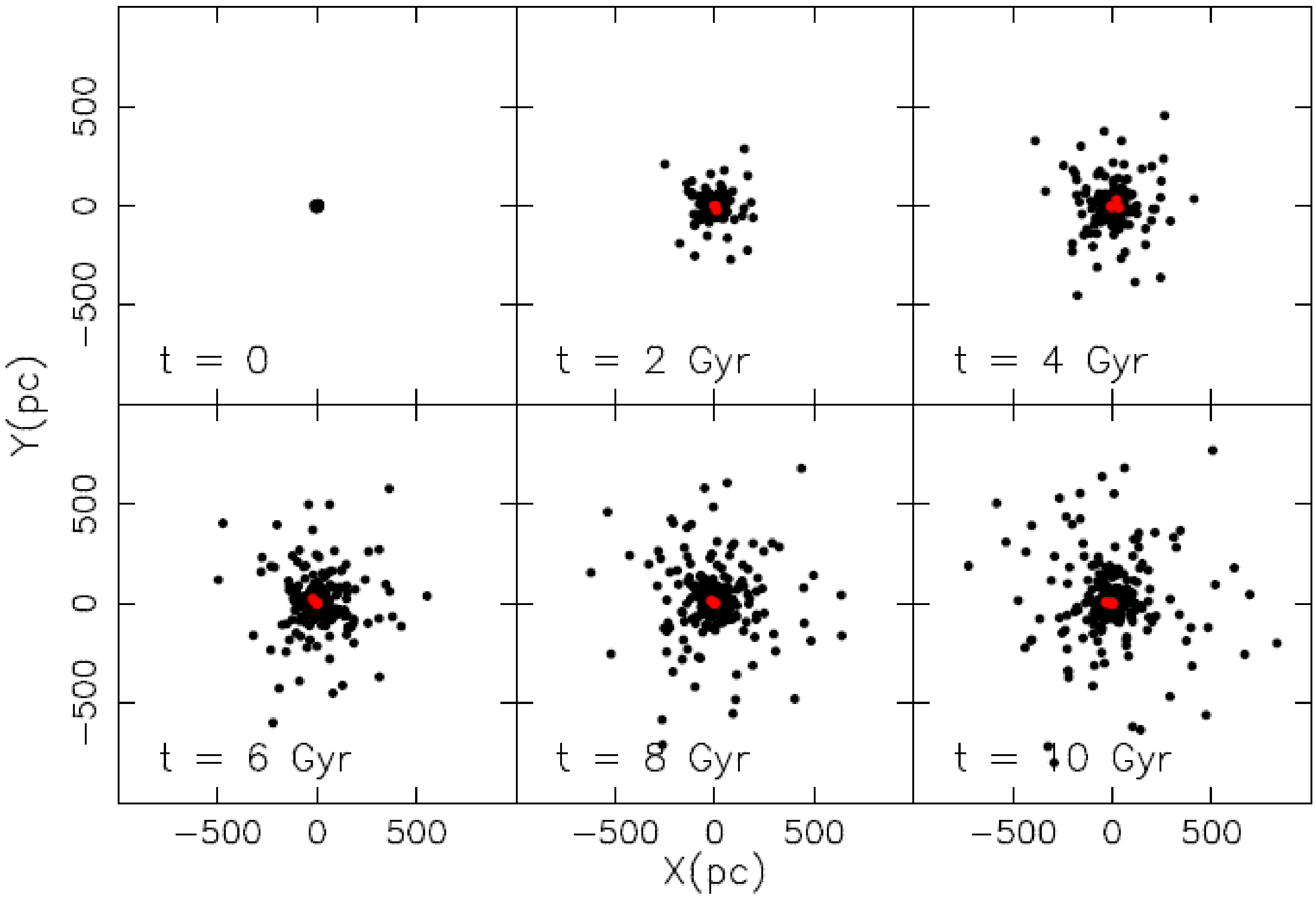


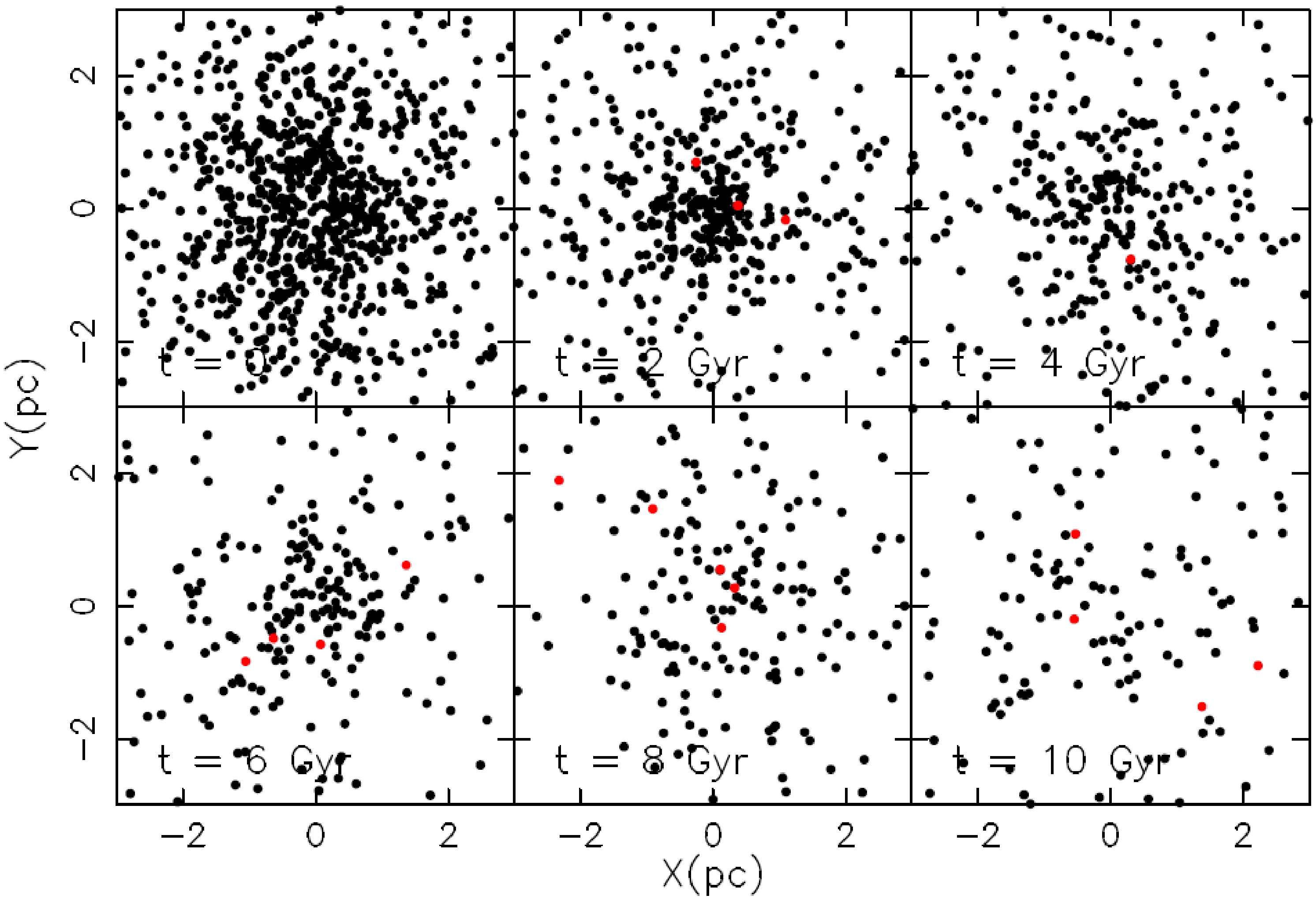
Zooming in shows that the core collapses inwards

Consequences of 2-body relaxation: **Core collapse**

