Stellar winds

From cool dwarfs to luminous supergiants

Jiří Krtička

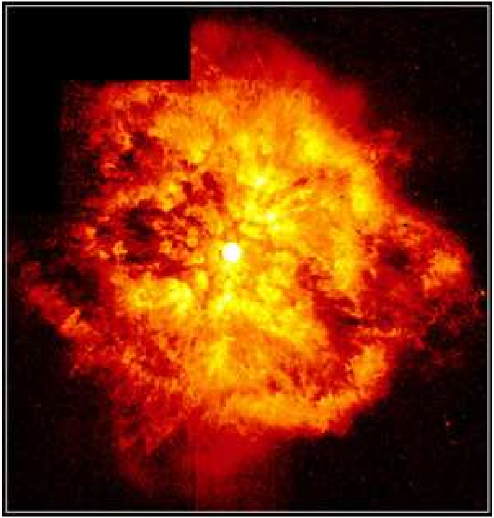
Masaryk University

Introduction: why mass-loss?

Evidence for mass-loss: shells around stars

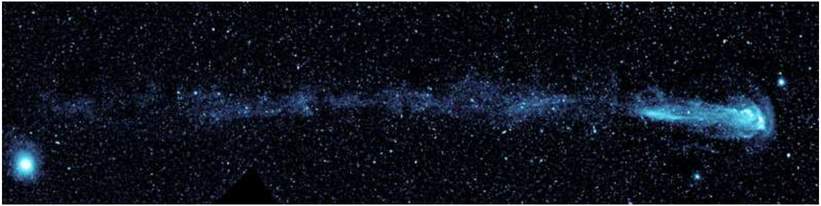
Abell 39 nebula

Evidence for mass-loss: shells around stars



nebula around WR 124

Evidence for mass-loss: shells around stars

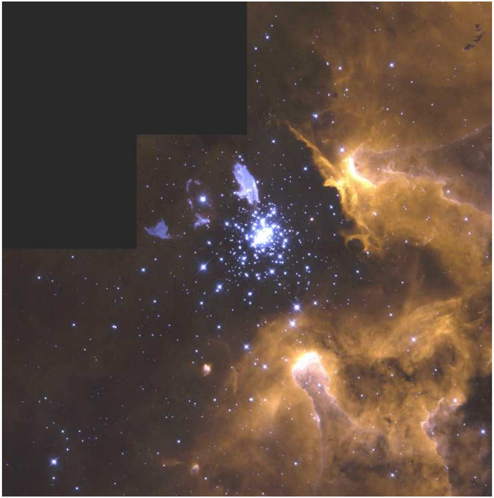


nebula around Mira (o Cet)

Bubbles are everywhere...

Image credit: Team Ciel Austral

Evidence for mass-loss: interstellar medium



NGC 3603 cluster

Evidence for mass-loss: heavy elements

After the period of primordial nucleosynthesys, the Universe was composed mostly form H and He (with a tiny amount of heavier elements like Li). Heavy elements (C, N, O, Fe, . . .) were completely missing.

However, there are heavy elements around us. Where do they come from? Heavier elements are synthethised during thermonuclear reactions in the stellar interiors. How do they get into the interstellar medium?

Hydrodynamical equations

Boltzmann equation

Particle distribution function $F(t, \boldsymbol{x}, \boldsymbol{\xi})$ gives the number of particles in the element of the phase space $\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{\xi} = \mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2\,\mathrm{d}\xi_3$ with coordinates $\boldsymbol{x}$ and momenta $\boldsymbol{\xi}$ as

$$F(t, \boldsymbol{x}, \boldsymbol{\xi})\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{\xi}.$$

The time evolution of the particle distribution function under the influence of external force $\boldsymbol{f}$ acting on partice with mass m and taking into account particle collisions is

$$\frac{\partial F}{\partial t} + \frac{\xi_h}{m}\frac{\partial F}{\partial x_h} + f_h\frac{\partial F}{\partial \xi_h} = \left(\frac{\mathrm{d}F}{\mathrm{d}t}\right)_{\mathrm{coll}},$$

which is the Boltzmann equation. Here used the Einstein summation convention for index h.

Momentum equations

The Boltzmann equation can be solved numerically to derive the particle distribution function. However, for most of practical applications, the distribution function is very close to the Maxwelian distribution expressed at given location in the frame comoving with the fluid. In such a case, just mean quantities are of real importance for the description of the flow. These are moments of the Boltzmann equation

$$m \int F \, \mathrm{d}\boldsymbol{\xi} = \rho, \qquad \text{(0th moment, flow density)},$$

$$\frac{1}{m} \int \boldsymbol{\xi} F \, \mathrm{d}\boldsymbol{\xi} = \boldsymbol{v}, \qquad \text{(1st moment, flow velocity)}.$$

These can be derived by multiplying the Boltzmann equation by m and $\boldsymbol{\xi}/m$ and by integrating. However, the equation for n-th moment contains $n+1$-th moment. Consequently, we shall close the equations somehow to avoid obtainig infinite set of equations. This is done for the equation for the 2nd moment using thermodynamical relations for pressure.

The continuity equation

Multiplicating the Boltzmann equation by particle mass m and integrating over the velocity space

$$\underbrace{\int m \frac{\partial F}{\partial t} \,\mathrm{d}\boldsymbol{\xi}}_{1} + \underbrace{\int m \frac{\xi_h}{m} \frac{\partial F}{\partial x_h} \,\mathrm{d}\boldsymbol{\xi}}_{2} + \underbrace{\int m\, f_h \frac{\partial F}{\partial \xi_h} \,\mathrm{d}\boldsymbol{\xi}}_{3} = \underbrace{\int m \left(\frac{\mathrm{d}F}{\mathrm{d}t}\right)_{\text{coll}} \mathrm{d}\boldsymbol{\xi}}_{4}$$

$$\begin{aligned}
1 =&\ m\frac{\partial}{\partial t} \int F \,\mathrm{d}\boldsymbol{\xi} = m\frac{\partial n}{\partial t} = \frac{\partial \rho}{\partial t}, \\
2 =&\ \frac{\partial}{\partial x_h} \int \xi_h F \,\mathrm{d}\boldsymbol{\xi} = m\frac{\partial}{\partial x_h} (n v_h) = \frac{\partial (\rho v_h)}{\partial x_h}, \\
3 =&\ \sum \int f_h [F]_{-\infty}^{\infty} \,\mathrm{d}\boldsymbol{\xi}' = 0 \text{ is } f_h \text{ does not depend on } \boldsymbol{\xi}, \\
4 =&\ 0 \text{ for conserved quantity } (m),
\end{aligned}$$

where

- $n = \int F \,\mathrm{d}\boldsymbol{\xi}$ is number density of particles,
- $\rho = mn$ is the density,
- $v_h = \frac{1}{N} \int \xi_h F \,\mathrm{d}\boldsymbol{\xi}$ is the mean speed.

