Stellar winds of hot stars

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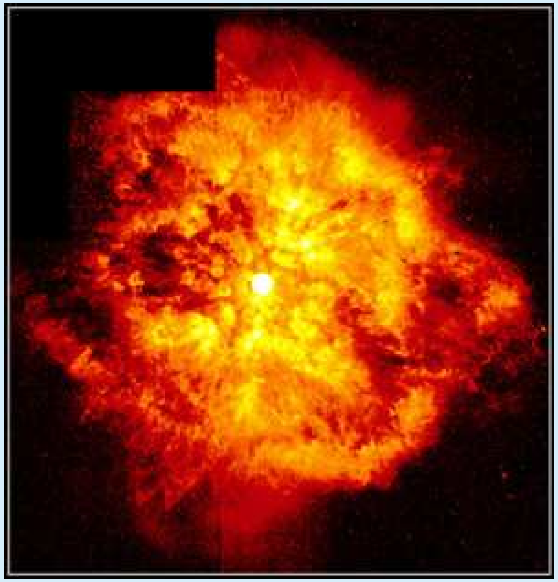
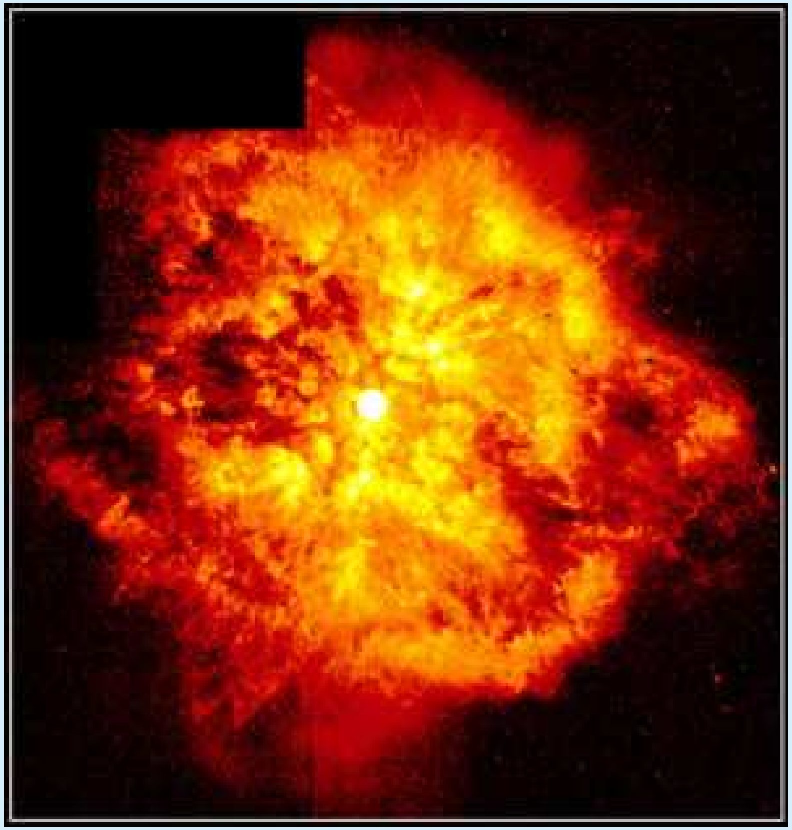


Observation of hot stars

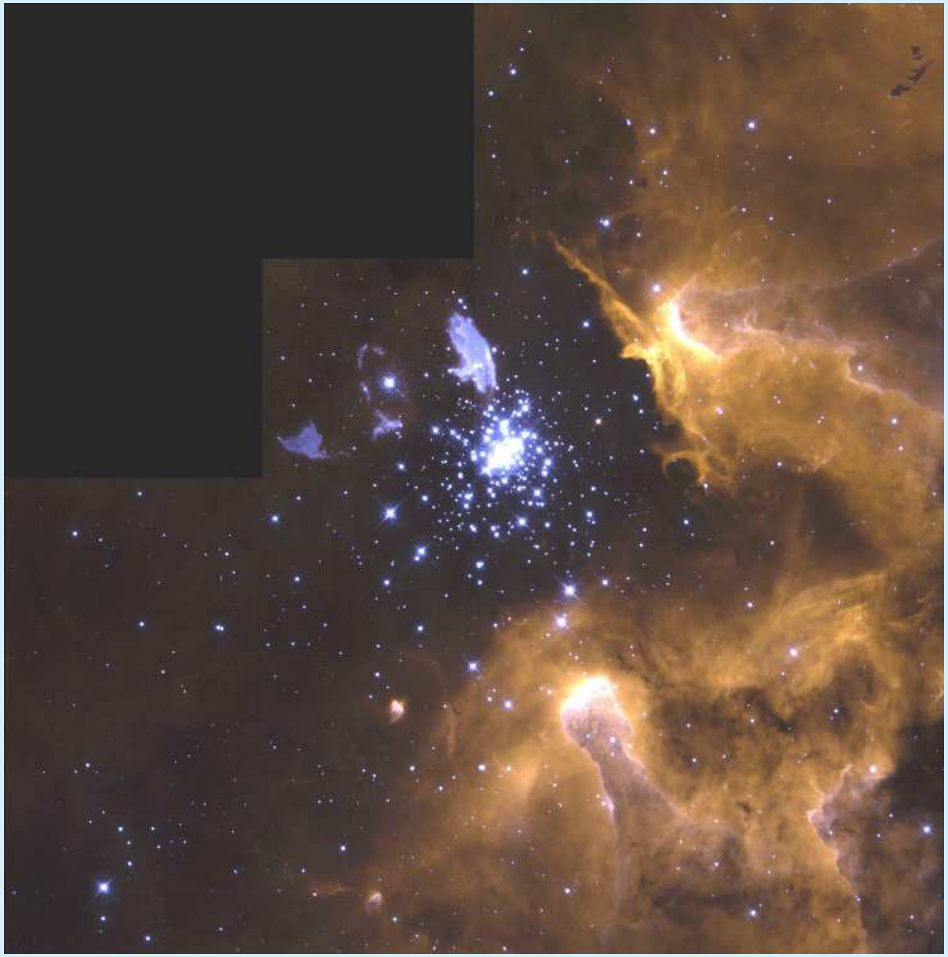
- shells in the surroundings of hot stars



nebula close to the star WR 124 (HST)

Observation of hot stars

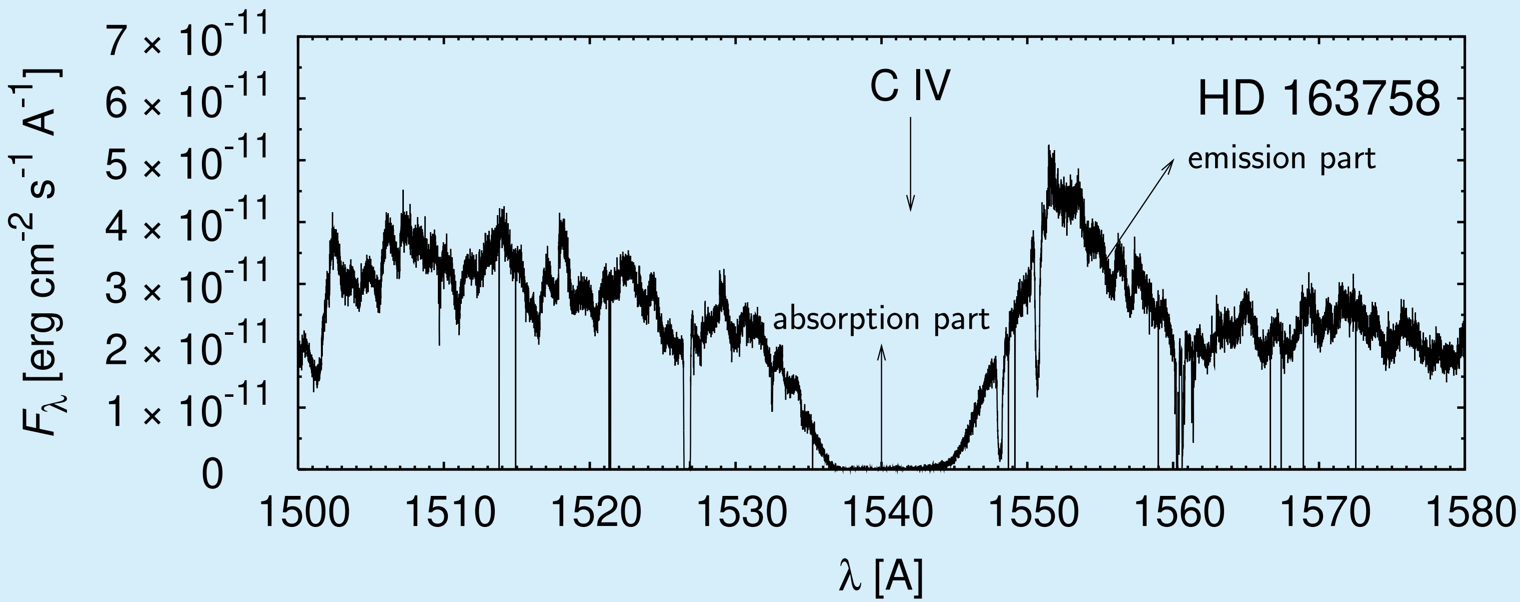
- the interstellar medium around hot stars



open cluster NGC 3603 (HST)

