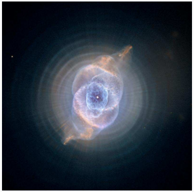


Dust-driven winds

Jiří Krtička

Masaryk University

Winds in cool luminous stars?

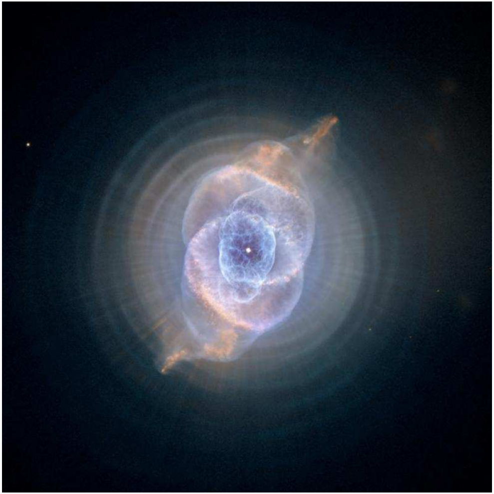
Evidence for wind in cool luminous stars

- envelopes around stellar remnants

planetary nebula Abell 39

Evidence for wind in cool luminous stars

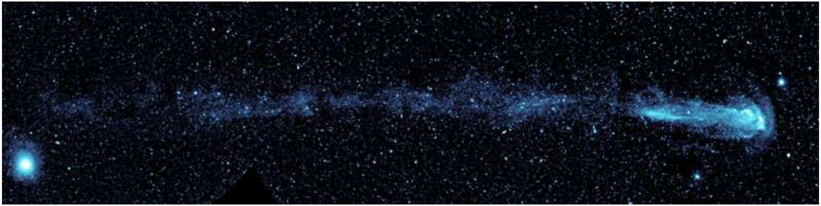
- envelopes around stellar remnants



Cat's Eye Nebula – NGC 6543 (HST)

Evidence for wind in cool luminous stars

- envelopes around stellar remnants



nebula around Mira (o Cet)

Driving the wind of cool luminous stars

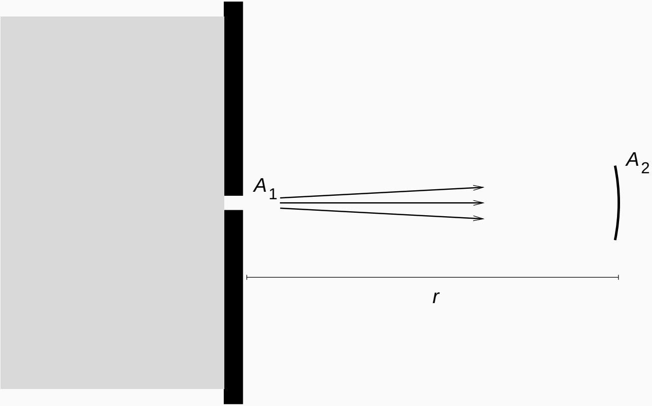
We have seen that there are observational indications of wind in cool luminous stars (AGB stars, red supergiants, $L \gtrsim 10^4\,L_\odot$). These winds are likely not connected with coronae, due to missing strong X-ray emission and chromospheric activity. On the other hand, these stars are luminous, consequently radiative force is capable to drive a wind in these stars. As a result of their low temperature, dust grains form in the envelopes of cool luminous stars. Dust grain absorb the stelar radiation and accelerate the wind giving rise to dust driven winds.

Intermezzo: Radiative transfer

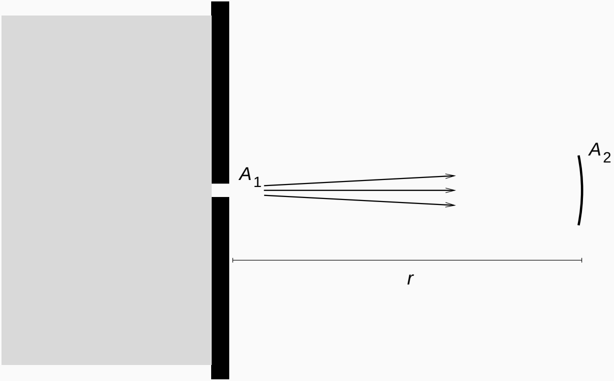
Definition of specific intensity

The *specific intensity* of radiation can be defined using an ideal apparatues (a pinhole camera). The energy collected by the detector during time Δt in a bandwidth $\Delta\nu$ is

$$\Delta E = I_\nu \frac{A_1 A_2}{r^2} \Delta t \Delta\nu.$$



Intricate dependences

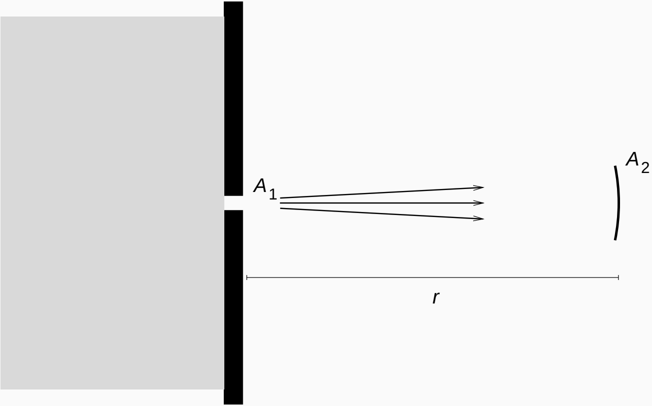


The specific intensity $I(\boldsymbol{r}, \boldsymbol{n}, \nu, t)$ depends on

- $\boldsymbol{r}$: location of the pinhole,
- t: time,
- $\boldsymbol{n}$: orientation of the screen,
- ν: frequency.

} tangent spaces $(\boldsymbol{n}, \nu)$ attached to fourdimensional spacetime manifold $(\boldsymbol{r}, t)$.

Radial dependency



$$\Delta E = I_\nu \frac{A_1 A_2}{r^2} \Delta t \Delta \nu.$$

Here $\Delta\Omega = A_2/r^2$ is the solid angle subtended by A_2 at the aperture. This means that the intensity does not depend on the location of the detector. Intensity does not change as the bundle of radiation moves.