The continuity equation

This gives

$$\frac{\partial \rho}{\partial t} + \frac{\partial \left(\rho v_h \right)}{\partial x_h} = 0,$$

or

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \boldsymbol{v} \right) = 0,$$

which is the continuity equation.

The continuity equation: interpretation

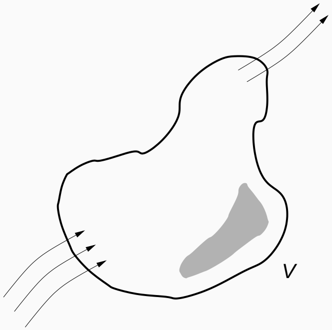
Integration over volume fixed in space gives

$$-\int_V \frac{\partial \rho}{\partial t}\,\mathrm{d}V = \int_V \nabla \cdot (\rho \boldsymbol{v})\,\mathrm{d}V,$$

or, using the Stokes theorem

$$-\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho\,\mathrm{d}V = \oint_{\partial V} \rho \boldsymbol{v}\,\mathrm{d}\boldsymbol{S},$$

which is the expression of the law of conservation of mass.



Equation of motion

Multiplying the Boltzmann equation by ξ_i and integrating

$$\underbrace{\int \xi_i \frac{\partial F}{\partial t}\,\mathrm{d}\boldsymbol{\xi}}_{1} + \underbrace{\int \xi_i \frac{\xi_h}{m}\frac{\partial F}{\partial x_h}\,\mathrm{d}\boldsymbol{\xi}}_{2} + \underbrace{\int \xi_i\, f_h \frac{\partial F}{\partial \xi_h}\,\mathrm{d}\boldsymbol{\xi}}_{3} = \underbrace{\int \xi_i \left(\frac{\mathrm{d}F}{\mathrm{d}t}\right)_{\text{coll}} \mathrm{d}\boldsymbol{\xi}}_{4}$$

$$\begin{aligned}
1 = {} & \frac{\partial}{\partial t}\int \xi_i F\,\mathrm{d}\boldsymbol{\xi} = m\frac{\partial}{\partial t}(nv_i) = \frac{\partial(\rho v_i)}{\partial t}, \\
2 = {} & \frac{1}{m}\frac{\partial}{\partial x_h}\int \xi_i\xi_h F\,\mathrm{d}\boldsymbol{\xi} = m\frac{\partial}{\partial x_h}\int (c_i+v_i)(c_h+v_h)F\,\mathrm{d}\boldsymbol{\xi} = \\
& m\frac{\partial}{\partial x_h}\left[v_iv_h\int F\,\mathrm{d}\boldsymbol{\xi} + v_h\int c_iF\,\mathrm{d}\boldsymbol{\xi} + v_i\int c_hF\,\mathrm{d}\boldsymbol{\xi} + \int c_ic_hF\,\mathrm{d}\boldsymbol{\xi}\right] = \\
& \frac{\partial}{\partial x_h}(mnv_iv_h + 0 + 0 + p_{hi}) = \frac{\partial}{\partial x_h}(\rho v_iv_h + p_{hi}), \\
3 = {} & \textstyle\sum_h \int f_h[\xi_iF]_{-\infty}^{\infty}\,\mathrm{d}\boldsymbol{\xi}' - \int \sum_h \delta_{ih} f_h F\,\mathrm{d}\boldsymbol{\xi} = -nf_i = -\rho g_i, \\
4 = {} & 0 \text{ for conserved quantity } (\xi)\text{, where}
\end{aligned}$$

- $c_h = \xi_h/m - v_h$ is the thermal speed,
- $p_{hi} = m\int c_ic_hF\,\mathrm{d}\boldsymbol{\xi}$ is the pressure tensor, $p_{hi} = p\delta_{hi}$,
- $g_i = f_i/m$ is force per unit of mass (acceleration).

Equation of motion

This gives

$$\frac{\partial\left(\rho v_i\right)}{\partial t}+\frac{\partial}{\partial x_h}\underbrace{\left(\rho v_i v_h+p\,\delta_{hi}\right)}_{\Pi_{ik}}=\rho g_i,$$

which is, after differencing and using the continuity equation,

$$\rho\frac{\partial v_i}{\partial t}+\rho v_h\frac{\partial v_i}{\partial x_h}=-\frac{\partial p}{\partial x_i}+\rho g_i,$$

where Π_{ik} is the momentum flux density tensor, or

$$\rho\frac{\partial \boldsymbol{v}}{\partial t}+\rho\boldsymbol{v}\cdot\nabla\boldsymbol{v}=-\nabla p+\rho\boldsymbol{g},$$

the momentum equation. Introducing the Lagrangian derivative the momentum equation has a form of Newton's second law

$$\rho\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t}=-\nabla p+\rho\boldsymbol{g}.$$

Energy equation

Multiplicating the Boltzmann equation by $\xi_i\xi_j/m$ and integrating

$$\underbrace{\int \frac{1}{m}\xi_i\xi_j\,\frac{\partial F}{\partial t}\,\mathrm{d}\boldsymbol{\xi}}_{1} + \underbrace{\int \frac{1}{m^2}\xi_i\xi_j\xi_h\frac{\partial F}{\partial x_h}\,\mathrm{d}\boldsymbol{\xi}}_{2} + \underbrace{\int \xi_i\xi_j\,\frac{f_h}{m}\frac{\partial F}{\partial \xi_h}\,\mathrm{d}\boldsymbol{\xi}}_{3} = \underbrace{\int \frac{1}{m}\xi_i\xi_j\left(\frac{\mathrm{d}F}{\mathrm{d}t}\right)_{\mathrm{coll}}\mathrm{d}\boldsymbol{\xi}}_{4}$$

$$1 = \frac{1}{m}\frac{\partial}{\partial t}\int \xi_i\xi_j F\,\mathrm{d}\boldsymbol{\xi} = m\frac{\partial}{\partial t}\int (c_i+v_i)(c_j+v_j)F\,\mathrm{d}\boldsymbol{\xi} = \frac{\partial}{\partial t}\left(\rho v_i v_j + p_{ij}\right),$$

$$2 = \frac{1}{m^2}\frac{\partial}{\partial x_h}\int \xi_i\xi_j\xi_h F\,\mathrm{d}\boldsymbol{\xi} = m\frac{\partial}{\partial x_h}\int (c_i+v_i)(c_j+v_j)(c_h+v_h)F\,\mathrm{d}\boldsymbol{\xi} = \frac{\partial}{\partial x_h}\left(\rho v_i v_j v_h + v_h p_{ij} + v_i p_{hj} + v_j p_{hi}\right),$$

$$3 = \begin{cases} 0, \text{ terms with } h \neq i \text{ and } h \neq j \text{ (direct integration)}, \\ -f_i n v_j - f_j n v_i, \text{ terms with } h = i \text{ or } h = j \text{ (per-partes)}, \end{cases}$$

$4 =$ 0 when contraction is performed, where

- $p_{hij} = \int c_h c_i c_j F\,\mathrm{d}\boldsymbol{\xi}/m$ is $p_{hij} = 0$ when neglecting viscosity.