Observation of hot stars

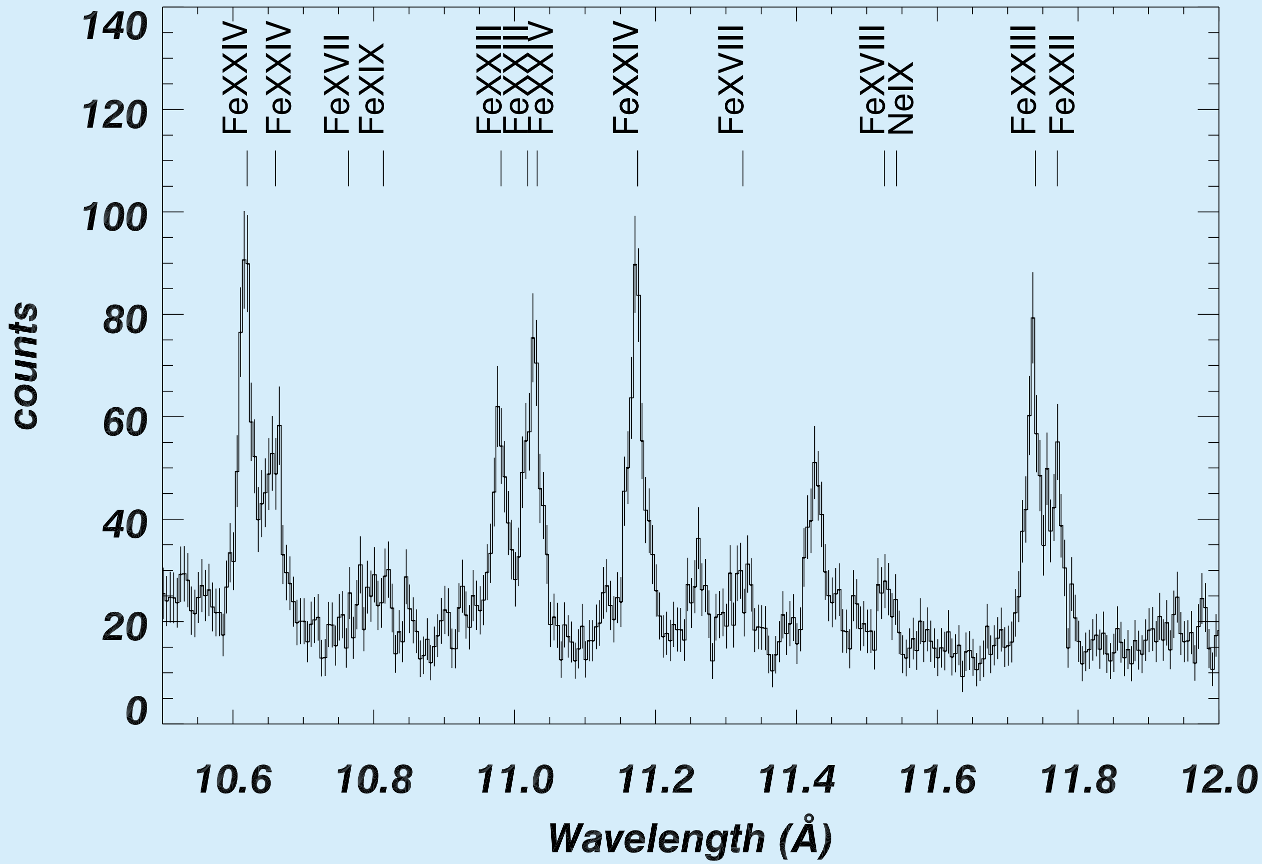
- P Cyg line profiles in UV



HD 163758 (HST)

Observation of hot stars

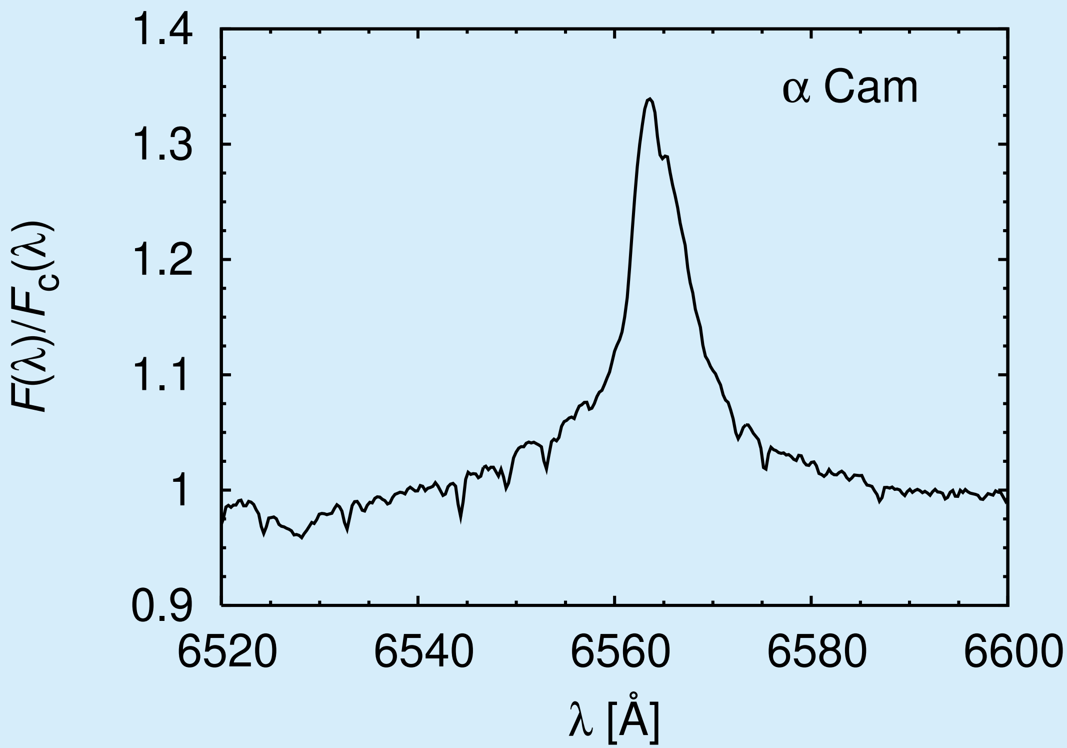
- X-ray emission



X-ray spectrum θ^1 Ori C
(CHANDRA, Schulz et al. 2003)

Observation of hot stars

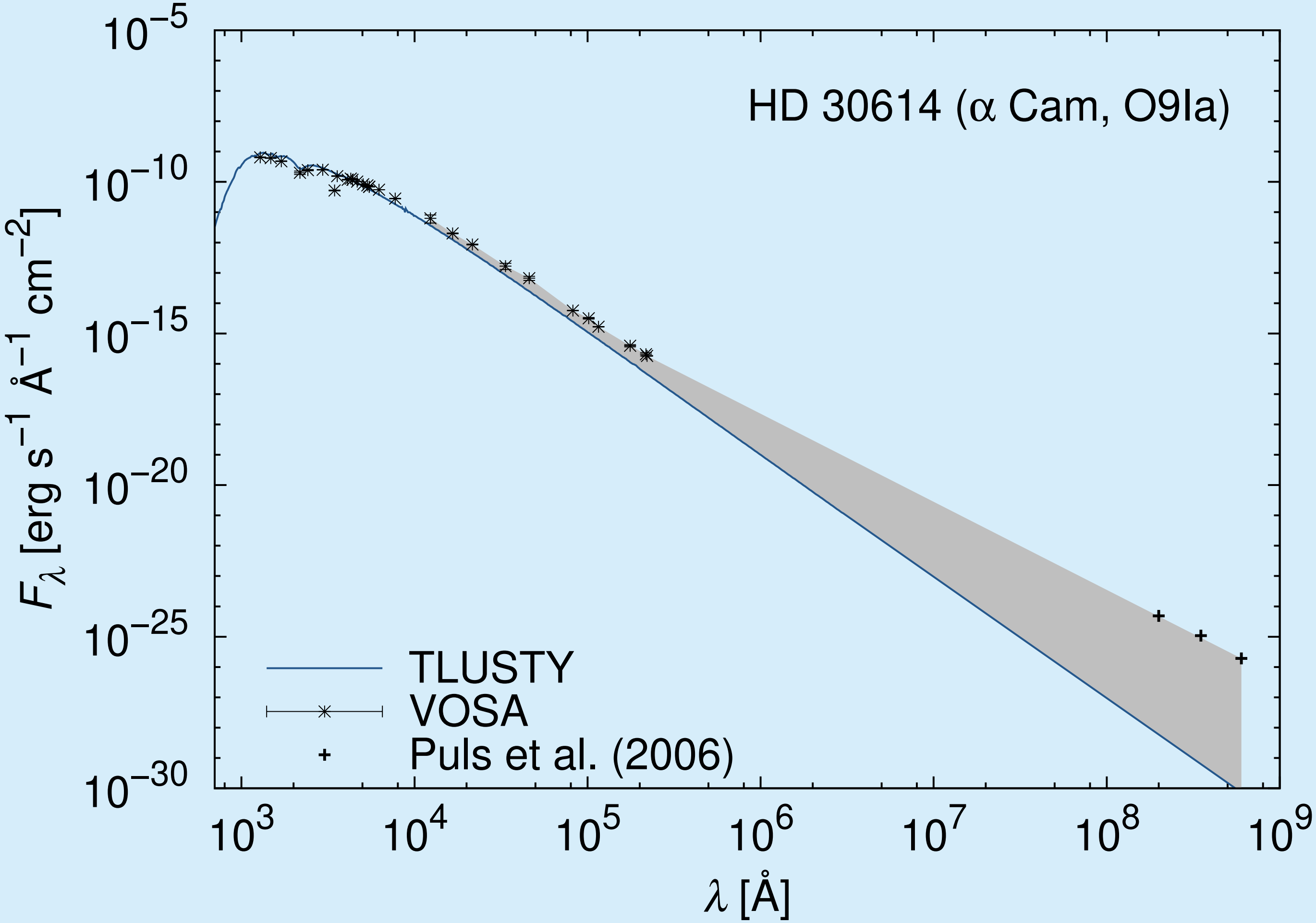
- Hα emission line



α Cam, 2m telescope in Ondřejov (Kubát 2003)

Observations of hot stars

- infrared excess



Hot star wind theory

- why is the wind blowing from hot stars?
- what are the main wind parameters (mass-loss rate, velocity)?
- how to predict the wind line profiles?
- how the wind influences the stellar evolution and the circumstellar environment?

Why is the wind blowing?