Red dots are binaries



Core Collapse is halted by binaries



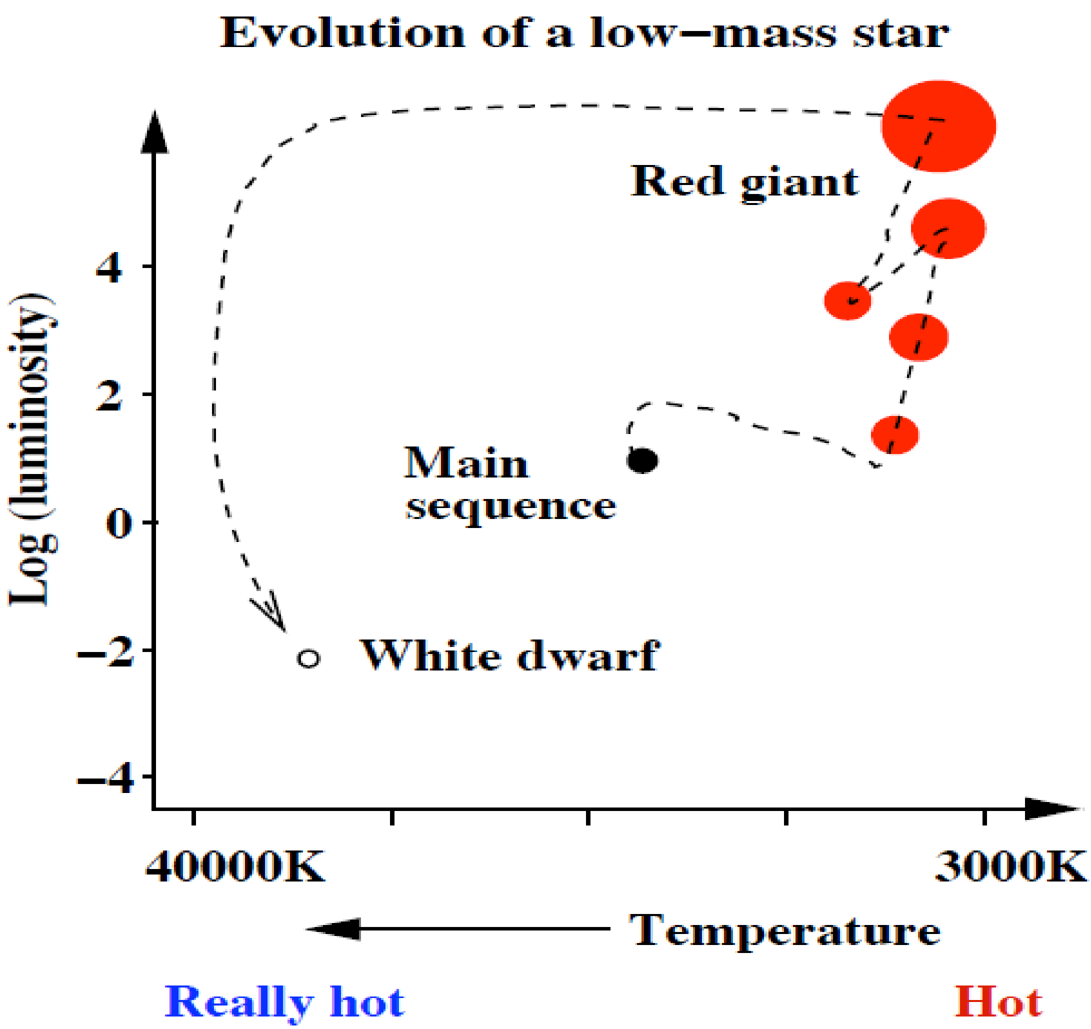
Binary stars become important sources of energy:
core of the cluster starts to re-expand