Energy equation

After the contraction and multiplication by $\frac{1}{2}$ we derive

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{3}{2}p\right) + \frac{\partial}{\partial x_h}\left(\frac{1}{2}\rho v_h v^2 + \frac{5}{2}p v_h\right) - \rho v_i g_i = 0,$$

or, introducing the specific energy $\rho\epsilon = \frac{3}{2}p$,

$$\frac{\partial}{\partial t}\left(\rho\epsilon + \frac{\rho v^2}{2}\right) + \nabla \cdot \left[\rho \boldsymbol{v}\left(\epsilon + \frac{v^2}{2}\right) + p\boldsymbol{v}\right] - \rho \boldsymbol{v}\boldsymbol{g} = 0,$$

which is the energy equation.

Collecting the nuggets: the hydrodynamical equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \rho \boldsymbol{g},$$

$$\frac{\partial}{\partial t}\left(\rho\epsilon + \frac{\rho v^2}{2}\right) + \nabla \cdot \left[\rho \boldsymbol{v}\left(\epsilon + \frac{v^2}{2}\right) + p\boldsymbol{v}\right] = \rho \boldsymbol{v} \boldsymbol{g}$$

- system of nonlinear first-order partial differential equations
- unknowns ρ, $\boldsymbol{v}$, p, and ϵ (+equation of state)
- initial and boundary conditions crucial
- inviscid flow, no magnetic field
- some special analytic solutions, general solution only numerically
- stationary solutions are important ($\partial/\partial t = 0$, but $\boldsymbol{v} \neq 0$)

The hydrodynamical equations in spherical coordinates

In spherical coordinate system, the components of the velocity vector are $\boldsymbol{v} = (v_r, v_\theta, v_\phi)$ and the components of force are $\boldsymbol{g} = (g_r, g_\theta, g_\phi)$. The equation of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \rho v_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

and the components of equation of motion take the form of

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r,$$

$$\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r} = -\frac{1}{r\rho}\frac{\partial p}{\partial \theta} + g_\theta,$$

$$\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\theta v_\phi\cot\theta}{r} = -\frac{1}{r\rho\sin\theta}\frac{\partial p}{\partial \phi} + g_\phi.$$

The hydrodynamical equations in spherical symmetry

In a spherical symmetry the hydrodynamic quantities do not depend on θ and ϕ coordinates, there is no flow in $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ directions ($\boldsymbol{v} = v(r)\boldsymbol{r}$) and the hydrodynamical equations are ($v \equiv v_r$)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v) = 0,$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g.$$

How can mass escape stars?

How can mass escape gravitational wells of stars?

Let us start with the momentum equation with gravity

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} = \rho g_\mathrm{w} - \frac{\partial p}{\partial r} - \frac{\rho G M}{r^2},$$

which can be simplified assuming stationary isothermal outflow ($p = a^2 \rho$)

$$v \frac{\mathrm{d}v}{\mathrm{d}r} = g_\mathrm{w} - \frac{a^2}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}r} - \frac{GM}{r^2},$$

where g_w gives the force that drives the wind (force per unit of mass, i.e., the acceleration) and a is the isothermal sound speed.

We can integrate the equation from the stellar surface R_* to infinity

$$\int_{R_*}^{\infty} v \frac{\mathrm{d}v}{\mathrm{d}r} \,\mathrm{d}r = \int_{R_*}^{\infty} g_\mathrm{w} \,\mathrm{d}r - \int_{R_*}^{\infty} \frac{a^2}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}r} \,\mathrm{d}r - \int_{R_*}^{\infty} \frac{GM}{r^2} \,\mathrm{d}r$$

yielding

$$\frac{1}{2} v_\infty^2 - \frac{1}{2} v_0^2 = \int_{R_*}^{\infty} g_\mathrm{w} \,\mathrm{d}r - a^2 \ln \frac{\rho_\infty}{\rho_0} - \frac{GM}{R_*}.$$

Three ways

The individual terms in

$$\frac{1}{2}v_\infty^2 - \frac{1}{2}v_0^2 = \int_{R_*}^{\infty} g_\mathrm{w}\,\mathrm{d}r - a^2 \ln\frac{\rho_\infty}{\rho_0} - \frac{GM}{R_*}$$

describe (from left to right) change of the kinetic energy (per unit of mass), work of driving forces, work of pressure force, and the potential energy (per unit of mass).

There are three ways to initiate the outflow. Either the initial velocity is larger than the escape speed v_esc,

$$\frac{1}{2}v_0^2 \geq \frac{GM}{R_*}, \quad v_0 \geq v_\mathrm{esc} = \sqrt{\frac{2GM}{R_*}}, \quad v_\mathrm{esc} = 620\,\mathrm{km\,s^{-1}}\sqrt{\frac{M}{M_\odot}\frac{R_\odot}{R}}.$$

The initial kinetic energy of the flow should be larger than the absolute value of the potential energy. This is fullfilled for **explosive outflows** like supernovae or supernova impostors (e.g., η Car).

Three ways

The individual terms in

$$\frac{1}{2}v_\infty^2 - \frac{1}{2}v_0^2 = \int_{R_*}^{\infty} g_\mathrm{w}\,\mathrm{d}r - a^2 \ln\frac{\rho_\infty}{\rho_0} - \frac{GM}{R_*}$$

describe (from left to right) change of the kinetic energy (per unit of mass), work of driving forces, work of pressure force, and the potential energy (per unit of mass).

The other possibility is that the driving force is large enough

$$\int_{R_*}^{\infty} g_\mathrm{w}\,\mathrm{d}r \geq \frac{GM}{R_*}.$$

This is true for **winds driven radiatively**, either due to the absorption in lines (in hot stars) or on dust particles (in luminous cool stars).

Three ways

The individual terms in

$$\frac{1}{2}v_\infty^2 - \frac{1}{2}v_0^2 = \int_{R_*}^{\infty} g_\mathrm{w}\,\mathrm{d}r - a^2 \ln\frac{\rho_\infty}{\rho_0} - \frac{GM}{R_*}$$

describe (from left to right) change of the kinetic energy (per unit of mass), work of driving forces, work of pressure force, and the potential energy (per unit of mass).

The last possibility is that the work done by pressure forces is large,

$$a^2 \ln\frac{\rho_0}{\rho_\infty} \geq \frac{GM}{R_*}.$$

Because $\ln(\rho_0/\rho_\infty)$ is of the order of ten at most, this implies that $a \approx v_\mathrm{esc}$. This happens in **coronal winds** of cool main-sequence stars.

Coronal winds

Is there any evidence for the wind of our Sun?