- some force accelerates the material from the stellar atmosphere to the circumstellar environment

Why is the wind blowing?

- hot stars are luminous: radiative force?

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\text{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu) F(r,\nu)\,\mathrm{d}\nu$$

  - spherically symmetric case
  - $\chi(r,\nu)$ absorption coefficient
  - $F(r,\nu)$ radiative flux

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu) F(r,\nu)\,\mathrm{d}\nu$$

  - radiative force due to the light scattering on free electrons

$$\chi(r,\nu) = \sigma_{\mathrm{Th}} n_{\mathrm{e}}(r)$$

    - σ_{Th} Thomson scattering cross-section
    - $n_{\mathrm{e}}(r)$ electron density

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$$f_{\text{rad}} = \frac{\sigma_{\text{Th}} n_{\text{e}}(r) L}{4\pi r^2 c}$$

$$\text{where } L = 4\pi r^2 \int_0^\infty F(r,\nu)\,\text{d}\nu$$

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$$f_{\mathrm{rad}} = \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}}(r) L}{4\pi r^2 c}$$

  - comparison with the gravity force

$$f_{\mathrm{grav}} = \frac{\rho(r) G M}{r^2}$$

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$$\Gamma \equiv \frac{f_{\mathrm{rad}}}{f_{\mathrm{grav}}} = \frac{\sigma_{\mathrm{T}} \frac{n_{\mathrm{e}}(r)}{\rho(r)} L}{4\pi c G M}$$

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  - comparison with the gravity force

$$\Gamma \approx 10^{-5} \left(\frac{L}{1\,\mathrm{L}_\odot}\right) \left(\frac{M}{1\,\mathrm{M}_\odot}\right)^{-1}$$

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  - comparison with the gravity force
  - example: α Cam, $L = 6.2 \times 10^5 \mathrm{L}_\odot$, $M = 43\,\mathrm{M}_\odot$, $\Gamma \approx 0.1$

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$$f_{\mathrm{rad}} = \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}}(r) L}{4\pi r^2 c}$$

  - comparison with the gravity force

$\Rightarrow$ radiative force due to the light scattering on free electrons is important, but it never (?) exceeds the gravity force

Why is the wind blowing?

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$$f_{\mathrm{rad}} = \frac{1}{c} \int_0^\infty \chi(r,\nu) F(r,\nu)\, \mathrm{d}\nu$$

  - radiative force due to the line transitions

$$\chi(r,\nu) = \frac{\pi e^2}{m_{\mathrm{e}} c} \sum_{\text{lines}} \varphi_{ij}(\nu) g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right)$$

    - $\varphi_{ij}(\nu)$ line profile, $\int_0^\infty \varphi_{ij}(\nu) = 1$
    - f_{ij} oscillator strength
    - $n_i(r)$, $n_j(r)$ level occupation number, g_i, g_j statistical weights

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- problem: influence of lines on $F(r,\nu)$?
- crude solution: $F(r,\nu)$ constant for frequencies corresponding to a given line, $\nu \approx \nu_{ij}$

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu) F(r,\nu)\,\mathrm{d}\nu$$

- radiative force due to the line transitions
  - maximum force

$$f_{\mathrm{lines}}^{\mathrm{max}} = \frac{\pi e^2}{m_\mathrm{e} c^2} \sum_{\mathrm{lines}} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) F(r,\nu_{ij})$$

  - ν_{ij} is the line center frequency

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\text{rad}} = \frac{1}{c} \int_0^\infty \chi(r,\nu) F(r,\nu)\, \mathrm{d}\nu$$

- radiative force due to the line transitions
  - maximum force: comparison with gravity

$$\frac{f_{\text{line}}^{\text{max}}}{f_{\text{grav}}} = \frac{Le^2}{4 m_{\text{e}} \rho G M c^2} \sum_{\text{line}} f_{ij} n_i(r) \frac{L_\nu(\nu_{ij})}{L}$$

  - neglect of $n_j(r) \ll n_i(r)$
  - $L_\nu(\nu_{ij}) = 4\pi r^2 F(r,\nu_{ij})$

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$$\frac{f_{\mathrm{lines}}^{\mathrm{max}}}{f_{\mathrm{grav}}} = \Gamma \sum_{\mathrm{lines}} \frac{\sigma_{ij}}{\sigma_{\mathrm{Th}}} \frac{n_i}{n_{\mathrm{e}}} \frac{\nu_{ij} L_\nu(\nu_{ij})}{L}$$

$$\sigma_{ij} = \frac{\pi e^2 f_{ij}}{\nu_{ij} m_{\mathrm{e}} c}$$

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  - hydrogen: mostly ionised in the stellar envelopes $\Rightarrow$ n_i/n_{e} very small $\Rightarrow$ negligible contribution to radiative force

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  - neutral helium: n_i/n_{e} very small $\Rightarrow$ negligible contribution to radiative force

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$$f_{\text{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu$$

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  - maximum force: which elements?

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  - ionised helium: nonnegligible contribution to the radiative force

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  - heavier elements (iron, carbon, nitrogen, oxygen, ...): large number of lines, $\sigma_{ij}/\sigma_{\mathrm{Th}} \approx 10^7 \Rightarrow f_{\mathrm{line}}^{\mathrm{max}}/f_{\mathrm{grav}}$ up to 10^3

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⇒ radiative force may be larger than gravity (for many O stars $f_{\rm lines}^{\rm max}/f_{\rm grav} \approx 2000$, Abbott 1982, Gayley 1995)

⇒ stellar wind

Radiative force?

- speculations of Kepler, Newton

Radiative force?

- predicted by James Clerk Maxwell (1873) in the book *A Treatise on Electricity and Magnetism*

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$\Rightarrow$ for $E_\mathrm{p} = E_\nu$ the momentum ratio is

$$\frac{p_\nu}{p_\mathrm{p}} \approx \frac{v}{c}$$

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  - particle with thermal energy $E_\mathrm{p} \approx kT$

$$\frac{p_\nu}{p_\mathrm{p}} \approx \frac{h\nu}{c\sqrt{mkT}} \approx 0.001 \left(\frac{\nu}{10^{15}\,\mathrm{s}^{-1}}\right) \left(\frac{T}{100\,\mathrm{K}}\right)^{-1/2}$$

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    - large $\nu \Rightarrow$ X-rays, Compton effect
    - minimise heating (as did Lebedev)

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    - line absorption followed by emission
    - Thomson scattering

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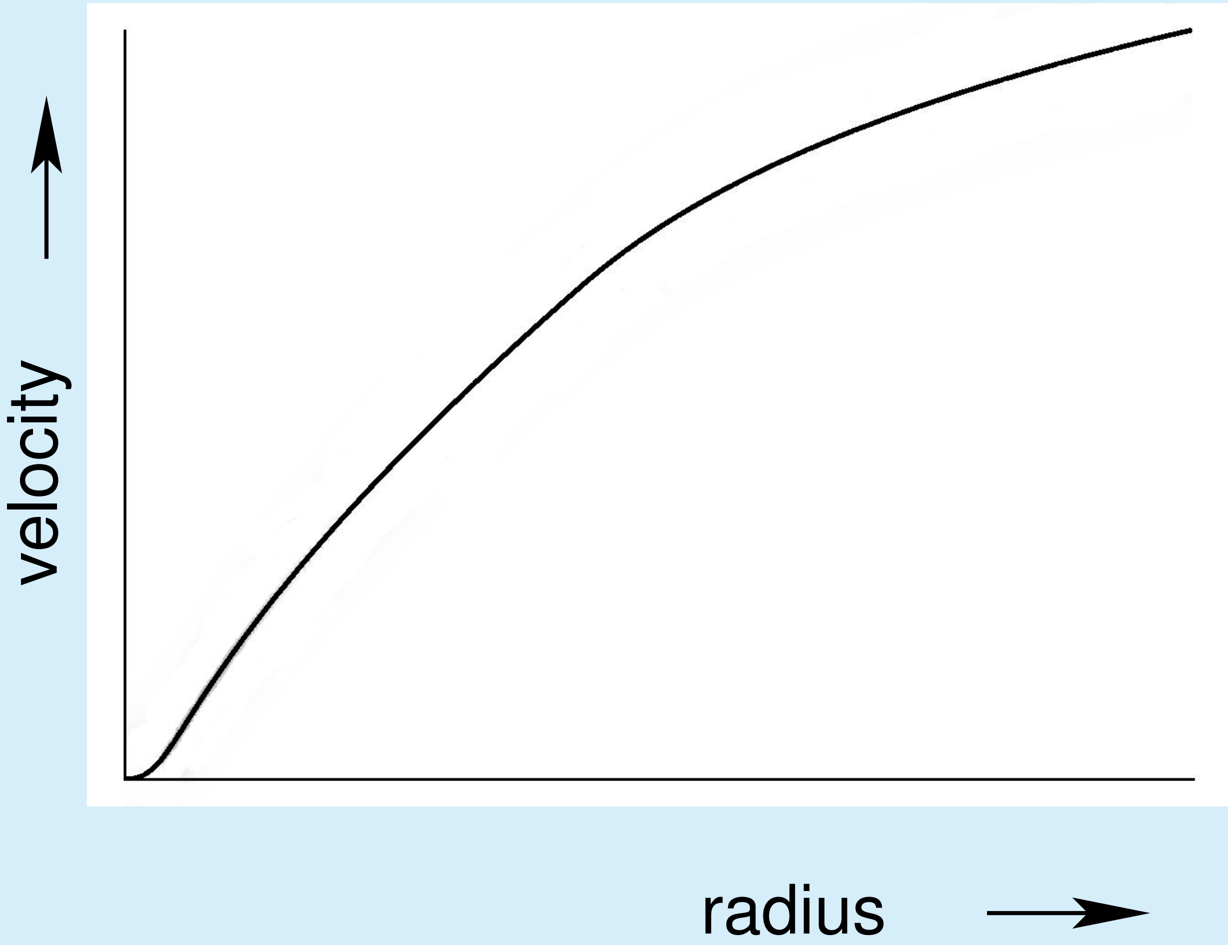
    - line absorption followed by emission
    - Thomson scattering
    - both processes important in hot star winds