The complexity of the physical processes involved in real clusters

Early gas expulsion

The complexity of the physical processes involved in real clusters

Particles evolve due to Stellar Evolution

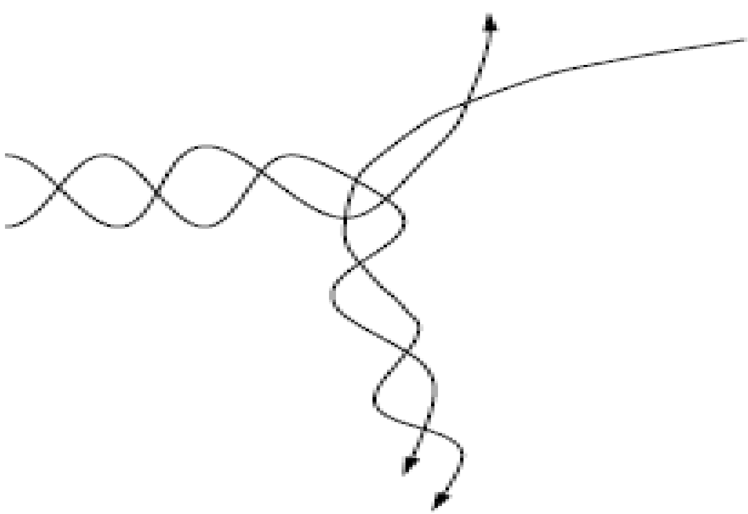


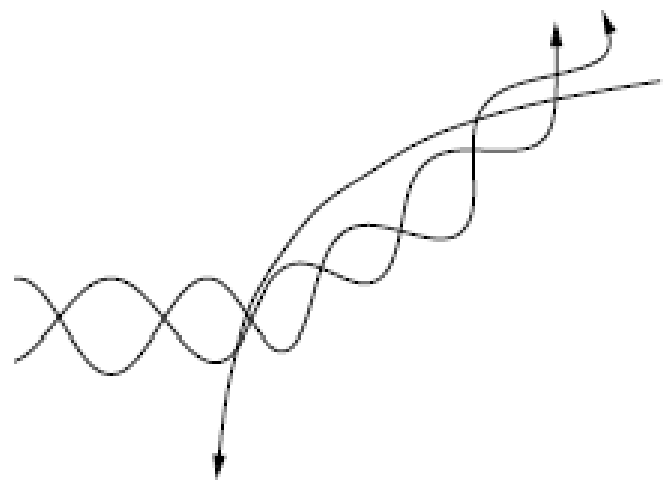
The complexity of the physical processes involved in real clusters