- two types of the comet tails (Biermann 1951)

Is there any evidence for the wind of our Sun?

- satellite observations
  - flux of particles streaming from our Sun (protons, electrons, He, ...)
  - speed about $\sim 500\,\mathrm{km\,s^{-1}}$
  - number density ($r = 1\,\mathrm{a.u.}$) $\sim 10^7$ particles $\mathrm{m^{-3}}$
  - mass-loss rate

$$\dot{M} = 4\pi r^2 \rho v \approx 2 \times 10^{-14}\,\mathrm{M_\odot\,yr^{-1}}$$

Thermally driven wind

The flux of matter cannot appear due to the escape of particles from the atmosphere. The root mean square of the total velocity of particles $v_{\mathrm{th}} = \sqrt{\frac{3kT}{m_{\mathrm{H}}}}$ in the atmosphere with $T = 6000\,\mathrm{K}$ is about $v_{\mathrm{th}} = 12\,\mathrm{km\,s^{-1}}$, which is significantly lower than the escape speed $v_{\mathrm{esc}} = 620\,\mathrm{km\,s^{-1}}$.

Thermally driven wind

The Sun has a large outer atmosphere called *corona*. The corona can be in optical light observed only during the solar eclipses or using satellites. The detection of lines of highly ionized atoms (Ca XII, Fe XIII, Ni XVI,. . . , "coronium", Grotrian 1939, Edlén 1942) shows that the temperature of the solar corona is about $10^5 - 10^6\,\mathrm{K}$. Corresponding root mean square of the total velocity of the order of $100\,\mathrm{km\,s^{-1}}$ is comparable with the escape speed. Consequently, the thermal expansion of the solar corona is thought to be the source of the solar wind (Parker 1958).

This coins the term coronal wind.

Hydrostatic equilibrium in spherical symmetry

- momentum equation
$$\rho\frac{\partial v}{\partial t}+\rho v\frac{\partial v}{\partial r}=-\frac{\partial p}{\partial r}-\frac{\rho GM}{r^2}$$
- stationary case ($v=0$)
$$\frac{\mathrm{d}p}{\mathrm{d}r}=-\frac{\rho GM}{r^2}$$
- using perfect gas equation of state $p=\rho a^2$ in isothermal atmosphere ($a=\text{konst.}$)
$$a^2\frac{\mathrm{d}\rho}{\mathrm{d}r}=-\frac{\rho GM}{r^2}$$
- the equation has a solution
$$\rho=\rho_0\exp\left[\frac{GM}{ra^2}\right]$$
- which implies nonzero density in infinity: $\lim_{r\to\infty}\rho=\rho_0$
- this means that at the thermal speed overcomes the escape speed at some point implying otlflow (wind)

Parker model of the coronal wind

Let us assume that the coronal wind wind can be described as a spherically symmetric, stationary, and isothermal outlow. Then the corresponding hydrodynamical equations are

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0,$$

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} = -a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} - \frac{\rho GM}{r^2}.$$

The integration of the continuity equation gives the **mass-loss rate** $\dot{M} \equiv 4\pi r^2\rho v = \mathrm{const.}$

Inserting $\mathrm{d}\rho/\mathrm{d}r$ from the continuity equation into the equation of motion gives ordinary differential equation for velocity

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} - \frac{GM}{r^2},$$

which can be solved analytically.

Parker model of the coronal wind

We shall study the momentum equation

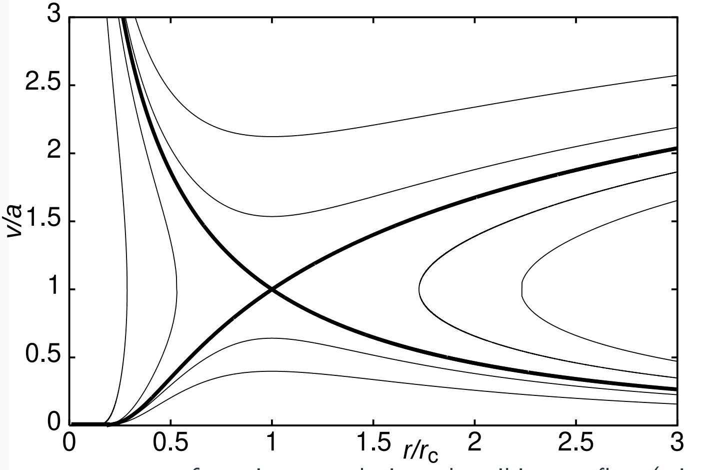
$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} - \frac{GM}{r^2}$$

in more detail.

At radius $r = r_\mathrm{c}$ given by $2a^2/r_\mathrm{c} = GM/r_\mathrm{c}^2$ the right-hand side of momentum equation is equal to zero. This implies either $v = a$ or $\mathrm{d}v/\mathrm{d}r = 0$. This resembles nozzle flow.

At the **sonic point** ($v = a$) either $r = r_\mathrm{c}$ or $\mathrm{d}v/\mathrm{d}r \to \infty$. At the sonic point the sound speed is equal to half of the escape speed.

Solution of the Parker equation



There are two types of continuous solutions describing outflow (wind) and inflow (accretion). There is one outflow solution that is supersonic at large distances from the Sun (wind). Other outflow solutions are subsonic ("breeze"). Observations show supersonic flow at the location of Earth.

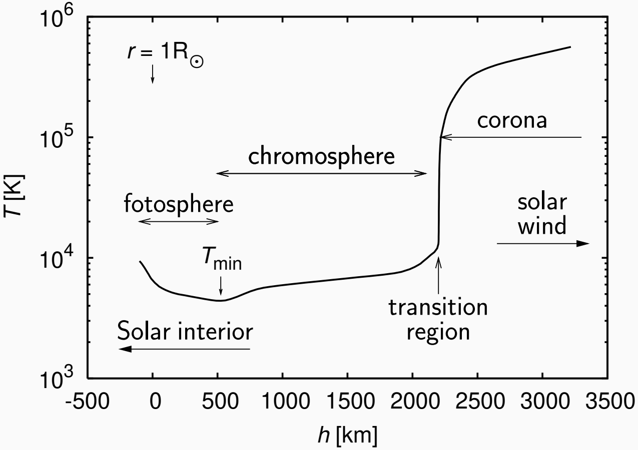
The coronal wind problem

- what accelerates the solar wind?

$\Rightarrow$ solar wind appears due to thermal expansion of corona

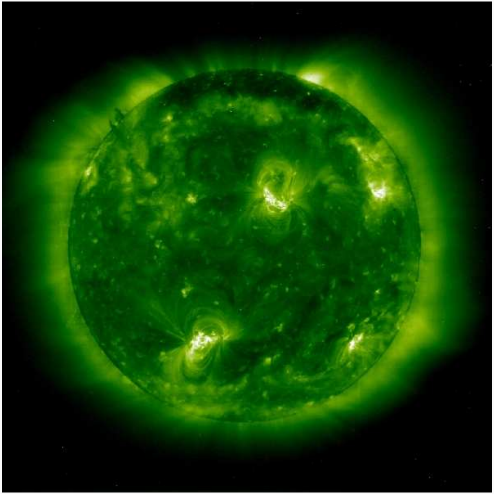
$\Rightarrow$ what heats the corona (energy equation!)