The Sobolev approximation

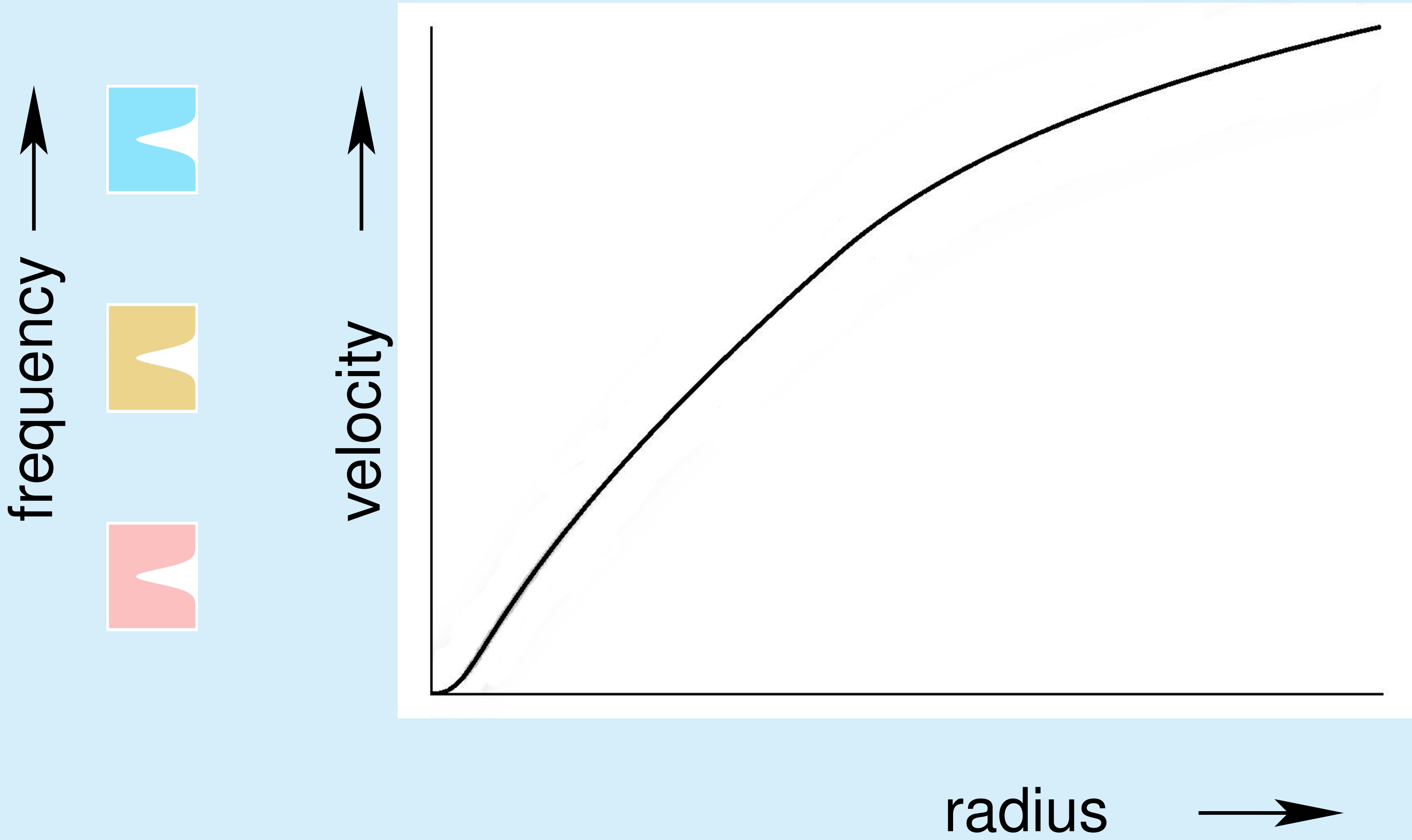
- the main problem: the line opacity (lines may be optically thick)