Radiation transport equation

Intensity does not change as the bundle of radiation moves over time τ:

$$\Delta I_\nu = I_\nu(\boldsymbol{r} + \boldsymbol{n}c\tau, \boldsymbol{n}, t + \tau) - I_\nu(\boldsymbol{r}, \boldsymbol{n}, t) = 0.$$

Taylor-expanding the left-hand side and discarding terms of order τ^2 and higher we obtain radiation transport equation in an empty space

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \boldsymbol{n} \cdot \nabla I_\nu = 0.$$

Change of the intensity due to absorption over time τ: $\Delta I_\nu = -k_\nu c\tau I_\nu$.
Change of the intensity due to emission over time τ: $\Delta I_\nu = j_\nu c\tau$.

Summing all the contributions we arive at the (nonrelativistic) radiation transport equation in the form of

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \boldsymbol{n} \cdot \nabla I_\nu = j_\nu - k_\nu I_\nu.$$

Radiation transport equation: the quantities

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \boldsymbol{n} \cdot \nabla I_\nu = j_\nu - k_\nu I_\nu$$

- I_ν is the *specific intensity*, which is connected with the phase space density of photons

$$f(\boldsymbol{r}, \boldsymbol{p}, t) = \frac{c^2}{h^4 \nu^3} I_\nu,$$

  where $f(\boldsymbol{r}, \boldsymbol{p}, t)$ appears in the Boltzmann equation. (Note: radiation transport equation can be derived from the Boltzmann equation.)
- k_ν is the *absorption* (extinction) *coefficient*
- j_ν is the *emission coefficient* (emissivity)

First moment of the radiation transport equation

Taking advantage of the constancy of $\boldsymbol{n}$:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \nabla \cdot (\boldsymbol{n} I_\nu) = j_\nu - k_\nu I_\nu.$$

Integrating over the angles:

$$\frac{1}{c}\frac{\partial}{\partial t}\oint I_\nu \, \mathrm{d}\Omega + \nabla \cdot \oint \boldsymbol{n} I_\nu \, \mathrm{d}\Omega = \oint (j_\nu - k_\nu I_\nu) \, \mathrm{d}\Omega.$$

Denoting the radiation energy density E_ν and the vector flux $\boldsymbol{F}_\nu$

$$E_\nu = \frac{1}{c}\oint I_\nu \, \mathrm{d}\Omega \qquad\qquad \boldsymbol{F}_\nu = \oint \boldsymbol{n} I_\nu \, \mathrm{d}\Omega$$

we rewrite the first moment of the radiation transport equation as

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \boldsymbol{F}_\nu = \oint (j_\nu - k_\nu I_\nu) \, \mathrm{d}\Omega.$$

First moment of the radiation transport equation

Integrating the momentum equation

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \boldsymbol{F}_\nu = \oint (j_\nu - k_\nu I_\nu)\, \mathrm{d}\Omega.$$

over frequencies we obtain

$$\frac{\partial E}{\partial t} + \nabla \cdot \boldsymbol{F} = \int \mathrm{d}\nu \oint (j_\nu - k_\nu I_\nu)\, \mathrm{d}\Omega,$$

where frequency-integrated radiation energy density is $E = \int \mathrm{d}\nu\, E_\nu$ and the vector flux is $\boldsymbol{F} = \int \mathrm{d}\nu \boldsymbol{F}_\nu$.

Derived equation represents the equation of energy conservation. The terms on the left-hand side represent the conservation law with energy density and energy flux. The terms on the right-hand side describe the rates of gain (due to emission) and loss (due to absorption) of radiation energy per unit of volume.

Second moment of the radiation transport equation

Multiplying the radiation transport equation

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \nabla \cdot (\boldsymbol{n} I_\nu) = j_\nu - k_\nu I_\nu$$

by $\boldsymbol{n}$ and integrating over the angles:

$$\frac{1}{c}\frac{\partial}{\partial t}\oint \boldsymbol{n} I_\nu \, \mathrm{d}\Omega + \nabla \cdot \oint \boldsymbol{n}\boldsymbol{n} I_\nu \, \mathrm{d}\Omega = \oint (\boldsymbol{n} j_\nu - k_\nu \boldsymbol{n} I_\nu) \, \mathrm{d}\Omega.$$

Denoting the vector flux $\boldsymbol{F}_\nu$ and the pressure tensor P_ν

$$\boldsymbol{F}_\nu = \oint \boldsymbol{n} I_\nu \, \mathrm{d}\Omega \qquad \mathrm{P}_\nu = \frac{1}{c}\oint \boldsymbol{n}\boldsymbol{n} I_\nu \, \mathrm{d}\Omega$$

and assuming isotropy of j_ν and k_ν we derive the second momentum equation

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_\nu}{\partial t} + c\nabla \cdot \mathrm{P}_\nu = -k_\nu \boldsymbol{F}_\nu.$$

Second moment of the radiation transport equation

Integrating the second momentum equation

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_\nu}{\partial t} + c\nabla \cdot \mathsf{P}_\nu = -k_\nu \boldsymbol{F}_\nu.$$

over frequencies wind dividing by c we obtain

$$\frac{1}{c^2}\frac{\partial \boldsymbol{F}}{\partial t} + \nabla \cdot \mathsf{P} = -\int \mathrm{d}\nu\, k_\nu c \frac{\boldsymbol{F}_\nu}{c^2},$$

where frequency-integrated vector flux is $\boldsymbol{F} = \int \mathrm{d}\nu \boldsymbol{F}_\nu$ and the pressure tensor is $\mathsf{P} = \int \mathrm{d}\nu \mathsf{P}_\nu$.

The radiation momentum density is $\boldsymbol{F}/c^2$ and the momentum flux is P. Therefore the terms on the left-hand side represent the conservation law with momentum density and momentum flux. The right-hand side represents the momentum lost per unit time.