Possible outcomes of encounters between a binary and a single star

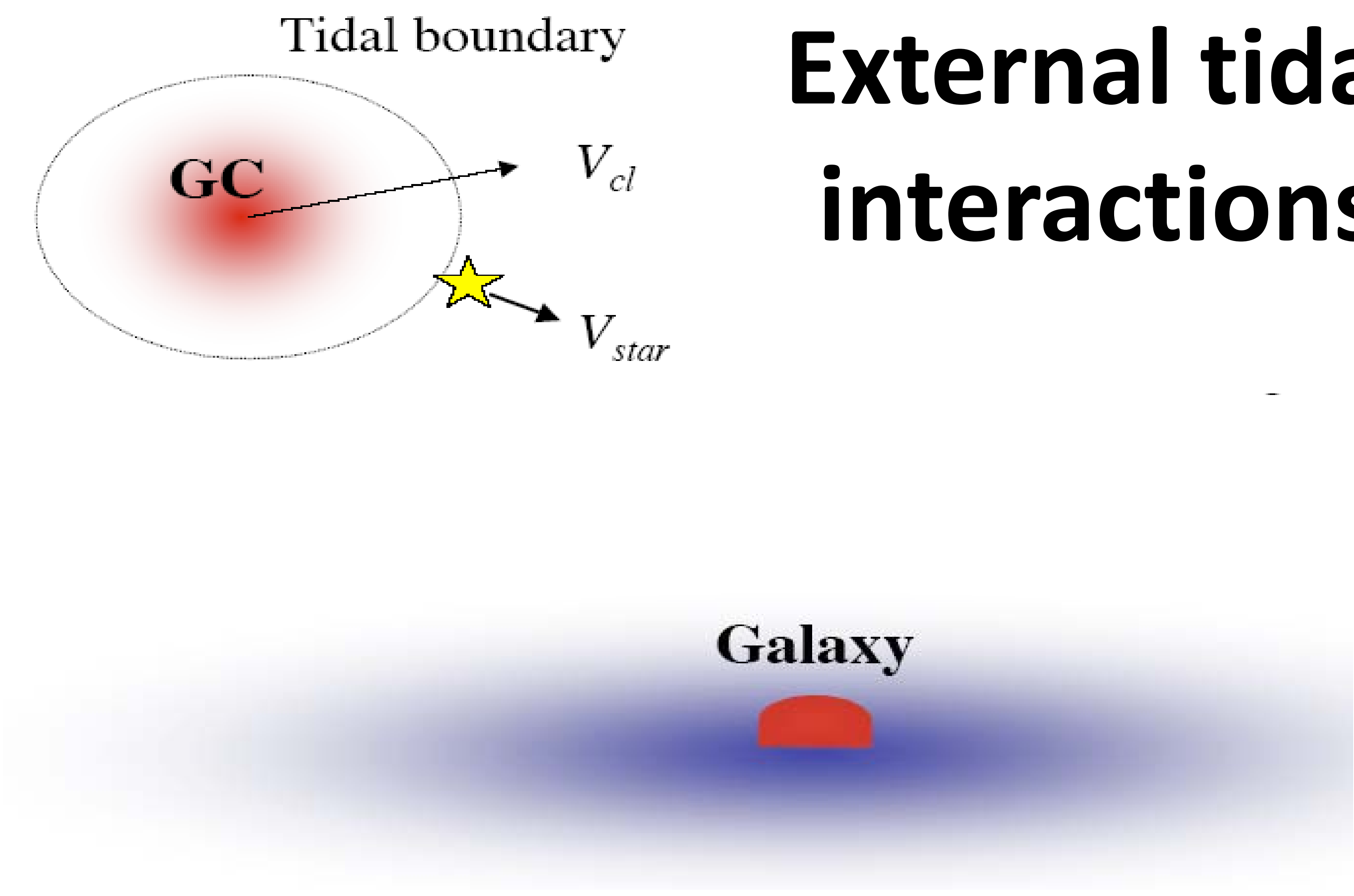
- Soft binaries get broken up
- Hard binaries get harder
- Clean exchanges: lowest-mass star ejected