$\Rightarrow$ necessary to solve the radiative transfer equation

The Sobolev approximation

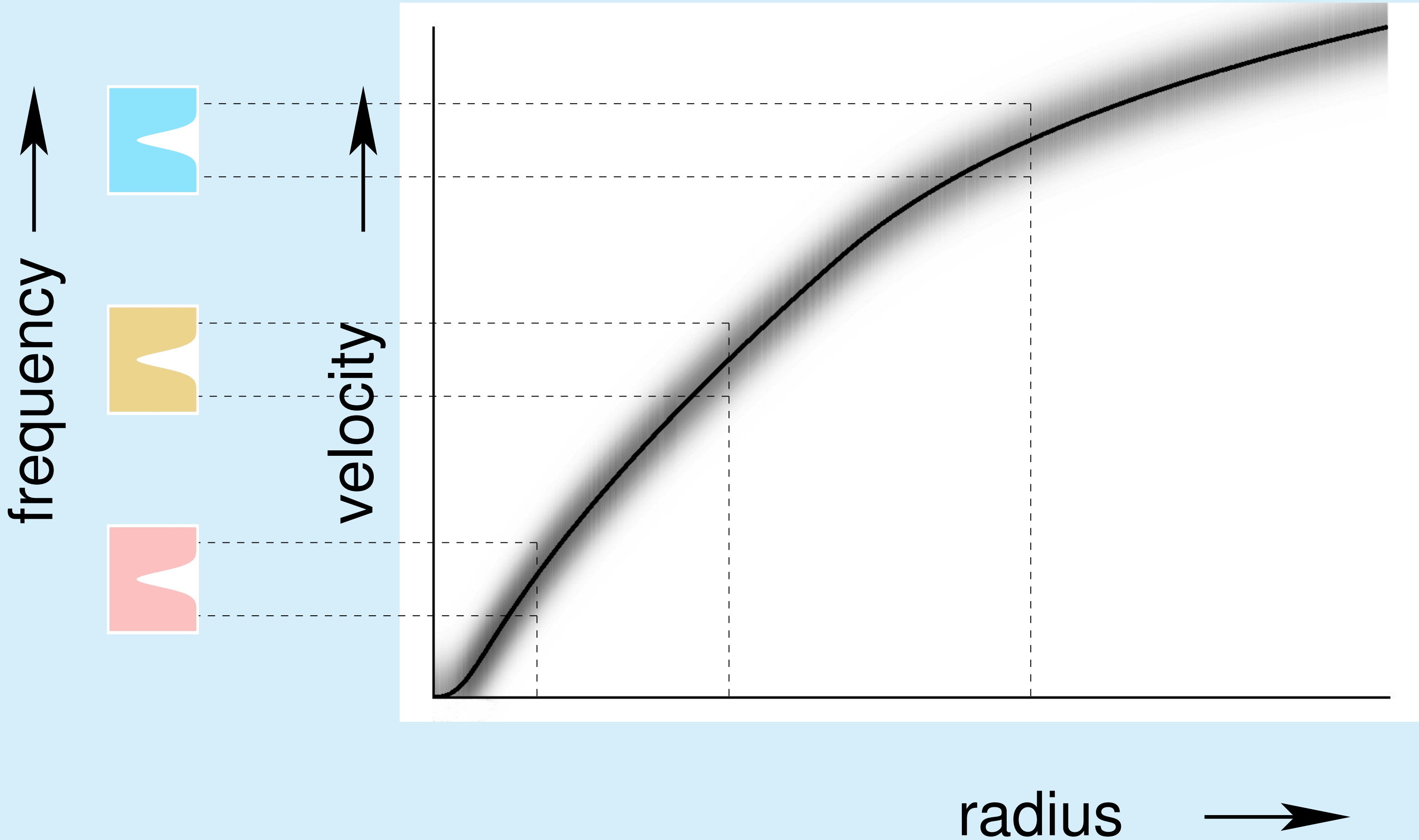


The Sobolev approximation



the Doppler effect in the wind

The Sobolev approximation



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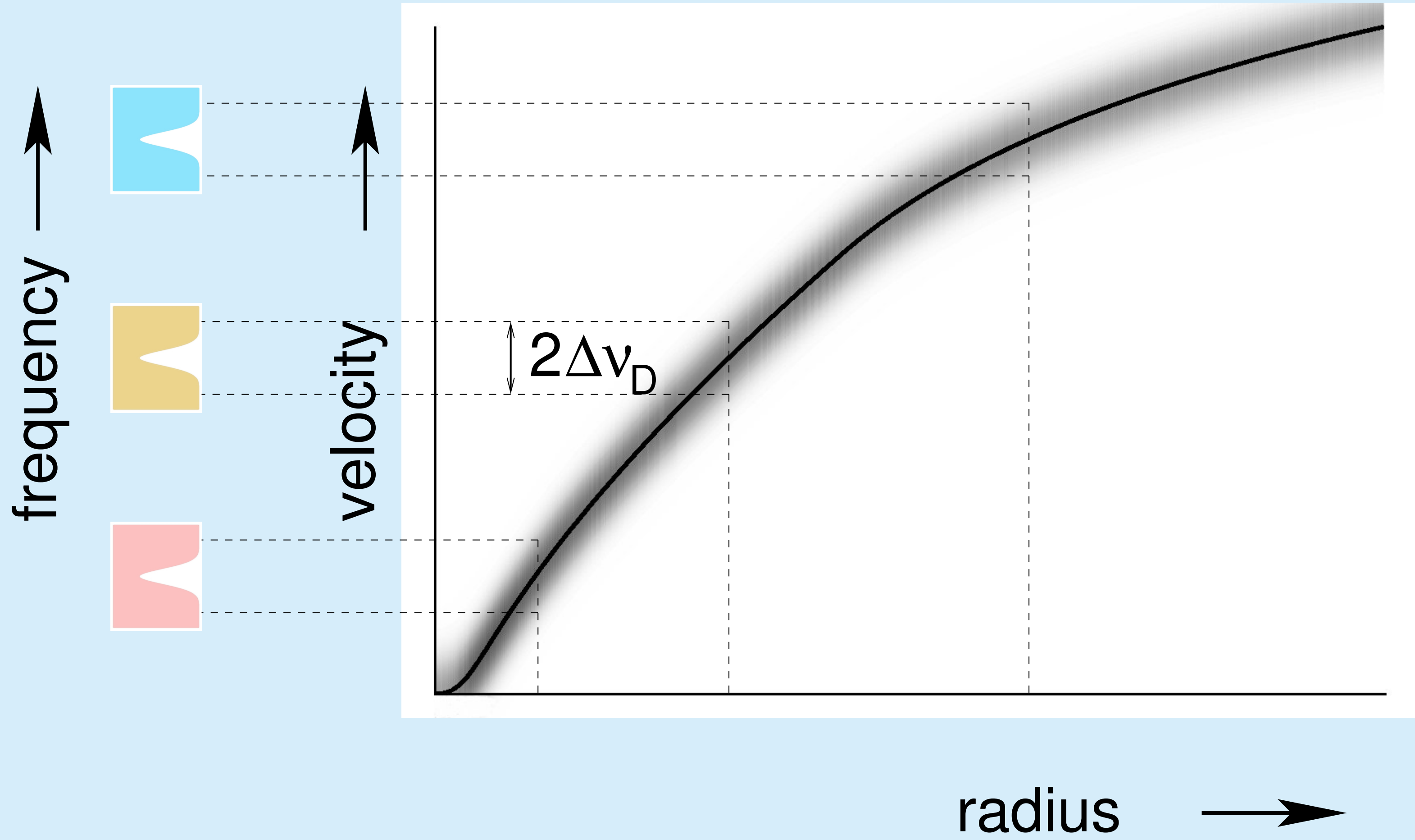
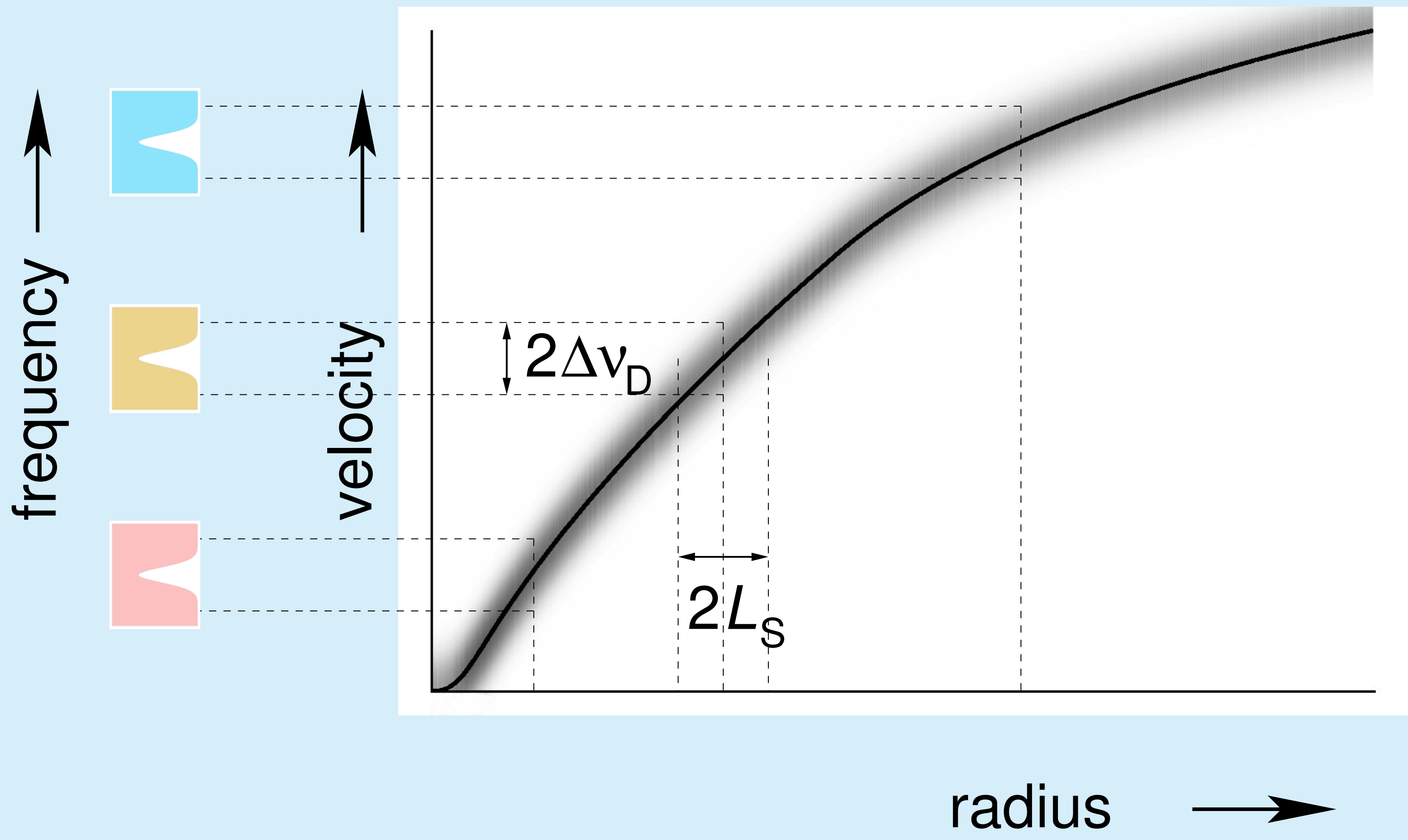


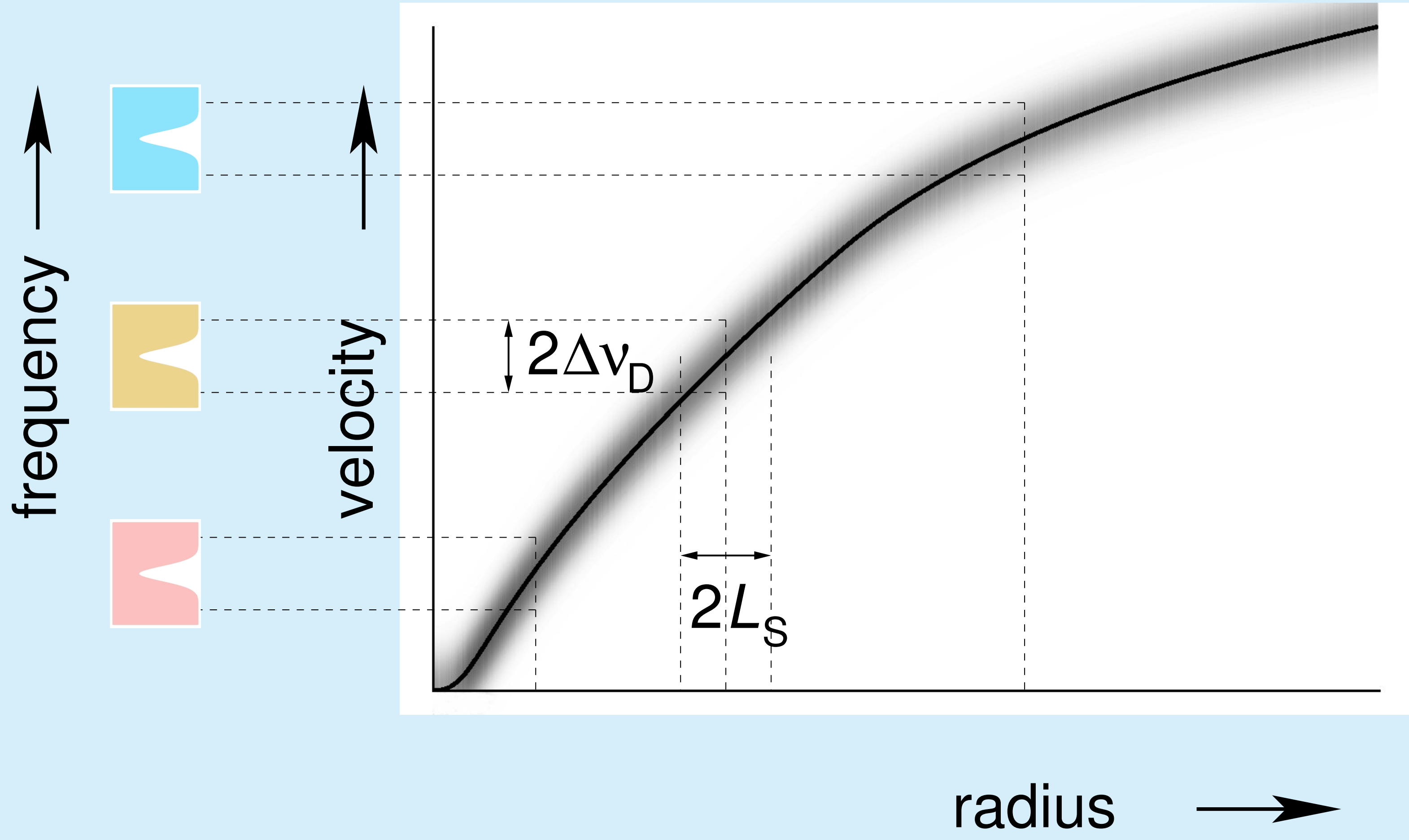
- $\Delta\nu_D$ is the Doppler width of the line