Coupling with Euler's equaions

The continuity equation remains to be the same:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0.$$

The loss of photon momentum is the gain of momentum of matter. Therefore, the negative of the photom momentum loss rate is the radiative force that shall be included in the momentum equation

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \rho \boldsymbol{g} + \frac{1}{c} \int \mathrm{d}\nu \, k_\nu \boldsymbol{F}_\nu.$$

Similarly, the loss of radiation energy is the gain of energy of matter. As a result, the negative of the energy loss rate shall be included in the equation for energy

$$\frac{\partial}{\partial t}\left(\rho\epsilon + \frac{\rho v^2}{2}\right) + \nabla \cdot \left[\rho \boldsymbol{v}\left(\epsilon + \frac{v^2}{2}\right) + p\boldsymbol{v}\right] = \rho \boldsymbol{v}\boldsymbol{g} - \int \mathrm{d}\nu \oint (j_\nu - k_\nu I_\nu)\, \mathrm{d}\Omega.$$

Driving by radiative force due to dust

Dust driven wind equations

Again, let us assume that the dust driven winds can be described as a spherically symmetric, stationary, and isothermal outlow. Then the corresponding hydrodynamical equations are

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0,$$

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} = -a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} - \frac{\rho GM}{r^2} + \rho g_{\mathrm{rad}},$$

where g_{rad} is the radiative force.

The integration of the continuity equation gives the **mass-loss rate**

$$\dot{M} \equiv 4\pi r^2\rho v = \mathrm{const.}$$

Inserting $\mathrm{d}\rho/\mathrm{d}r$ from the continuity equation into the equation of motion gives ordinary differential equation for velocity

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} - \frac{GM}{r^2} + g_{\mathrm{rad}}.$$

In cool dust driven winds, one can neglect $\frac{2a^2}{r} \ll \frac{GM}{r^2}$.

Sonic point condition

From the momentum equation

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = g_{\mathrm{rad}} - \frac{GM}{r^2}$$

follows that at the sonic point $v = a$ the radiative force equals the gravity (in magnitude)

$$g_{\mathrm{rad}} = \frac{GM}{r^2}.$$

The gravity is stronger than the radiative force in subsonic region $v < a$

$$g_{\mathrm{rad}} < \frac{GM}{r^2},$$

while the radiation dominates in the supersonic region $v > a$

$$g_{\mathrm{rad}} > \frac{GM}{r^2}.$$

Radiative force due to the dust

The radiative force due to absorption of radiation on dust particles is

$$f_{\mathrm{rad}} = \rho g_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu,$$

where $\chi(r,\nu)$ is the absorption coefficient and $F(r,\nu)$ is the radiative flux. Because $\kappa(r,\nu) = \chi(r,\nu)/\rho(r)$ varies only due to a change of the dust fraction and $F(r,\nu)/F(r)$ varies mostly with frequency, the radiative acceleration is

$$g_{\mathrm{rad}} = \frac{1}{c}\,\overline{\kappa}(r)F(r),$$

where the mean opacity is

$$\overline{\kappa}(r) = \int_0^\infty \kappa(r,\nu)\frac{F(r,\nu)}{F(r)}\,\mathrm{d}\nu$$

and $F(r)$ is the integrated flux, $F(r) = L/(4\pi r^2)$.

Eddington parameter due to dust

Introducing the ratio of the radiative and gravity acceleration,

$$\Gamma_{\rm d}(r) = \frac{\overline{\kappa}(r)L}{4\pi cGM},$$

the momentum equation can be rewritten as

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{GM}{r^2}\left(1 - \Gamma_{\rm d}(r)\right).$$

There are three regions of the wind:

- subsonic region, $v < a$, $\Gamma_{\rm d}(r) < 1$,
- sonic point, $v = a$, $\Gamma_{\rm d}(r) = 1$ (formation of the dust),
- supersonic region $v > a$, $\Gamma_{\rm d}(r) > 1$.

How to create the dust?

easy job...

How to create the dust?

- the condensation core forms containing about ~ 10 atoms
- another molecules join the condensation core (Gail & Sedlmayr 1987)
- typical dimension of dust particles is about $\sim 0.1\,\mu$m
- chemical composition of dust particles depends on the chemical composition of the star
  - available carbon and oxygen create CO – stable molecule, some oxygen of carbon remain
  - **oxygen rich stars**: silicates (Mg_2SiO_4, $MgSiO_3$) – more frequent,higher abundance of oxygen in Universe, for example in M supergiants
  - **carbon rich stars**: carbon dust grains (amorphous carbon), SiC, MgS, Fe – result of "dredge-up" process, products of nuclear burning na povrchu

Mass-loss rate

Limiting mass-loss rate I.

- every photon carries momentum

$$\frac{h\nu}{c}$$

- total momentum of photons leaving a star per unit of time

$$\frac{L}{c}$$

- total momentum of wind leaving a star per unit of time denoting v_∞ as terminal wind speed

$$\dot{M} v_\infty$$

- this gives the condition

$$\eta_{\rm mom} = \frac{\dot{M} v_\infty}{L/c} \leq 1$$

- or for maximum mass-loss rate corresponding to $\eta_{\rm mom} = 1$

$$\dot{M}_{\rm max} = \frac{L}{v_\infty c} = 2 \times 10^{-5} {\rm M}_\odot\,{\rm yr}^{-1} \left(\frac{L}{10^4\,{\rm L}_\odot}\right) \left(\frac{v_\infty}{10\,{\rm km\,s^{-1}}}\right)^{-1}$$

Limiting mass-loss rate II.

However, the momentum is a vector and the total momentum loss via spherically symmetric wind is zero. The previous estimate corresponds to one scattering per photon (single scattering limit), but there might be more scattering events per photon.