E_{bin} / kT_c



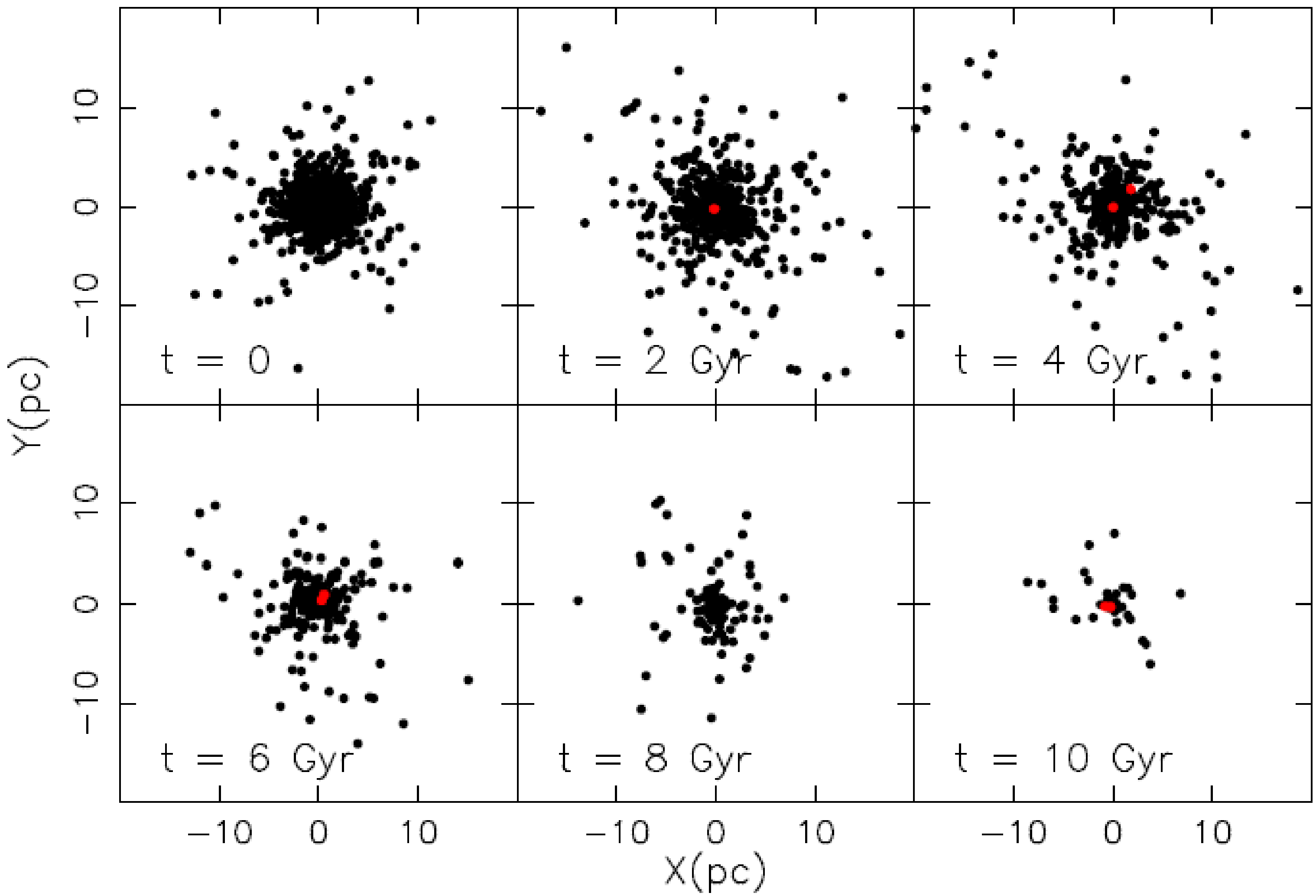


The complexity of the physical processes involved in real clusters

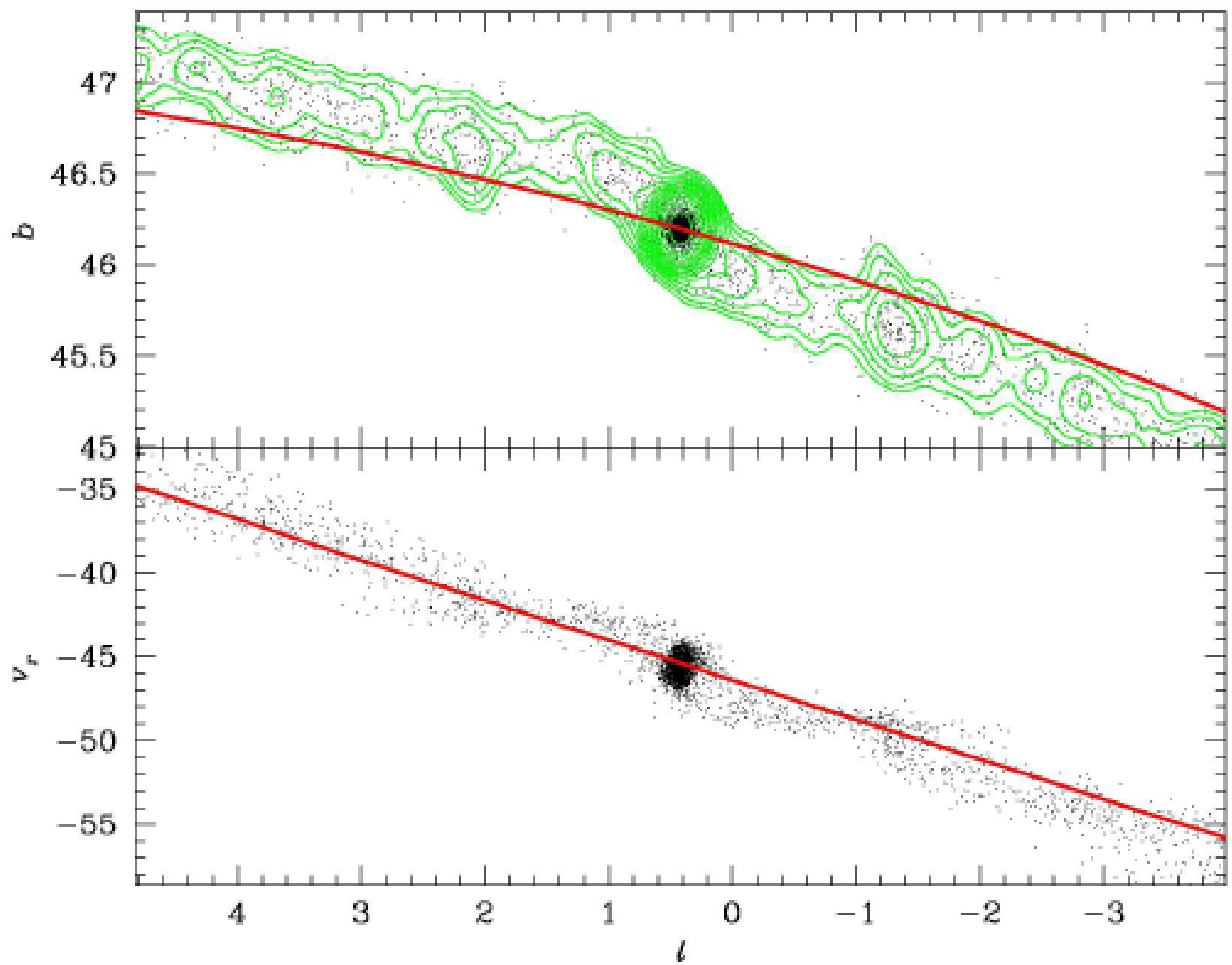


External tidal interactions

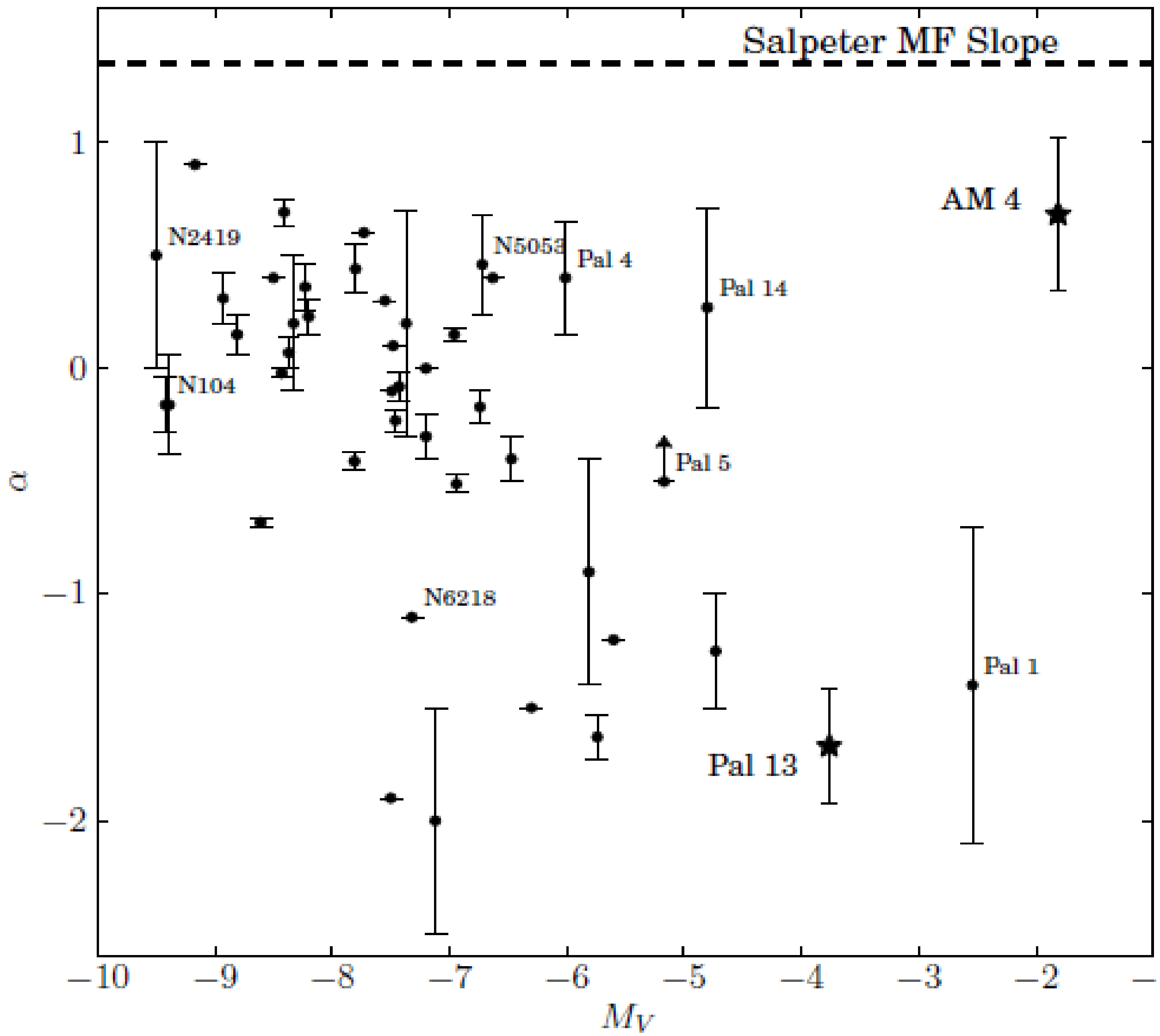
External tidal interactions



External tidal interactions



MF changes through long-term evolution
MF-slope is a tracer of mass loss