The Sobolev approximation



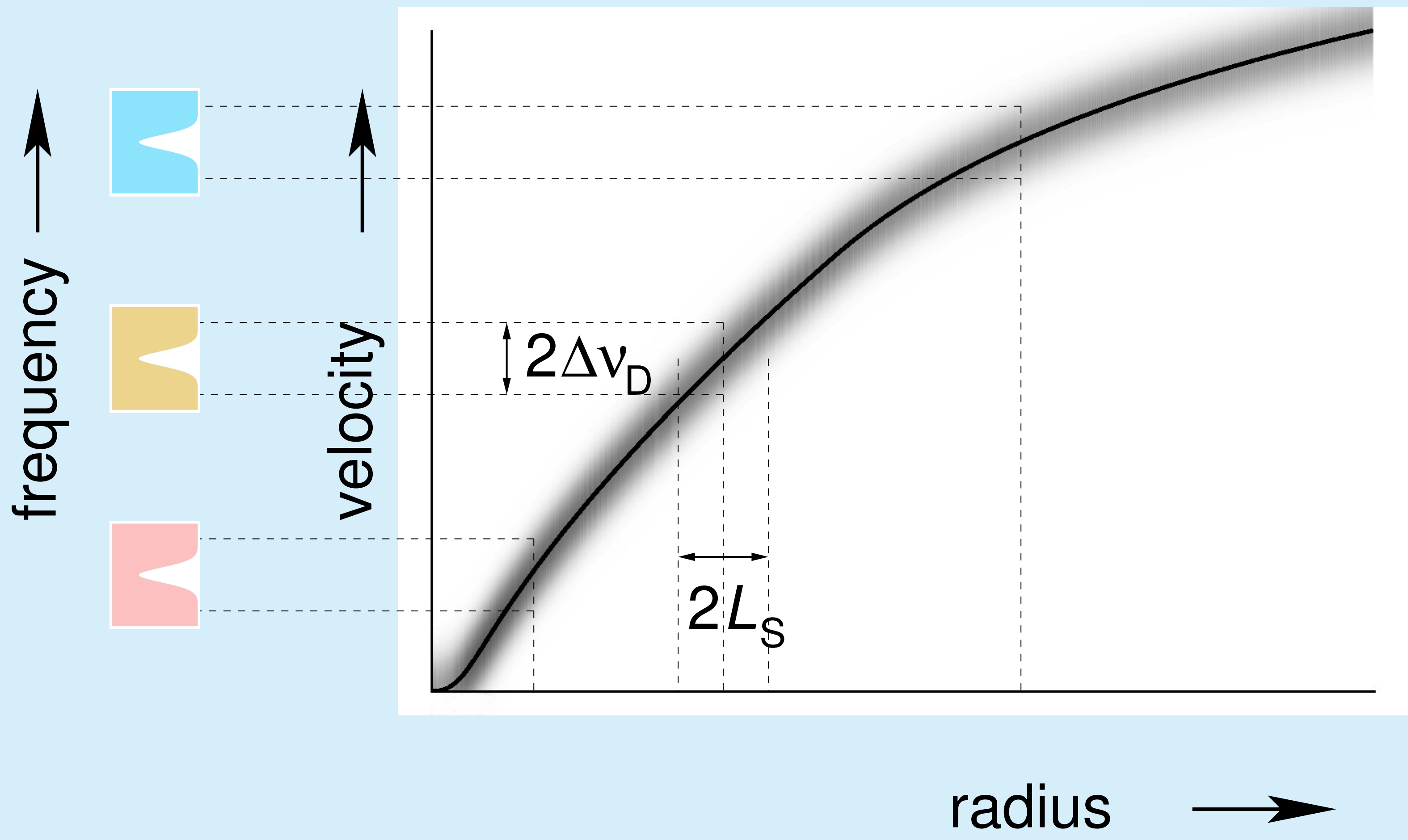
- $L_S \equiv \frac{v_{th}}{\frac{dv}{dr}} = c\frac{\Delta\nu_D}{\nu_{ij}}\frac{1}{\frac{dv}{dr}}$ is the Sobolev length

The Sobolev approximation



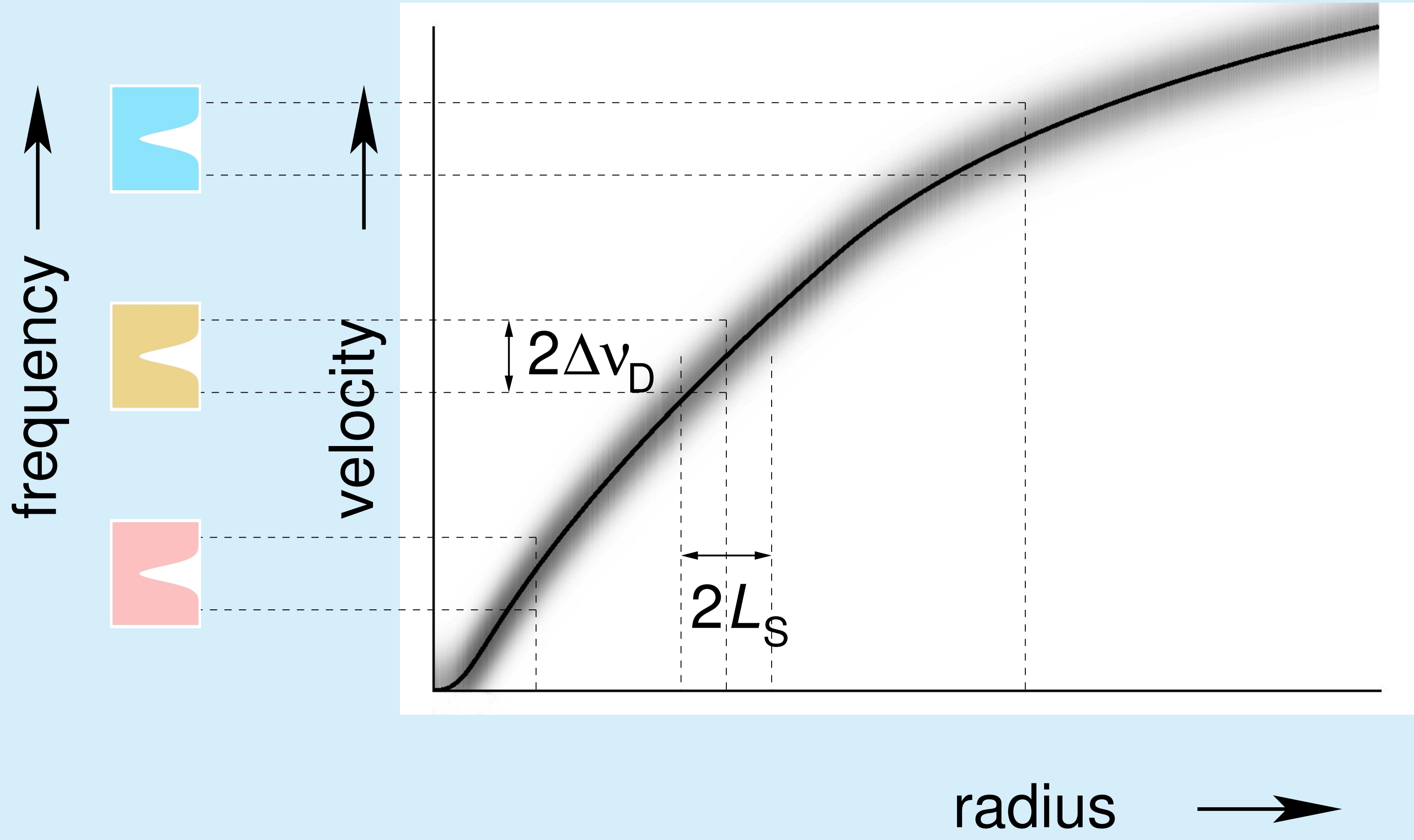
- structure does not significantly vary over L_S ⇒ simplification of the calculation of f^{rad} possible

The Sobolev approximation



- opacity nonnegligible only over $L_S \Rightarrow$ solution of RTE in the "gray" zone only

The Sobolev approximation



- $\boldsymbol{H} \equiv \dfrac{\boldsymbol{\rho}}{\left(\frac{d\boldsymbol{\rho}}{d\boldsymbol{r}}\right)} \approx \dfrac{\boldsymbol{v}}{\left(\frac{d\boldsymbol{v}}{d\boldsymbol{r}}\right)} \gg \dfrac{\boldsymbol{v}_{\mathrm{th}}}{\left(\frac{d\boldsymbol{v}}{d\boldsymbol{r}}\right)} \equiv \boldsymbol{L}_{\mathrm{S}} \; (\boldsymbol{v} \gg \boldsymbol{v}_{\mathrm{th}})$

Our assumptions

- spherical symmetry

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- spherical symmetry
- stationary (time-independent) flow

The Sobolev line force I.

- the radiative transfer equation

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r} \frac{\partial}{\partial \mu} I(r,\mu,\nu) = \\ = \eta(r,\mu,\nu) - \chi(r,\mu,\nu) I(r,\mu,\nu)$$

- frame of static observer
- stationarity, spherical symmetry
- μ is frequency, $\mu = \cos\theta$
- $I(r,\mu,\nu)$ is specific intensity
- $\chi(r,\mu,\nu)$ is absorption (extinction) coefficient
- $\eta(r,\mu,\nu)$ is emissivity (emission coefficient)

The Sobolev line force I.

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$$= \eta(r,\mu,\nu) - \chi(r,\mu,\nu) I(r,\mu,\nu)$$

- problem: $\chi(r,\mu,\nu)$ and $\eta(r,\mu,\nu)$ depend on μ due to the Doppler effect

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- problem: $\chi(r,\mu,\nu)$ and $\eta(r,\mu,\nu)$ depend on μ due to the Doppler effect
- solution: use comoving frame!

The Sobolev line force I.