Energy conditions should give more reliable upper limit

$$\frac{1}{2}v_\infty^2\dot{M} \leq L,$$

which gives for the maximum mass-loss rate

$$\dot{M}_{\rm max} = \frac{L}{v_\infty^2} = 1\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}\left(\frac{L}{10^4\,\mathrm{L}_\odot}\right)\left(\frac{v_\infty}{10\,\mathrm{km\,s}^{-1}}\right)^{-2}.$$

We also note that for the single-scattering limit $\dot{M} \approx L/\left(v_\infty c\right)$ the total energy of the wind is much lower than the stellar luminosity,

$$\frac{1}{2}v_\infty^2\dot{M} \approx \frac{1}{2}\frac{v_\infty}{c}L \ll L.$$

Estimating the mass-loss rate

Let us start with the momentum equation

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{\rho G M}{r^2} + \Gamma_\mathrm{d} \frac{\rho G M}{r^2}$$

and let us assume that dust starts to form close to the sonic point r_c, where Γ_d changes from $\Gamma_\mathrm{d} \ll 1$ to $\Gamma_\mathrm{d} \gg 1$. We shall multiply the momentum equation by $4\pi r^2$ and integrate the equation from R_* to ∞

$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r + \int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho G M}{r^2}\right] \mathrm{d}r = \int_{R_*}^{\infty} 4\pi r^2 \Gamma_\mathrm{d} \frac{\rho G M}{r^2} \mathrm{d}r.$$

The term in the first integral can be taken out assuming constant mass-loss rate $\dot{M} = 4\pi r^2 \rho v$. The integral can be evaluated assuming hydrostatic equilibrium in the atmosphere $v(R_*) \approx 0$ and using wind terminal velocity $v_\infty = v(r = \infty)$. Therefore,

$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r = \dot{M} \left[v\right]_{R_*}^{\infty} \approx \dot{M} v_\infty.$$

Estimating the mass-loss rate

Let us start with the momentum equation

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{\rho GM}{r^2} + \Gamma_\mathrm{d} \frac{\rho GM}{r^2}$$

and let us assume that dust starts to form close to the sonic point r_c, where Γ_d changes from $\Gamma_\mathrm{d} \ll 1$ to $\Gamma_\mathrm{d} \gg 1$. We shall multiply the momentum equation by $4\pi r^2$ and integrate the equation from R_* to ∞

$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r + \int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r = \int_{R_*}^{\infty} 4\pi r^2 \Gamma_\mathrm{d} \frac{\rho GM}{r^2} \mathrm{d}r.$$

The second integral gives the hydrostatic equilibrium density distribution for $r < r_\mathrm{c}$, while the pressure gradient becomes neglibible for $r < r_\mathrm{c}$. Therefore,

$$\int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r = \int_{R_*}^{r_\mathrm{c}} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r + \\ + \int_{r_\mathrm{c}}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r \approx 4\pi GM \int_{r_\mathrm{c}}^{\infty} \rho \, \mathrm{d}r.$$

Estimating the mass-loss rate

Let us start with the momentum equation

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{\rho GM}{r^2} + \Gamma_\mathrm{d} \frac{\rho GM}{r^2}$$

and let us assume that dust starts to form close to the sonic point r_c, where Γ_d changes from $\Gamma_\mathrm{d} \ll 1$ to $\Gamma_\mathrm{d} \gg 1$. We shall multiply the momentum equation by $4\pi r^2$ and integrate the equation from R_* to ∞

$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r + \int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r = \int_{R_*}^{\infty} 4\pi r^2 \Gamma_\mathrm{d} \frac{\rho GM}{r^2} \mathrm{d}r.$$

The right-hand side integral can be evaluated using wind optical depth $\tau_\mathrm{W} = \int_{r_\mathrm{c}}^{\infty} \overline{\kappa}(r) \rho \, \mathrm{d}r$,

$$\int_{R_*}^{\infty} 4\pi r^2 \Gamma_\mathrm{d} \frac{\rho GM}{r^2} \mathrm{d}r = 4\pi GM \int_{r_\mathrm{c}}^{\infty} \Gamma_\mathrm{d} \rho \, \mathrm{d}r = \frac{L}{c} \int_{r_\mathrm{c}}^{\infty} \overline{\kappa}(r) \rho \, \mathrm{d}r = \tau_\mathrm{W} \frac{L}{c}.$$

Estimating the mass-loss rate

Putting all the terms together we derive

$$\dot{M}v_\infty = 4\pi GM \int_{r_\mathrm{c}}^{\infty} \rho \, \mathrm{d}r + \tau_\mathrm{W}\frac{L}{c}.$$

For $\Gamma_\mathrm{d} \gg 1$ the gravity term can be neglected with respect to the radiative acceleration and we derive the mass-loss rate estimate

$$\dot{M}v_\infty = \tau_\mathrm{W}\frac{L}{c}.$$

We see that by assuming multiple scattering ($\tau_\mathrm{W} > 1$) the mass-loss rate can be significantly higher than the single-scattering limit $\dot{M}v_\infty = L/c$ (Gail a Sedlmayr 1986, Netzer a Elitzur 1993).

Dust and gas

Opacity due to dust particles

Cross-section for the **light extinction** (or true absorption) due to the dust particles can be written as

$$C^{\mathsf{A}} = \pi a^2 Q^{\mathsf{A}}(a, \lambda),$$

- a is particle radius,
- Q^{A} is the efficiency of absorption,
- important for both the radiative force and temperature.

Cross-section for the **light scattering** is

$$C^{\mathsf{S}} = \pi a^2 Q^{\mathsf{S}}(a, \lambda),$$

- a is particle radius,
- Q^{S} is the efficiency of scattering with $Q^{\mathsf{S}} \sim \lambda^{-4}$,
- influences the radiative force but not the temperature.