Hamren et al., 2013, ApJ

End of Lecture 1

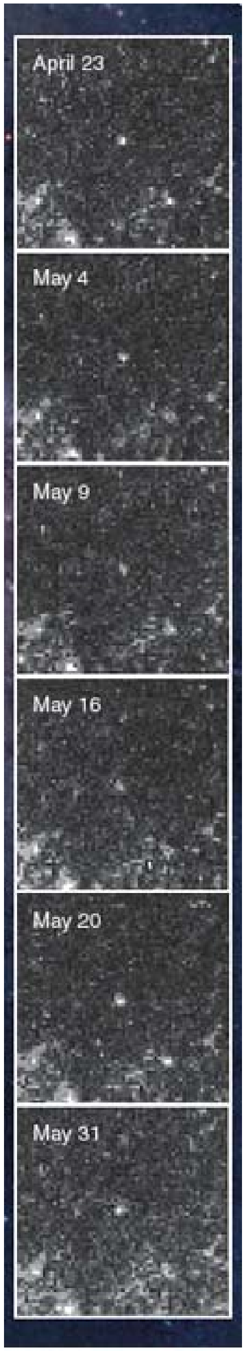
Need a tool to measure distances to stars throughout the Milky Way…

Need to be able to measure distances greater than what stellar parallax can do (about 100 pc = 325 light yrs maximum).

The Milky Way is 100,000 light years across!

Turns out… there is a method VARIABLE STARS!