- CMF radiative transfer equation

$$\mu\frac{\partial}{\partial r}I(r,\mu,\nu)+\frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}I(r,\mu,\nu)-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$

$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

- comoving frame (CMF) equation
- $v(r)$ is the fluid velocity
- $\chi(r,\nu)$ and $\eta(r,\nu)$ do depend on μ

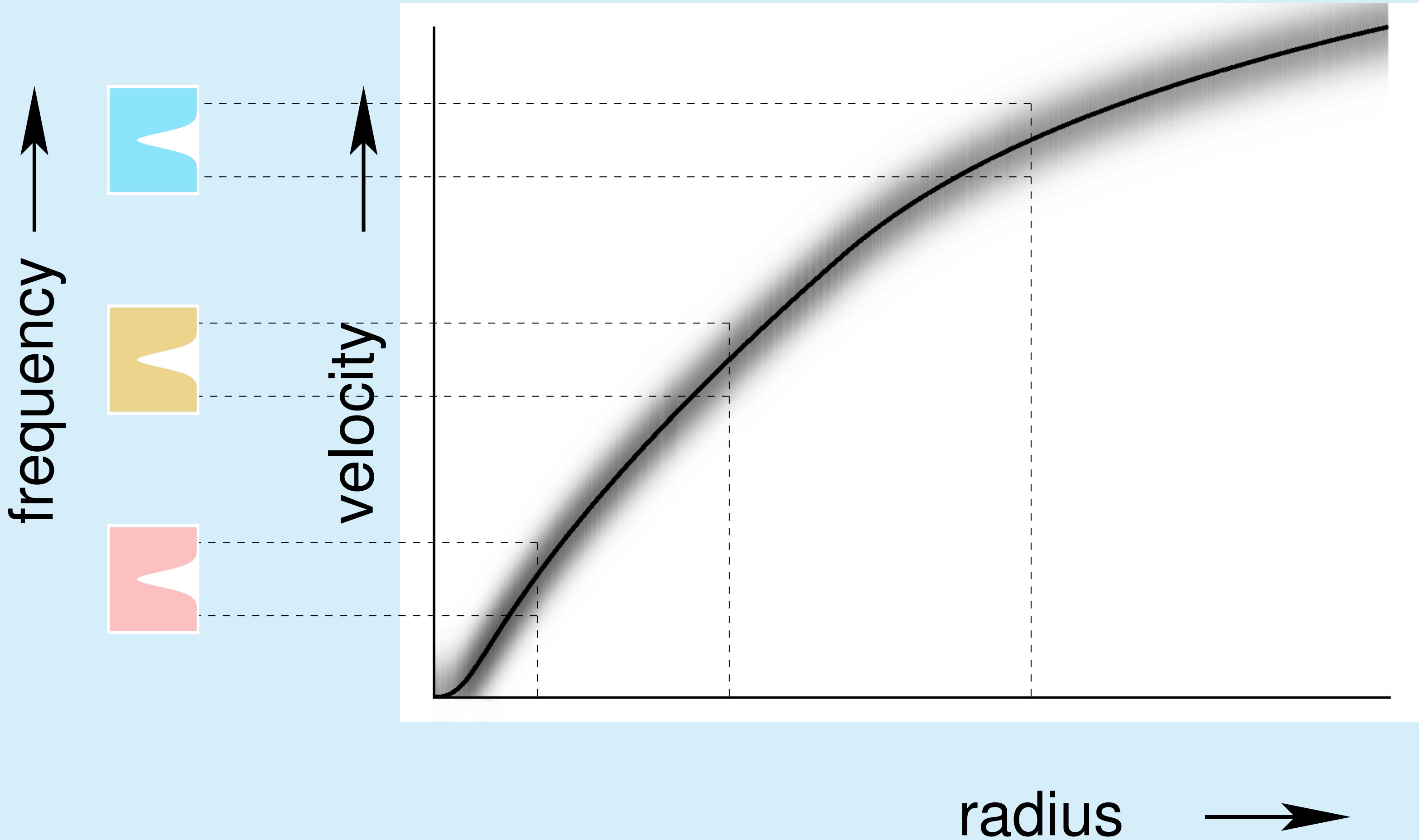
The Sobolev line force I.

- CMF radiative transfer equation

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu} I(r,\mu,\nu) -$$
$$\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial \nu} I(r,\mu,\nu) =$$
$$= \eta(r,\nu) - \chi(r,\nu) I(r,\mu,\nu)$$

- neglected aberration, advection (unimportant for $v \ll c$, e.g., Korčáková & Kubát 2003)
- neglect of the transformation of $I(r,\mu,\nu)$ between individual inertial frames

Intermezzo: the interpretation



- in CMF: continuous redshift of a given photon

The Sobolev line force II.

- the Sobolev transfer equation (Castor 2004)

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu} I(r,\mu,\nu) -$$

$$- \frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial \nu} I(r,\mu,\nu) =$$

$$= \eta(r,\nu) - \chi(r,\nu) I(r,\mu,\nu)$$

The Sobolev line force II.

- the Sobolev transfer equation (Castor 2004)

$$\mu\frac{\partial}{\partial r}I(r,\mu,\nu)+\frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}I(r,\mu,\nu)-$$

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$

$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

- possible when $\frac{\nu v(r)}{cr}\frac{\partial}{\partial\nu}I(r,\mu,\nu)\gg\frac{\partial}{\partial r}I(r,\mu,\nu)$
- dimensional arguments:
  - $\frac{\partial}{\partial r}I(r,\mu,\nu)\sim\frac{I(r,\mu,\nu)}{r}$,
  - $\frac{\partial}{\partial\nu}I(r,\mu,\nu)\sim\frac{I(r,\mu,\nu)}{\Delta\nu}$,
    $\Delta\nu=\nu\frac{v_{\mathrm{th}}}{c}$ is the line Doppler width

The Sobolev line force II.

- the Sobolev transfer equation (Castor 2004)

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu} I(r,\mu,\nu) -$$

$$- \frac{\nu v(r)}{c r}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial \nu} I(r,\mu,\nu) =$$

$$= \eta(r,\nu) - \chi(r,\nu) I(r,\mu,\nu)$$

- possible when $v(r) \gg v_{\mathrm{th}}$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

- line absorption and emission coefficients are

$$\chi(r,\nu)=\frac{\pi e^2}{m_\mathrm{e}c}\varphi_{ij}(\nu)g_i f_{ij}\left(\frac{n_i(r)}{g_i}-\frac{n_j(r)}{g_j}\right)$$
$$\eta(r,\nu)=\frac{2h\nu^3}{c^2}\frac{\pi e^2}{m_\mathrm{e}c}\varphi_{ij}(\nu)g_i f_{ij}\frac{n_j(r)}{g_j}$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

- the line opacity and emissivity are

$$\chi(r,\nu)=\chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)$$

$$\eta(r,\nu)=\chi_{\mathrm{L}}(r)S_{\mathrm{L}}(r)\varphi_{ij}(\nu)$$

where $\chi_{\mathrm{L}}(r)=\frac{\pi e^2}{m_{\mathrm{e}}c}g_i f_{ij}\left(\frac{n_i(r)}{g_i}-\frac{n_j(r)}{g_j}\right)$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)\left(S_{\mathrm{L}}(r)-I(r,\mu,\nu)\right)$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)\left(S_{\mathrm{L}}(r)-I(r,\mu,\nu)\right)$$

- introduce a new variable

$$y=\int_{\nu}^{\infty}\mathrm{d}\nu'\varphi_{ij}(\nu')$$

- where
    - $y=0$: the incoming side of the line
    - $y=1$: the outgoing side of the line

The Sobolev line force III.

- solution of the transfer equation for one line

$$\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial y}I(r,\mu,y)=$$
$$=\chi_\mathrm{L}(r)\left(S_\mathrm{L}(r)-I(r,\mu,y)\right)$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial y}I(r,\mu,y) =$$
$$= \chi_\mathrm{L}(r)\left(S_\mathrm{L}(r)-I(r,\mu,y)\right)$$

- assumptions:
  - variables do not significantly vary with r within the "resonance zone"

  $\Rightarrow$ fixed r, $\frac{\partial}{\partial y} \to \frac{\mathrm{d}}{\mathrm{d}y}$

  - $\nu \to \nu_0$

$\Rightarrow$ integration possible

The Sobolev line force III.

- solution of the transfer equation for one line

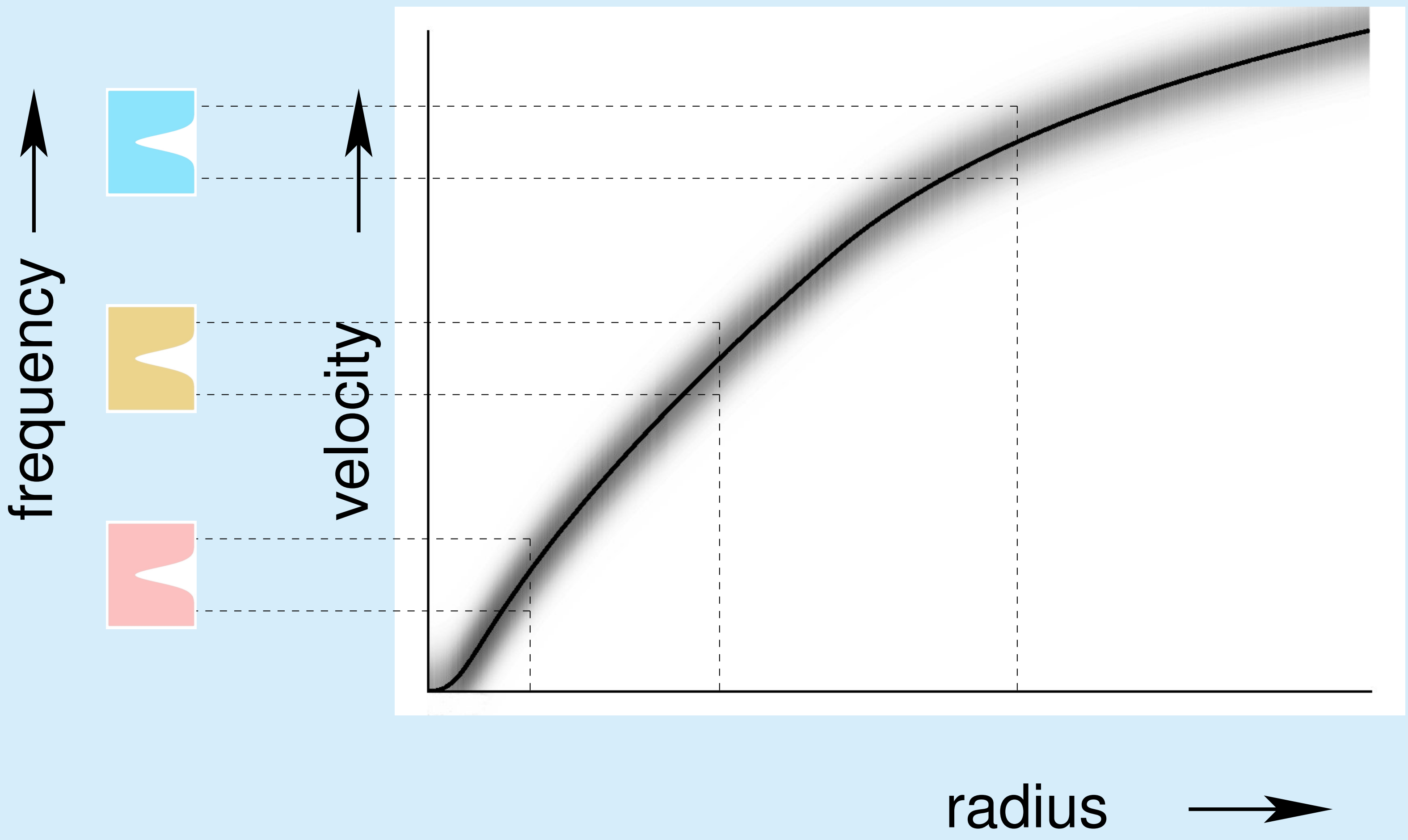
$$I(y) = I_c(\mu) \exp\left[-\tau(\mu) y\right] + S_L \left\{1 - \exp\left[-\tau(\mu) y\right]\right\}$$