Temperature distribution in the solar atmosphere



- based on semiempirical models of solar atmosphere (quiet regions, Fontenla, Avrett a Loeser 1993)
- thin transition region duet to Field criterion of thermal instability

Observation of solar corona



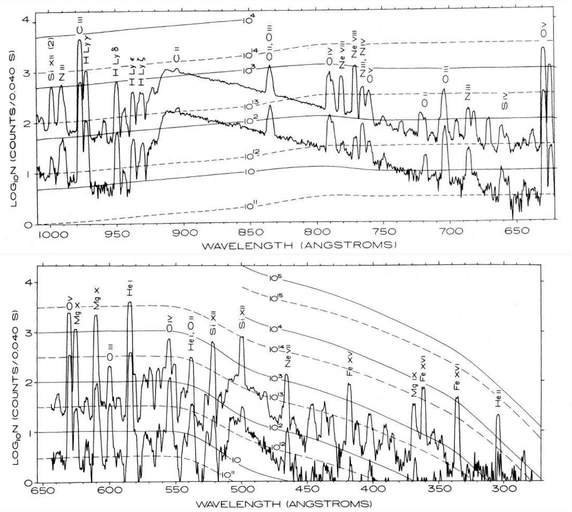
SOHO satellie, Fe XII lines, 195 Å

Observation of solar corona

We observe three types of regions:

- **region of closed magnetic fieldlines**: hot matter confined by magnetic field, typically close to the active regions, source of solar activity (flares, coronal mass ejections), typical temperature $2 \cdot 10^6$ K
- **quiet regions**: medium brightness on the X-ray image, source of slow ($\sim 300\,\mathrm{km\,s^{-1}}$) solar wind, typical temperature $1 \cdot 10^6$ K
- **coronal holes**: dark region in X-rays, source of fast ($\sim 700\,\mathrm{km\,s^{-1}}$) solar wind

Structure of the corona: X-ray and UV region



Spectrum of quiet region (lower spectrum) and active region (upper spectrum): emission lines of highly ionized elements (Dupree a kol. 1973)

Structure of the corona: optical region

magnetic field: close magnetic fieldlines (trapped particles) and open magnetic fieldlines (outflow of particles)

Structure of the corona: optical region

- **K corona**: region close to the Sun $r \lesssim 2\,\mathrm{R}_\odot$, originates due to the scattering of photosheric radiation on free electrons, as a results of fast speed of electrons most of photospheric lines are blurred, continuum polarized (K – "kontinuerlich")
- **F corona** (Fraunhofer's): difraction of photospheric radiation on dust particles in the interplanetary matter (zodiacal light), particles slower, it is possible to observe photospheric Fraunhofer lines

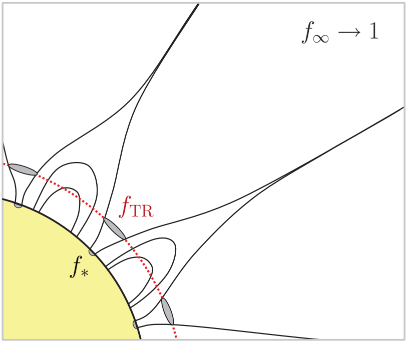
Structure of the corona: radio domain

- solar radio emission with s $\lambda \sim 1\,\mathrm{m}$ produced in corona and transition region with temperature $10^5 - 10^6$ K
- origin: thermal free-free emission and synchrotron emission in regions with strong fields

Influence of the magnetic field

- structure of the corona determined by the magnetic field

Influence of the magnetic field



- sketch of the structure of base corona (Cranmer & Saar 2011)

Influence of the magnetic field

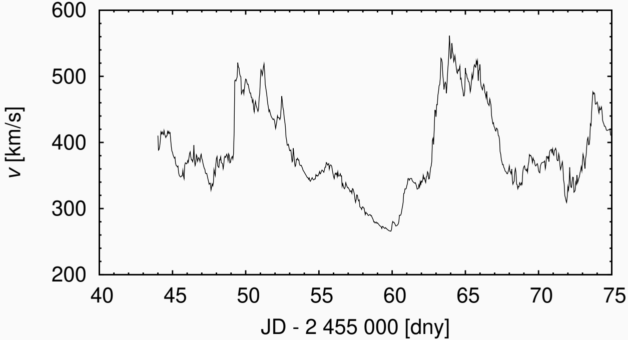
- plasma beta parameter

$$\beta = \frac{p}{p_{\text{mag}}} = \frac{8\pi p}{B^2}$$

  - p is pressure (ram pressure in moving media ρv^2)
  - $p_{\text{mag}} = B^2/(8\pi)$ is the magnetic field pressure
  - $\beta \gg 1$: magnetic field determined by gas dynamics
  - $\beta \ll 1$: gas dynamics determined by magnetic field
- $\beta \gg 1$: typical solar photosphere
- $\beta \sim 1$: photospheric regions with strong fields (typically spots)
- $\beta \ll 1$: corona near Sun

Measurements "in situ" at 1 au: velocity

- Mariner 2 (1962): wind is supersonic (confirmation of Parker model)

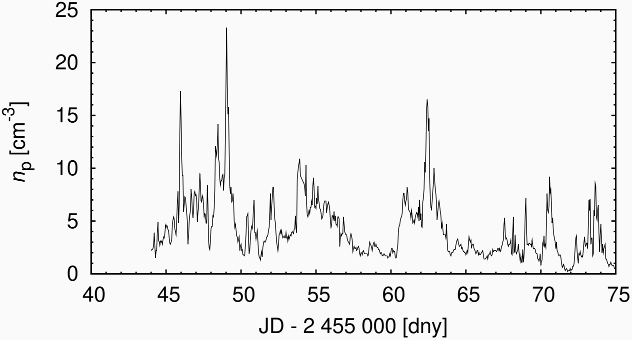


strong variability: wind velocity (September 2009, ACE satellite – NASA)

- slow wind ($\sim 300\,\mathrm{km\,s^{-1}}$), originates typically in quiet regions
- fast wind ($\sim 700\,\mathrm{km\,s^{-1}}$), originates typically in coronal holes

Feldman a kol. (1977)