Opacity due to dust particles: accounting for dust distribution

Accounting for the distribution of dust sizes, the total extinction coefficient due to true absorption is

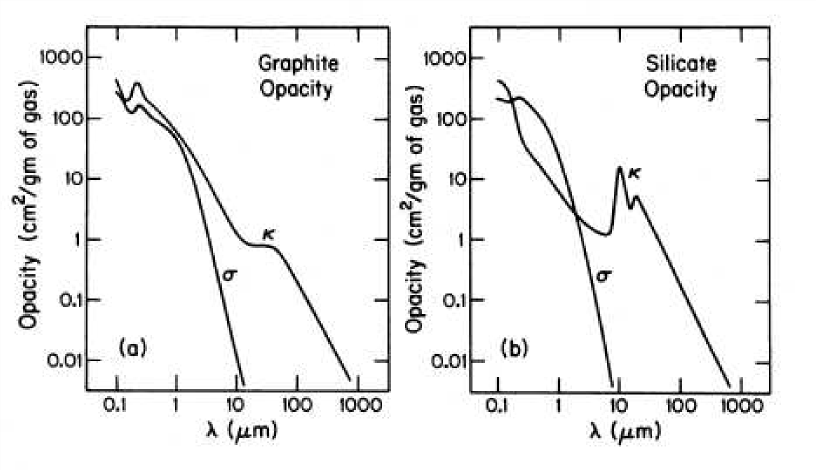
$$\chi^{\mathsf{A}} = \kappa_\lambda \rho = \int_{a_{\min}}^{a_{\max}} n(a) \pi a^2 Q^{\mathsf{A}}(a, \lambda) \, \mathrm{d}a.$$

Similarlym the total extinction coefficient due to light scattering is

$$\chi^{\mathsf{S}} = \sigma_\lambda \rho = \int_{a_{\min}}^{a_{\max}} n(a) \pi a^2 Q^{\mathsf{S}}(a, \lambda) \, \mathrm{d}a.$$

Here $n(a)$ je is the distrubution function of radii, $n(a) \sim a^{-3.5}$ (Mathis, Rumpl & Nordsieck 1977), with $a_{\min} = 0.005\,\mu$m, $a_{\max} = 0.25\,\mu$m.

Opacity due to dust particles



Wolfire a Cassinelli (1986)

Coupling dust and gas

Volatile particles (H, H_2, He) do not condensate into dust particles. Therefore, dust driven wind contains also gas and has a two-component structure:

- dust particles accelerated due to absorption of photons,
- dust particles transfer momentum by scattering.

Momentum equation of dust and gas

Assuming just single radius of dust particles a and neglecting scattering $\sigma_\lambda = 0$ the equation of motion of dust particles has the form of

$$\rho_d v_d \frac{dv_d}{dr} = -\frac{\rho_d GM}{r^2} + \frac{\overline{\chi}(r)L}{4\pi c r^2} - n_d f_{drag}$$

- $\rho_d = n_d m_d$ is dust density,
- f_{drag} is the friction force,
- $\overline{\chi}(r) = \int_0^\infty \chi(r,\nu) \frac{F(r,\nu)}{F(r)} \, d\nu$,
- $\chi(r,\nu) = n_d \pi a^2 Q^A(a,\lambda)$.

Momentum equation of gas is

$$\rho_g v_g \frac{dv_g}{dr} = -\frac{dp}{dr} - \frac{\rho_g GM}{r^2} + n_d f_{drag}.$$

Solving for the two-component flow

Neglecting inertial terms and gravity ($\Gamma_\mathrm{d} \gg 1$) in the equation for the dust, we arrive at the balance between radiative and frictional force (Gilman 1972)

$$\frac{\pi a^2 Q L}{4\pi c r^2 m_\mathrm{d}} = \frac{f_\mathrm{drag}}{m_\mathrm{d}},$$

where $Q = \int_0^\infty Q^\mathrm{A}(a,\lambda)\frac{F(r,\nu)}{F(r)}\,\mathrm{d}\nu$.

Introducing relative velocity of dust and gas $w_\mathrm{drift} = v_\mathrm{d} - v_\mathrm{g}$, the frictional force can be written as $f_\mathrm{drag} = \pi a^2 \rho_\mathrm{g} w_\mathrm{drift}^2$ (with ram pressure $\rho_\mathrm{g} w_\mathrm{drift}^2$ for $w_\mathrm{drift} \gg a_\mathrm{th}$). This gives for the drift velocity

$$w_\mathrm{drift} = \sqrt{\frac{QL}{4\pi r^2 \rho_\mathrm{g} c}} = \sqrt{\frac{QLv_\mathrm{g}}{\dot{M}c}}$$

Momentum equation for the gas

Inserting the frictional force from the momentum equation for dust the gas equation reads

$$v_{\rm g}\frac{{\rm d}v_{\rm g}}{{\rm d}r}=-\frac{1}{\rho_{\rm g}}\frac{{\rm d}p}{{\rm d}r}-\frac{GM}{r^2}+\frac{n_{\rm d}}{\rho_{\rm g}}\frac{\pi a^2 QL}{4\pi c r^2}.$$

This can be rewritten as

$$v_{\rm g}\frac{{\rm d}v_{\rm g}}{{\rm d}r}=-\frac{1}{\rho_{\rm g}}\frac{{\rm d}p}{{\rm d}r}-\frac{GM}{r^2}\left(1-\Gamma_{\rm d}\right)$$

with the Eddington ration

$$\Gamma_{\rm d}=\frac{n_{\rm d}}{\rho_{\rm g}}\frac{\pi a^2 QL}{4\pi cGM}\equiv\frac{\overline{\kappa}(r)L}{4\pi cGM}.$$

Therefore, equation for the gas remains the same, but the interaction of gas and dust introduces nonzero drift velocity between these components. Large drift velocity may lead to destruction of duist particles. Low-density winds may not be able to accelerate gas leading to purely dust wind.

Wind radial velocity

Neglecting the gas pressure term, the momentum equation

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = (\Gamma_\mathrm{d}(r) - 1)\frac{GM}{r^2}$$

can be integrated. Substituting $v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{1}{2}\frac{\mathrm{d}(v^2)}{\mathrm{d}r}$, we have for the radial wind velocity

$$v^2(r) = v^2(r_\mathrm{c}) + 2GM\int_{r_\mathrm{c}}^{r} \frac{\Gamma_\mathrm{d} - 1}{{r'}^2}\,\mathrm{d}r'.$$

The wind speed at the critical point can me typically neglected. Moreover, one can assume that the Eddingtin parameter is constant $\Gamma_\mathrm{d} = \mathrm{const}$. This yelds for the wind velocity "beta" velocity law in the form of

$$v(r) = v_\infty\sqrt{1 - \frac{r_\mathrm{c}}{r}}$$

with **wind terminal velocity**

$$v_\infty = \sqrt{2GM\,(\Gamma_\mathrm{d} - 1)/r_\mathrm{c}}$$

propotional to he escape speed.