(Hershel didn't know about this either)



Cepheid Variable Stars

Distance Indicator beyond Parallax

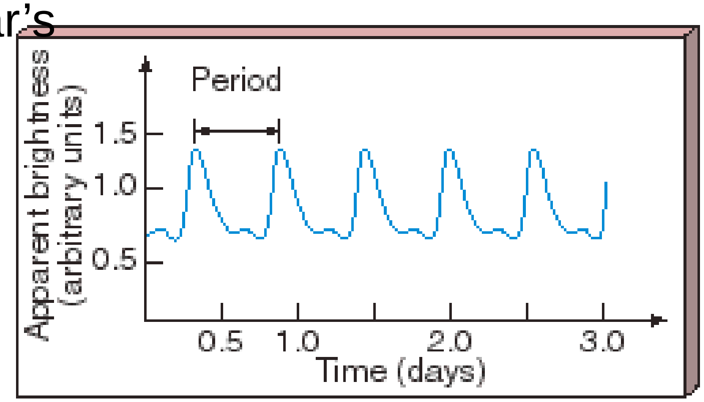
- The brightness of these stars varies in a very periodic way.
- The period is directly proportional to the star's luminosity!

So…

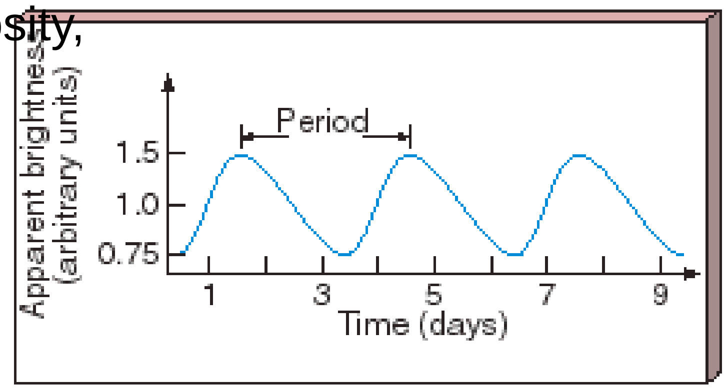
- Measure the period and get luminosity
- Measure the average brightness

Once you have average brightness and luminosity, you solve for distance from

Brightness = Luminosity/(distance)^2



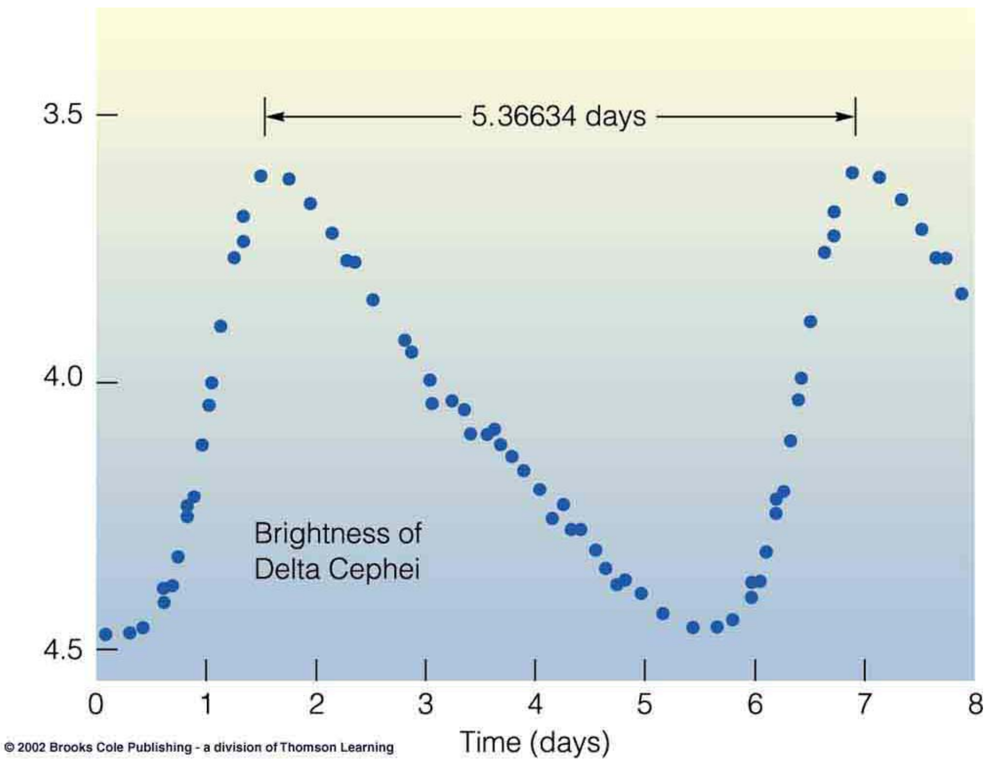
(a)