Measurements "in situ" at 1 au: density

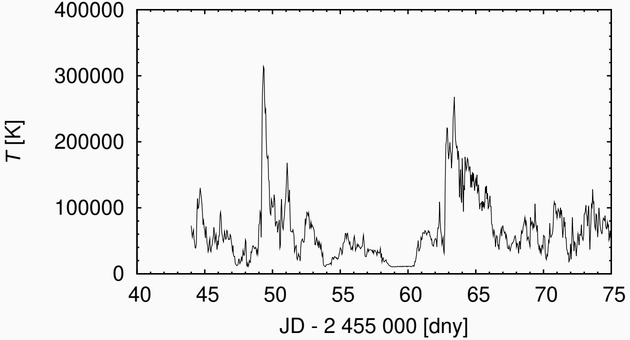


proton number density (September 2009, ACE satellite – NASA)

- slow wind component ($\sim 12\,\mathrm{cm}^{-3}$)
- fast wind component ($\sim 4\,\mathrm{cm}^{-3}$)
- variable He abundance, enhanced abundance of elemens with low first ionization potential (Si, Fe)

Feldman a kol. (1977)

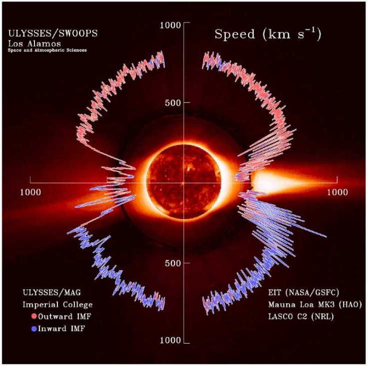
Measurements "in situ" at 1 au: temperature



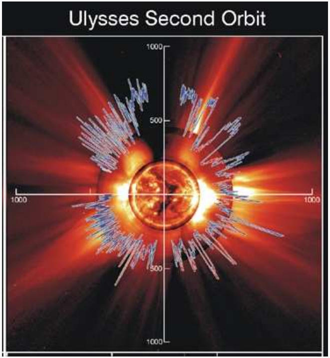
temperature of ions (September 2009, ACE satellite – NASA)

- different mean temperatures of the fast and slow component
- different temperatures of individual particles
- protons $T_{\mathrm{p}} \approx 1.2 \cdot 10^5$ K
- electrons $T_{\mathrm{e}} \approx 1.4 \cdot 10^5$ K
- helium nuclei $T_\alpha \approx 5.8 \cdot 10^5$ K

Measurements "in situ" – Ulysses



solar minimum



solar maximum

- variability of the solar wind during the solar cycle
- fast wind originates mostly from the coronal holes around poles

McComas a kol. (2003)

What is the cause of coronal heating?

- energetics:
    - loss rate of coronal energy due to radiation, conduction, and advection: $3 \cdot 10^{28}\,\mathrm{erg\,s^{-1}}$
    - this corresponds to about 1% of power needed to heat chromosphere
    - about $10^{-5}\,L_{\odot}$
- possibly two different sources of heating needed:
    - coronal holes (open magnetic fieldlines)
    - loops

What is the cause of coronal heating? MHD waves?

dissipation of mechanical and electromagnetic energy of MHD waves

- cool stars have deep subsurface convective zone
- convective zone triggers surface oscilations
- generated sound waves heat lower part of the chromosphere, but their damping is strong and they do not disseminate into corona
- sound waves interact with magnetic field in the corona creating MHD waves
- dissipation of MHD waves may heat the corona
- hybrid waves or Alfvén waves (narrow frequency range) most promissing
- another types of waves strongly damped or reflected

What is the cause of coronal heating? Electric currents?

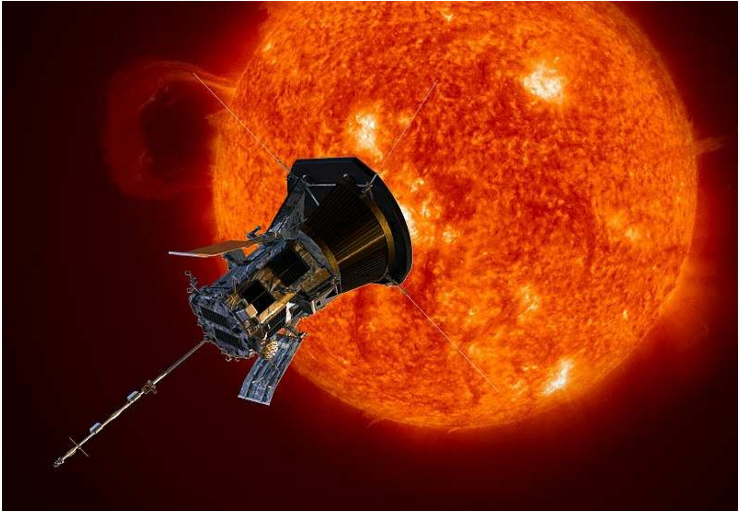
ohmic heating due to electric currents flowind along the magnetic field lines

- problem: resistivity too low
- possible solution: plasma turbulence triggerred by instabilities

$\Rightarrow$ ohmic heating possibly important just for low volumes of plasma ("nanoflares")

$\Rightarrow$ important just for small loops

Parker Solar Probe may tell...



- launched in 2018, should approch Sun within 8.5 $R_\odot$

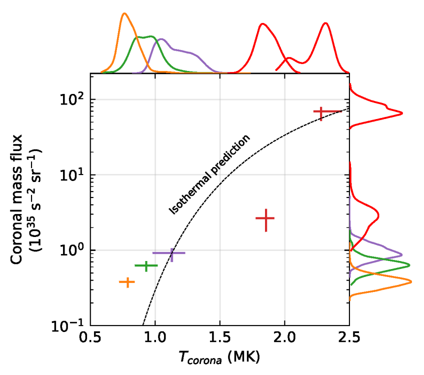
Sensitivity of the mass-flux to the base temperature

Integrating the momentum equation

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} - \frac{GM}{r^2}$$

over radius we arrive at

$$\frac{v^2}{2} - a^2 \ln v = 2a^2 \ln r + \frac{GM}{r} + \text{const.}$$



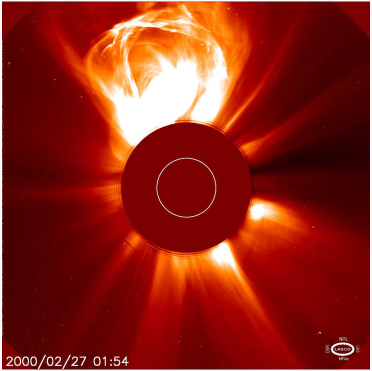
The constant can be evaluated requiring for the wind velocity at the critical point $v(r_c) = a$.

Assuming $v \ll a$ at the base of the wind, the mass-flux is

$$\dot{m} = a\rho\left(\frac{r_c}{r}\right)^2 \exp\left[-\frac{GM}{a^2}\left(\frac{1}{r} - \frac{1}{r_c}\right)\right].$$

Therefore, the mass-flux is very sensitive to temperature as shown from observations of the Parker Solar Probe (Stansby et al. 2021).