Minimum stellar luminosity to drive a wind

As a necessary condition for the wind existence, the radiative force should overcome the gravity,

$$\Gamma_\mathrm{d} > 1.$$

Inserting the formula for the Eddington parameter,

$$\frac{\overline{\kappa} L}{4\pi c G M} > 1,$$

this gives the condition for the stellar luminosity,

$$L > \frac{4\pi c G M}{\overline{\kappa}}.$$

Assuming a typical mean opacity $\overline{\kappa} \approx 30\,\mathrm{cm^2\,g^{-1}}$, the minimum stellar luminosity to drive a wind is (in scaled quantities)

$$L > 400\,\mathrm{L_\odot} \left(\frac{M}{1\,\mathrm{M_\odot}}\right).$$

This means, that evolved solar-mass stars with $L > 400\,\mathrm{L_\odot}$ may drive a dust driven wind.

There are more things in wind

Condensation radius of dust particles

The radius where the dust forms is given by the condensation temperature of dust particles T_c

- for $T < T_c$: dust particles condensed,
- for $T > T_c$: dust particles do not form,
- radius r_c where $T = T_c$ defines condensation radius.

Temperature of dust particles given by radiative equilibrium

$$\int_0^\infty \kappa_\lambda \, B_\lambda(T) \, \mathrm{d}\lambda = \int_0^\infty \kappa_\lambda \, J_\lambda \, \mathrm{d}\lambda,$$

- left hand side gives radiative cooling,
- right hand side gives radiative heating,
- $B_\lambda(T)$ is the Planck function,
- J_λ is the mean inensity.

Radiative equilibrium

Approximating the mean intensity by

$$J_\lambda \approx W(r) B_\lambda(T_{\mathrm{eff}})$$

with the dilution factor $W = \frac{1}{2}\{1 - [1 - \left(\frac{R_*}{r}\right)^2]^{1/2}\}$, the radiative equilibrium for each particle with radius a

$$\int_0^\infty \pi a^2 Q^{\mathrm{A}}(a,\lambda)\, B_\lambda(T)\, \mathrm{d}\lambda = \int_0^\infty \pi a^2 Q^{\mathrm{A}}(a,\lambda)\, W(r) B_\lambda(T_{\mathrm{eff}})\, \mathrm{d}\lambda,$$

$$\int_0^\infty Q^{\mathrm{A}}(a,\lambda)\, B_\lambda(T)\, \mathrm{d}\lambda = \int_0^\infty Q^{\mathrm{A}}(a,\lambda)\, W(r) B_\lambda(T_{\mathrm{eff}})\, \mathrm{d}\lambda.$$

Introducing Planck mean absorption efficiency

$$Q_{\mathrm{P}}^{\mathrm{A}}(a,T) = \frac{\int_0^\infty Q^{\mathrm{A}}(a,\lambda)\, B_\lambda(T)\, \mathrm{d}\lambda}{\int_0^\infty B_\lambda(T)\, \mathrm{d}\lambda}$$

gives for each particle

$$T^4 Q_{\mathrm{P}}^{\mathrm{A}}(a,T) = T_{\mathrm{eff}}^4 W(r) Q_{\mathrm{P}}^{\mathrm{A}}(a,T_{\mathrm{eff}}).$$

Condensation radius

From this we can estimate the temperature of dust particles

$$T = T_{\text{eff}} \left[W(r)\right]^{1/4} \left[\frac{Q_{\text{P}}^{\text{A}}(a, T_{\text{eff}})}{Q_{\text{P}}^{\text{A}}(a, T)}\right]^{1/4} .$$

This equation gives in the photosphere with $r \approx R_*$ the value of $W(r) = 2$ and $T \approx T_{\text{eff}} > T_{\text{c}}$. This means that the dust is destroyed close to the star.

For $r \gtrsim 2\, R_*$ one can approximate $W(r) \approx (R_*/2r)^2$, which for $T = T_{\text{c}}$ gives the condensation radius

$$r_{\text{c}} \approx \frac{R_*}{2} \left(\frac{T_{\text{eff}}}{T_{\text{c}}}\right)^2 \sqrt{\frac{Q_{\text{P}}^{\text{A}}(a, T_{\text{eff}})}{Q_{\text{P}}^{\text{A}}(a, T_{\text{c}})}}.$$

The condensation radius grows with luminosity, $r_{\text{c}} \sim L^{1/2}$.

Problem with too high condensation radii

The condition of radiative equilibrium on dust particles

$$\int_0^\infty \kappa_\lambda B_\lambda(T)\,\mathrm{d}\lambda = \int_0^\infty \kappa_\lambda J_\lambda\,\mathrm{d}\lambda$$

predicts too high dust temperature in the atmosphere. The temperature is higher than the condensation temperature T_c. The condensation may appar only at larger distances from the star, which are higher than the condensation radius, $r > r_\mathrm{c}$. This is a problem for radiative driving.

	silicate	graphite	amorphous carbon
	$T_\mathrm{c} = 1500\,\mathrm{K}$	$T_\mathrm{c} = 1500\,\mathrm{K}$	$T_\mathrm{c} = 1500\,\mathrm{K}$
T_eff	r_c/R_*	r_c/R_*	r_c/R_*
3000	2.99	4.03	3.42
2500	1.85	2.34	2.12
2000	1.15	1.29	1.24

(Lamers a Cassinelli 1999)

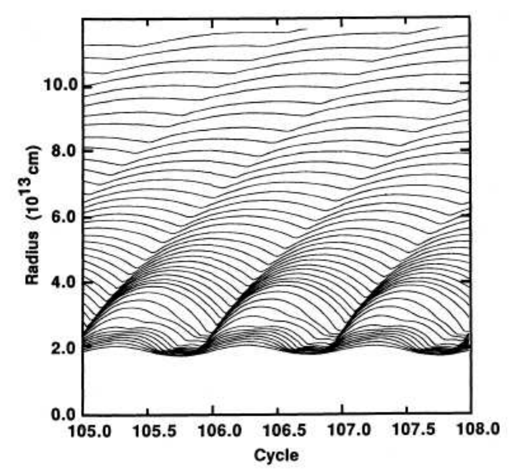
Pulsations: the missing ingredient

Pulsations in cool stars

Asymptotic giant branch stars show pulsations. They belong the the class of variables known as **Mira variables**. These stars pulsate due to instability connected with hydrogen ionization zone. The resulting shock wave dissociates molecules (TiO) leading to a significant decrease of opacity and huge associated brightness variations in the optical.