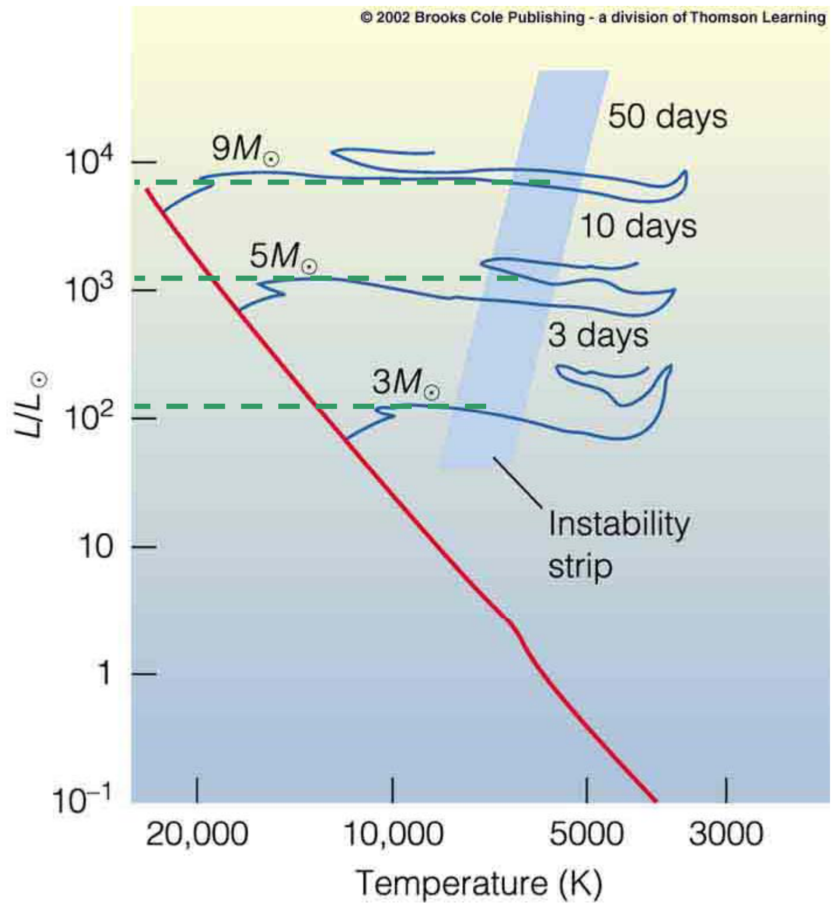


(b)

Cepheid Variable Stars

These stars are aging stars. They lie in a region of the HR diagram called the Instability Strip. A star becomes a Cepheid variable star (unstable to oscillations) several times before it dies. These stars are more massive than the sun.

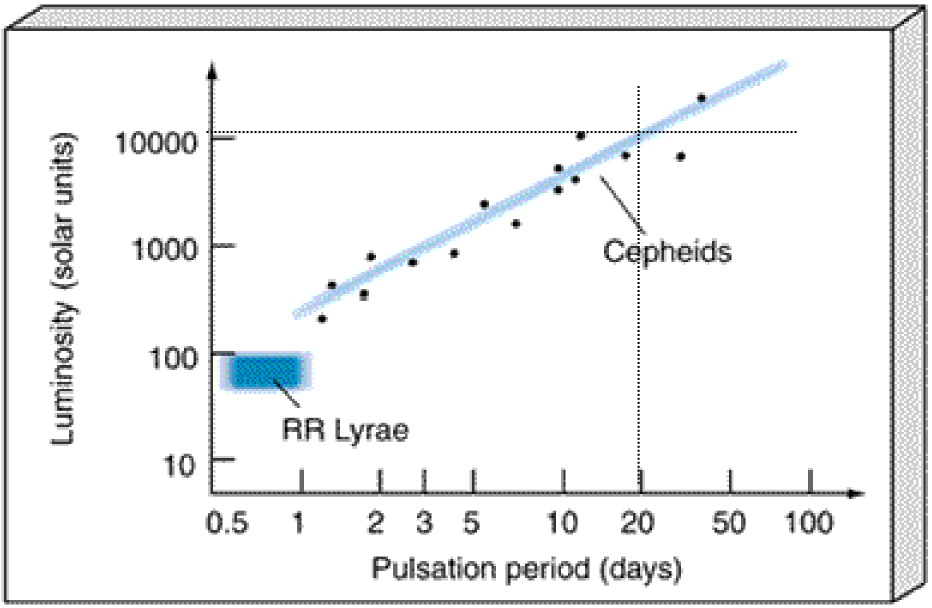




How it Works…

Luminosity-Period Relation

Once you measure the period of variability you can then just read off the luminosity.



Example. A star with a 20 day period has a luminosity 10,000 times that of the sun.

Star Cluster's Evolve

Internal Processes

- Stellar Evolution
- Two-body Interactions
- Binary Stars
- Triple / Quadruple Body Interactions
- Collisions

External Processes

- Gravitational Field
  - Stripping, heating, and shocks
- Complex dynamical histories?

Star Cluster's Evolve

High-Mass Clusters:
- Survive to present day
- Only form at early times
    - 10-12 billion years ago

Low-Mass Clusters:
- Easily dissolve within 12 billion years
- Keep forming as galaxy evolves

The two time scales of star cluster dynamics

1. The crossing time t_{cr}
   - $\sim$ 1Myr for the globular star clusters of the Milky Way
   - $\sim$ "period" of orbital motions of stars
   - $\sim$ time scale of virialisation
2. The relaxation time t_{rh}
   - $\sim \dfrac{N}{\log N} t_{cr}$
   - $\gg t_{cr}$
   - A fraction of the age of the universe for the globular clusters
   - The time scale of evolution of a virialised cluster
   - Escape rate is $\sim 100 t_{rh}$[1]