CME – coronal mass ejections



(SOHO, coronograph)

- CME: about 1-10% of solar mass loss (Cranmer 2017)
- typically result from flares of filament decay
- occur together with radio storms and bursts of energetic particles
- interact with magnetosphere of Earths leading to auroras

Is the solar wind important at all?

mass-loss

- Sun main-sequence star for $\sim 11 \times 10^9\,\mathrm{yr}$
- solar wind mass-loss rate $2 \times 10^{-14}\,\mathrm{M_\odot\,yr^{-1}}$
- total main-sequence mass-loss of about $\sim 10^{-4}\,\mathrm{M_\odot}$

$\Rightarrow$ too low amount to influence the solar evolution

angular momentum loss

- Sun has magnetic field
- the solar wind is ionized

$\Rightarrow$ solar wind moves flows the magnetic field lines ($\beta \ll 1$)

- solar magnetosphere rotates as a solid bolid up to radius of about $r_\mathrm{A} \approx 15\,\mathrm{R_\odot}$

$\Rightarrow$ this leads to angular momentum loss and rotational braking

(Weber a Davis 1967)

Braking the rotation of Sun

The magnitude of solar angular momentum is

$$L = \eta M_\odot R_\odot^2 \Omega,$$

where Ω is the angular frequency, $\eta \approx 0.1$. Differentiating with respect to time gives

$$\dot{L} = \eta M_\odot R_\odot^2 \dot{\Omega}.$$

The angular momentum loss via spherically-symmetric wind is

$$\dot{L} = -\frac{2}{3}\dot{M} r_{\mathrm{A}}^2 \Omega,$$

where r_{A} is the radius of effective wind corotation and $\dot{M} = 4\pi r^2 \rho v$ is the mass-loss rate.

Radius of effective corotation

Close to the Sun, magnetic field dominates and wind corotates with magnetic field, $\beta \ll 1$. Further out, the magnetic field becomes weak and wind starts to dominates, $\beta \gg 1$. There is a radius, where both effects are balanced and where $\beta \approx 1$ for $r = r_{\mathrm{A}}$.

At radius r_{A} the magnetic field energy density just balances the wind kinetic energy density,

$$\frac{1}{2}\rho v^2 = \frac{B^2}{8\pi}.$$

This can be rewritten in terms of wind velocity,

$$v = v_{\mathrm{A}} \equiv \frac{B}{\sqrt{4\pi\rho}},$$

where v_{A} is so-called Alfvén speed. For slow wind close to the Sun, $v < v_{\mathrm{A}}$ and the magnetic field dominates, while for fast wind at large radii $v > v_{\mathrm{A}}$ and the matter dominates.

Alfvén radius

For polar magnetic field $B = B_\odot \left(r/R_\odot\right)^{-n}$ the Alfvén radius is

$$r_\mathrm{A} = R_\odot \left(\frac{B_\odot R_\odot}{\sqrt{v\dot{M}}}\right)^{\frac{1}{n-1}}.$$

We can assume that the dipole-generated magnetic field depends on the rotational frequency via

$$B_\odot = k_\odot \Omega^a,$$

where $k_\odot$ and a are constants. Collecting all terms, this gives equation for angular momentum loss

$$\dot{L} = -\frac{2}{3} k_\odot^{\frac{2}{n-1}} R_\odot^{\frac{2n}{n-1}} v^{-\frac{1}{n-1}} \dot{M}^{\frac{n-2}{n-1}} \Omega^{1+\frac{2a}{n-1}} = \eta M_\odot R_\odot^2 \dot{\Omega}.$$

Dropping all unnecessary constants,

$$\dot{\Omega} \sim -\Omega^{1+\frac{2a}{n-1}}.$$

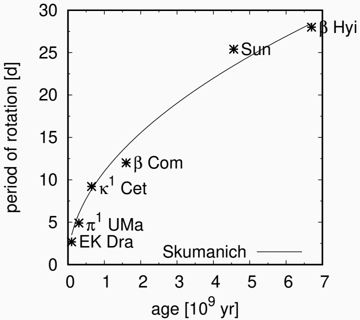
Rotational braking

The angular momentum equation has a solution

$$\Omega \sim t^{-\frac{n-1}{2a}}.$$

For a typical dynamo generated magnetic field $a = 1$ (Saar 1996), which gives for $n = 2$ so-called Skumanich law

$$\Omega \sim t^{-1/2}.$$



However, one should note that there is a saturation in early phases for $\Omega > 10\,\Omega_\odot$, where $a \approx 0$. Anyway, we have seen that the coronal wind is able to significantly break the rotation of our Sun and the Skumanich law nicely agrees with observations (Ribas a kol. 2005).

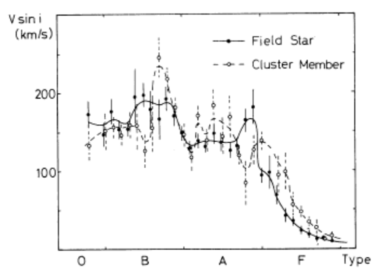
There are also other stars than our Sun

Cool stars with $T_{\rm eff} \lesssim 7000\,\mathrm{K}$ have deep subsurface convective zones. This means that cool main-sequence stars should also have coronal winds. This can be tested by

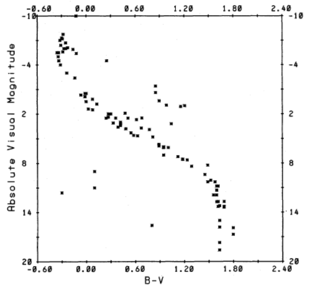
- X-ray emission,
- braking of the stellar rotation.

One can expect that young stars are fast rotators leading strong activity, which can be manifested by X-ray emission. They lose angular momentum due to winds. This leads to decrease of rotational velocity, the subsurface dynamo becomes less effective leading to the decrease of the stellar activity and X-ray emission.

Another stars: rotation & X-ray emission

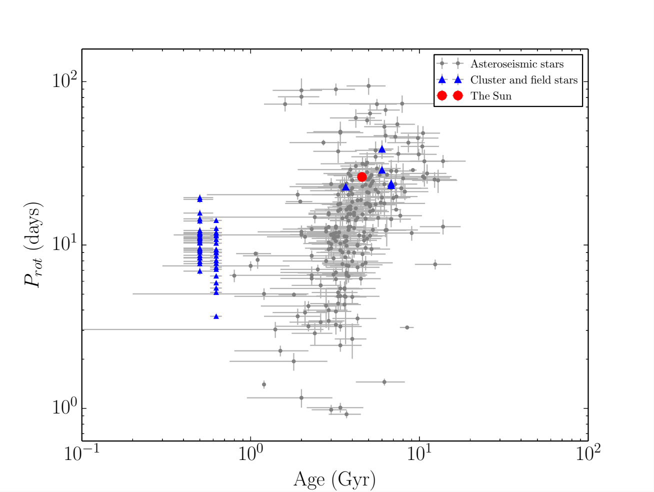


Cool stars later than F5 type show slower rotation on average (Fukuda 1982). Note the difference between field and cluster stars.