Solution of the problem with too high condensation radii

Luminous cool stars pulsate, consequently, pulsations may transfer the stellar matter to a large distances from the star.



trajectories of pulsating particles without radiative acceleration on dust particles (Bowen 1988)

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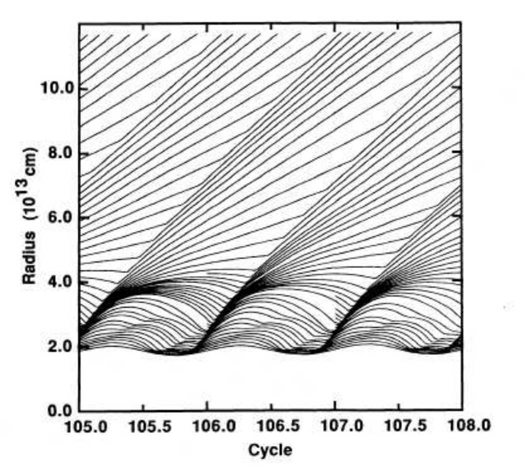

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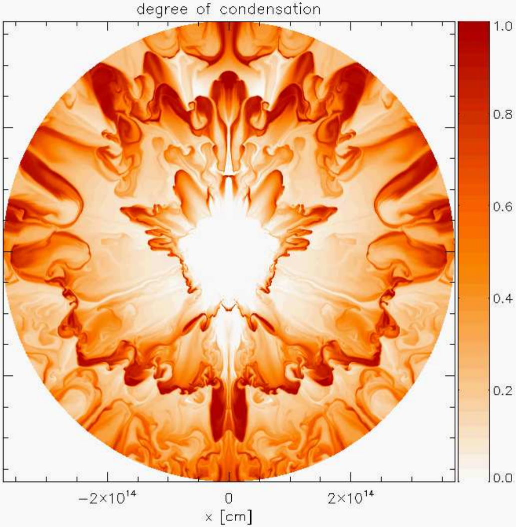
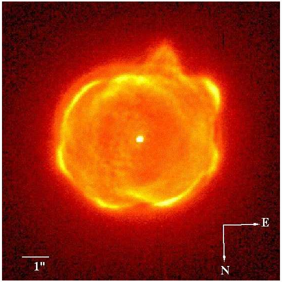
The realistic model

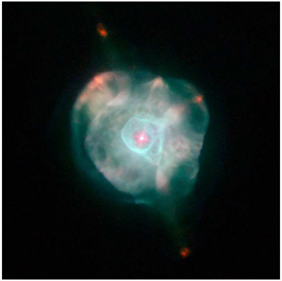


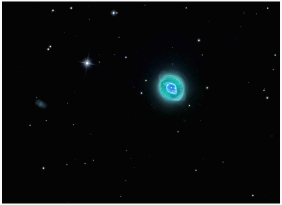
(Woitke 2005)

Observational tests

Planetary nebulae



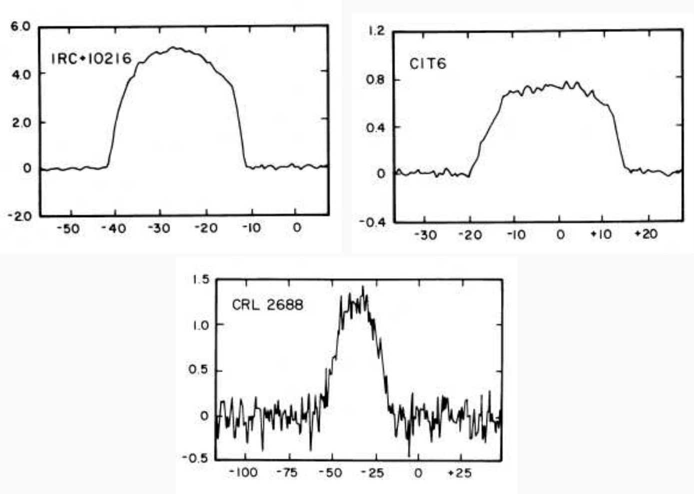




- central stars after AGB phase when it lost its envelope
- hot central star able to ionize matter lost in AGB phase

Infrared emission lines from winds

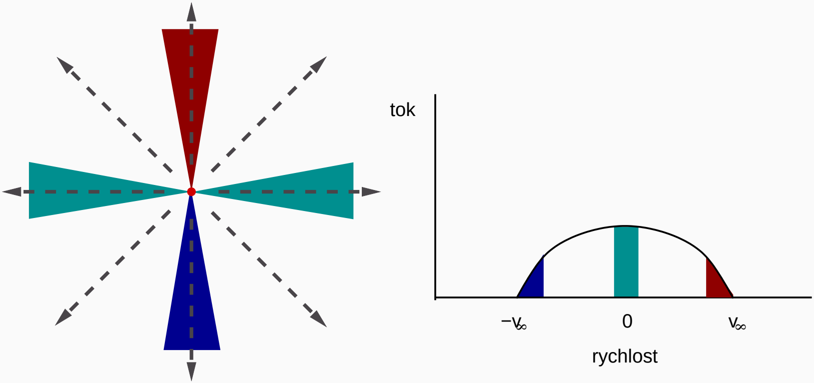
CO line ($\lambda = 2.6\,\mathrm{mm}$) corresponding to rotational transition $J = 2 - 1$