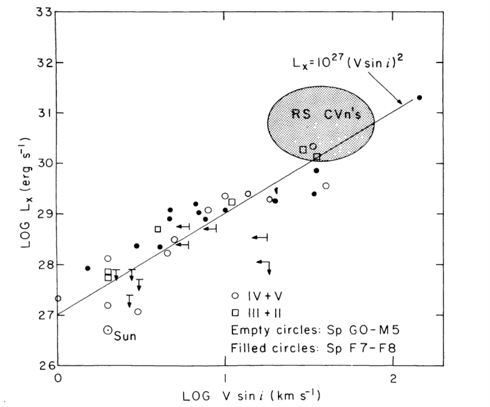
Cool stars are soft X-ray sources (Vaiana 1983, Rosner et al. 1985, EINSTEIN satellite).

Rotational braking from Kepler photometry



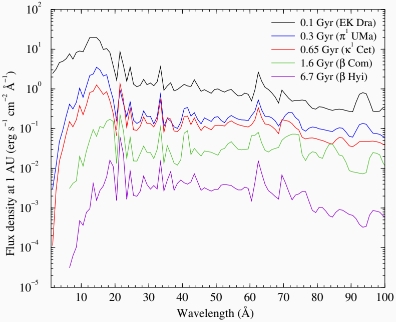
- the dependence of rotational period on stellar age for Kepler asteroseismic targets (Angus et al. 2015)

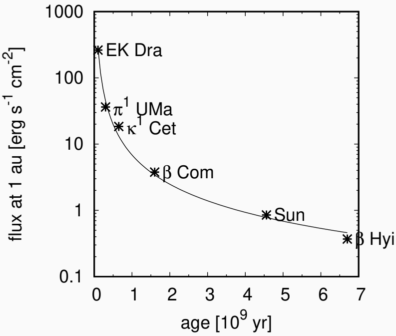
Relation between the X-ray emission and rotation speed



- relation between X-ray liminosity and rotational velocity projection (Vaiana 1983, Rosner et al. 1985, EINSTEIN satellite)

Decrease of X-ray luminosity with age



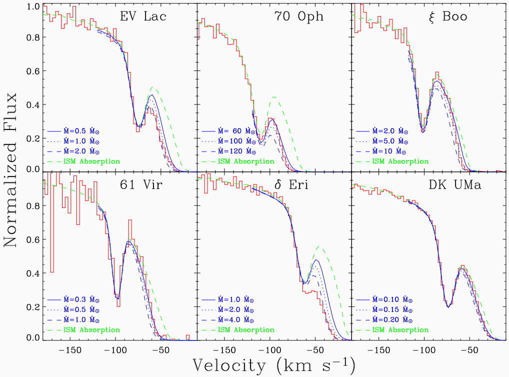


- fit of the X-rat flux (ASCA and ROSAT satellites) for stars with different age: decrease by three orders during the evolution

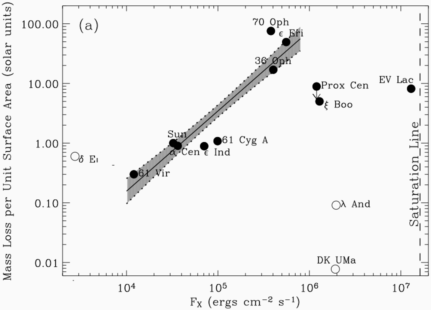
- relation between X-ray luminosity and age (Ribas a kol. 2005)

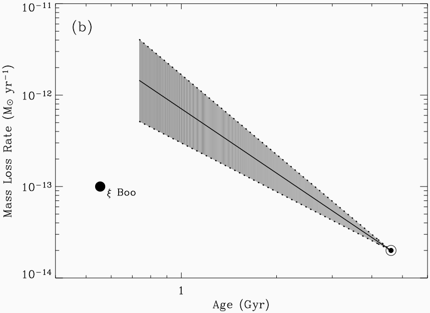
Mass-loss rate measurements

Direct mass-loss rate measurements are not available in cool stars. There are some indirect measurements, for instance, thanks to interaction of winds with interstallar enironment, which creates astrosphere (or heliosphere) around stars. The winds plough the neutral hydrogen from interstellar environment, what is detectable in Hα line. This provides a possibility to determine $\dot{M}$. (Wood and Linsky 1998, Wood et al. 2002)



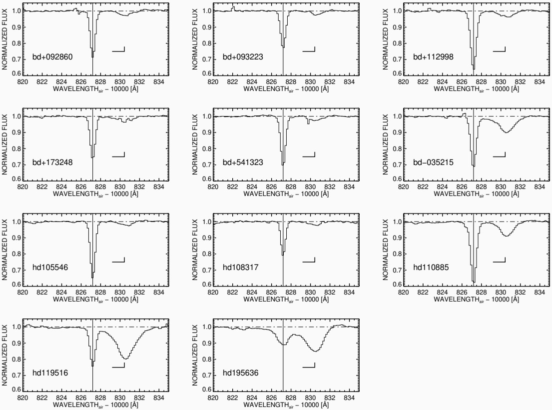
Relation of $\dot{M}$ and X-ray flux and age





(Wood a kol. 2005)

Coronal wind in cool giants?



- blueshifted IR helium lines: $\dot{M} \sim 3 \times 10^{-10} - 6 \times 10^{-8}\,\mathrm{M}_{\odot}\,\mathrm{yr}^{-1}$ (Dupree et al. 2009, mark denotes lab frame wavelength)
- possibly sound-wave driven winds (Holzer et al. 1983)?

Conclusions

Coronal winds of main-sequence stars are weak and do not influence stellar evolution significantly. In earlier phases, they might be responsible for evolution of interplanetary medium. Coronal winds are important for the interaction of stars with exoplanets and for rotational braking of main-sequence stars.

The nature of coronal heating is one of the most important open problems in astrophysics. It is most likley related to a deep hydrogen convective zone in cool stars. The convection magnifies seed magnetic fields. Sound waves generated by the convection interact with coronal magnetic field and create MHD waves. Hybrid MHD waves or Alfvén waves possibly heat the corona.

In cool gaints, the coronal winds could be important for the mass-loss (Cranmer & Saar 2011, Suzuki 2013). It is expected that our Sun will lose a fraction of its mass by this mechanism (about 0.2).

Suggested reading

S. R. Cranmer, A. R. Winebarger, 2019, ARA&A, 57, 157

H. J. G. L. M. Lamers, & J. P. Cassinelli: Introduction to Stellar Winds

S. Owocki: Stellar wind mechanisms and instabilities

F. H. Shu: The physics of astrophysics: II. Hydrodynamics