- dependence of emission temperature on velocity in $\mathrm{km\,s^{-1}}$ (Knapp a Morris 1985)

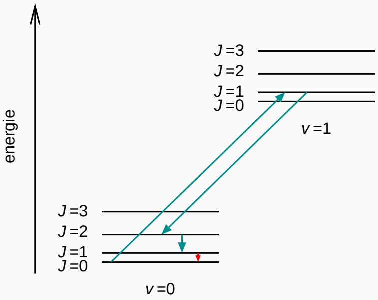
Origin of emission lines

Analysis of emission lines ($I \sim e^{-\tau}$):



- origin: envelope with radius $\sim 10^4\,R_*$
- wind velocity $\sim 10\,\mathrm{km\,s^{-1}}$
- mass-loss rates $10^{-8} - 10^{-4}\,\mathrm{M_\odot\,yr^{-1}}$

Maser lines in spectra



- levels out of LTE, kinetic equilibrium needed

$$n_j B_{ij} J_\nu = n_j A_{ji} + n_j B_{ji} J_\nu$$

- J_ν mean intensity of radiation,
- B_{ij}, A_{ji}, B_{ji} Einstein coefficients,
- B_{ij} absorption rate,
- A_{ji} spontaneous emission rate,
- B_{ji} stimulated emission rate,

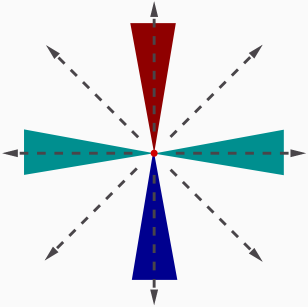
$$g_i B_{ij} = g_j B_{ji}.$$

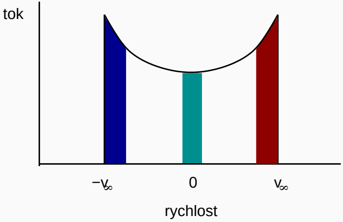
Origin of maser lines

The statistical equilibrium equation

$$n_i B_{ij} J_\nu = n_j A_{ji} + n_j \frac{g_i}{g_j} B_{ij} J_\nu$$

in the case of population inversion $n_j/n_i > g_j/g_i$ has a negative coefficient for stimulated emission, $1 - \frac{g_i n_j}{g_j n_i} < 0$ (photon passing by produce adittional photon) giving negative optical depths. This leads to typical maser lines ($I \sim \mathrm{e}^{-\tau}$).





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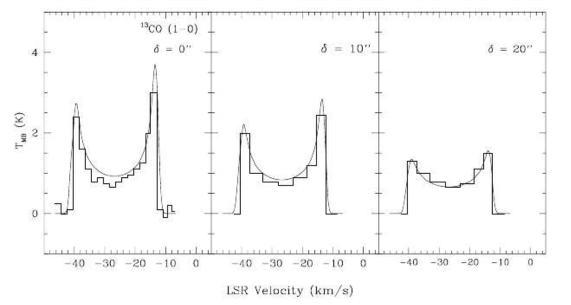
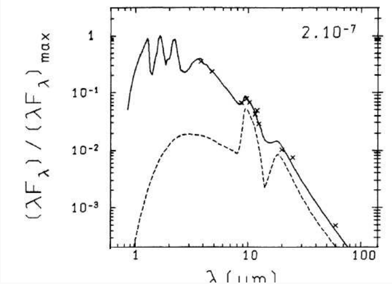


maser CO lines (Crosas a Menten 1997)

Spectral energy distribution: low mass-loss rate

For low mass-loss rate, the extinction due to dust is low. The optical and near-IR region is dominated by the photosphere, while dust contributes to the radiation at longer wavelengths.

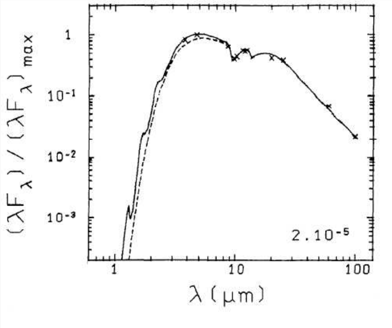
Figure shows observed SED (solid line), while dashed lines gives contribution due to the dust in the wind with $\dot{M} = 2 \cdot 10^{-7}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$ (Bedijn 1987).



Spectral energy distribution: high mass-loss rate

For high mass-loss rate the photosphere becomes invisible and only dust is visible. This appears in OH/IR stars showing OH masers and IR excess.

Figure shows observed SED (solid line), while dashed lines gives contribution due to the dust in the wind with $\dot{M} = 2 \cdot 10^{-5}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$ (Bedijn 1987). Fitting SED gives a possibility to determine $\dot{M}$ (knowing the dust fraction).



The bigger picture

Mass-loss rate formulas

- empirical Reimers law

$$\dot{M} = 4 \cdot 10^{-13}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}\,\eta\,\frac{\left(\frac{L}{L_\odot}\right)\left(\frac{R}{R_\odot}\right)}{\left(\frac{M}{M_\odot}\right)}$$

  with a correction factor η

- predictions based on numerical models (Mattsson a kol. 2009)

Dust driven winds: importance

Dust driven winds appear during the AGB phase of low-mass stars with initial mass $0.4\,\mathrm{M}_\odot \lesssim M_0 \lesssim 8\,\mathrm{M}_\odot$. A typical wind mass-loss rate due to dust driven wind is of the order of $10^{-7}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1} - 10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$. Given a typical duration of AGB phase (which is of the order of $10 \times 10^{6}\,\mathrm{yr}$), low-mass stars lose a significant amount of their mass via dust driven winds. Dust driven wind could be important also for other types of stars (perhaps red supergiants).

Part of the material lost during the AGB phase is ionized in a subsequent evolutionary phase and form a planetary nebula. Dust driven winds carry freshly synthethised s-process elements, consequently AGB stars are an important source of elements heavier than iron.

Suggested reading

L. Decin: Annual Review of Astronomy and Astrophysics, 2021, 59, 337

H. J. G. L. M. Lamers, & J. P. Cassinelli: Introduction to Stellar Winds