- where
  - the Sobolev optical depth is

$$\tau(\mu) = \frac{\chi_L(r) c r}{\nu_0 v(r) \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)}$$

  - the boundary condition is $I(y = 0) = I_c(\mu)$

Intermezzo: the interpretation



- $\boldsymbol{\tau}$ is given by the slope $\Rightarrow \boldsymbol{\tau} \sim \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\rm rad} = \frac{1}{c} \int_0^\infty \chi(r,\nu) F(r,\nu)\, \mathrm{d}\nu$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \mathrm{d}\nu\, \chi(r,\nu) \oint \mathrm{d}\Omega\, \mu I(r,\mu,\nu)$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi}{c} \int_0^\infty \mathrm{d}\nu \, \chi_{\mathrm{L}}(r) \varphi_{ij}(\nu) \int_{-1}^{1} \mathrm{d}\mu \, \mu I(r,\mu,\nu)$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_0^1 \mathrm{d}y \int_{-1}^{1} \mathrm{d}\mu\, \mu I(r,\mu,y)$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_0^1 \mathrm{d}y \times$$

$$\int_{-1}^{1} \mathrm{d}\mu\,\mu\left\{I_{\mathrm{c}}(\mu)\exp\left[-\tau(\mu)y\right] + S_{\mathrm{L}}\left\{1-\exp\left[-\tau(\mu)y\right]\right\}\right\}$$

- where the Sobolev optical depth is

$$\tau(\mu) = \frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)}$$

- $\tau(\mu)$ is an even function of μ

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c} \int_0^1 \mathrm{d}y \int_{-1}^{1} \mathrm{d}\mu\, \mu I_{\mathrm{c}}(\mu) \exp\left[-\tau(\mu) y\right]$$

- no net contribution of the emission to the radiative force (S_{L} is isotropic in the CMF)

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_{-1}^{1}\mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\frac{1-\exp\left[-\tau(\mu)\right]}{\tau(\mu)}$$

- inserting

$$\tau(\mu) = \frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)}$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\text{rad}} = \frac{2\pi\nu_0 v(r)}{rc^2} \int_{-1}^{1} \mathrm{d}\mu\, \mu I_{\text{c}}(\mu) \left[1 + \mu^2\sigma(r)\right] \times$$
$$\times \left\{1 - \exp\left[-\frac{\chi_{\text{L}}(r) c r}{\nu_0 v(r)\left(1 + \mu^2\sigma(r)\right)}\right]\right\}$$

- where $\sigma(r) = \frac{r}{v(r)} \frac{\mathrm{d}v(r)}{\mathrm{d}r} - 1$
- Sobolev (1957), Castor (1974), Rybicki & Hummer (1978)

Optically thin lines

- optically thin line:

$$\frac{\chi_{\mathrm{L}}(\boldsymbol{r}) c \boldsymbol{r}}{\nu_0 \boldsymbol{v}(\boldsymbol{r})\left(1+\boldsymbol{\mu}^2 \boldsymbol{\sigma}(\boldsymbol{r})\right)} \ll 1$$

Optically thin lines

- optically thin line:

$$\frac{\chi_{\mathrm{L}}(r) c r}{\nu_0 v(r)\left(1+\mu^2 \sigma(r)\right)} \ll 1$$

- the radiative force proportional to

$$f_{\mathrm{rad}} \sim 1-\exp\left[-\frac{\chi_{\mathrm{L}}(r) c r}{\nu_0 v(r)\left(1+\mu^2 \sigma(r)\right)}\right]$$

Optically thin lines

- optically thin line:

$$\frac{\chi_\mathrm{L}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \ll 1$$

- the radiative force proportional to

$$f_\mathrm{rad} \sim 1-\exp\left[-\frac{\chi_\mathrm{L}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right]$$

$$\approx \frac{\chi_\mathrm{L}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}$$

Optically thin lines

$$f_{\rm rad} = \frac{2\pi}{c} \int_{-1}^{1} {\rm d}\mu \, \mu \, I_{\rm c}(\mu) \chi_{\rm L}(r)$$

Optically thin lines

$$f_{\mathrm{rad}} = \frac{1}{c} \chi_{\mathrm{L}}(r) F(r)$$

Optically thin lines

$$f_{\mathrm{rad}} = \frac{1}{c}\chi_{\mathrm{L}}(r)F(r)$$

- optically thin radiative force proportional to the radiative flux $F(r)$
- optically thin radiative force proportional to the normalised line opacity $\chi_{\mathrm{L}}(r)$ (or to the density)
- the same result as for the static medium

Optically thick lines

- optically thick line:

$$\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \gg 1$$

Optically thick lines

- optically thick line:

$$\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \gg 1$$

- the radiative force proportional to

$$f_{\mathrm{rad}} \sim 1 - \exp\left[-\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right]$$

Optically thick lines

- optically thick line:

$$\frac{\chi_\mathrm{L}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \gg 1$$

- the radiative force proportional to

$$f_\mathrm{rad} \sim 1-\exp\left[-\frac{\chi_\mathrm{L}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right]$$
$$\approx 1$$

Optically thick lines

$$f_{\mathrm{rad}} = \frac{2\pi\nu_0 v(r)}{r c^2} \int_{-1}^{1} \mathrm{d}\mu \, \mu I_{\mathrm{c}}(\mu) \left[1 + \mu^2 \sigma(r)\right]$$

Optically thick lines

$$f_{\text{rad}} = \frac{2\pi\nu_0 v(r)}{rc^2}\int_{-1}^{1} \mathrm{d}\mu\,\mu I_{\text{c}}(\mu)\left[1+\mu^2\sigma(r)\right]$$

- neglect of the limb darkening:

$$I_{\text{c}}(\mu) = \begin{cases} I_{\text{c}} = \text{const.}, & \mu \geq \mu_*, \\ 0, & \mu < \mu_* \end{cases},$$

where $\mu_* = \sqrt{1-\frac{R_*^2}{r^2}}$

Optically thick lines

$$f_{\mathrm{rad}} = \frac{2\pi\nu_0 v(r)}{r c^2} \int_{\mu_*}^{1} \mathrm{d}\mu \, \mu I_{\mathrm{c}} \left[1 + \mu^2 \sigma(r)\right]$$

Optically thick lines

$$f_{\text{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2} \right) \right]$$

where $F = 2\pi \int_{\mu_*}^{1} \mathrm{d}\mu \, \mu I_{\text{c}} = \pi \frac{R_*^2}{r^2} I_{\text{c}}$

Optically thick lines

$$f_{\text{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2} \right) \right]$$

- large distance from the star: $r \gg R_*$

Optically thick lines

$$f_{\text{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2} \right) \right]$$

- large distance from the star: $r \gg R_*$

$$f_{\text{rad}} \approx \frac{\nu_0 F(r)}{c^2} \frac{\mathrm{d}v(r)}{\mathrm{d}r}$$

Optically thick lines

$$f_{\text{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2} \right) \right]$$

- large distance from the star: $r \gg R_*$

$$f_{\text{rad}} \approx \frac{\nu_0 F(r)}{c^2} \frac{\mathrm{d}v(r)}{\mathrm{d}r}$$

- optically thick radiative force proportional to the radiative flux $F(r)$
- optically thick radiative force proportional to $\frac{\mathrm{d}v}{\mathrm{d}r}$
- optically thick radiative force does not depend on the level populations or the density

Wind driven by thick lines

- continuity and momentum equation of isothermal spherically symmetric wind

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v\right) = 0$$

$$\frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} = -a^2 \frac{\partial \rho}{\partial r} + f_{\text{rad}} - \frac{\rho G M (1-\Gamma)}{r^2}$$

- ρ, v are the wind density and velocity
- a is the sound speed

Wind driven by thick lines

- continuity and momentum equation of isothermal spherically symmetric wind

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0$$

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} + f_{\mathrm{rad}} - \frac{\rho G M(1-\Gamma)}{r^2}$$

- assumption: stationary flow

Wind driven by thick lines

- continuity equation

$$\frac{1}{\boldsymbol{r}^2}\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{r}}\left(\boldsymbol{r}^2\boldsymbol{\rho}\boldsymbol{v}\right) = 0 \Rightarrow \dot{\boldsymbol{M}} \equiv 4\boldsymbol{\pi}\boldsymbol{r}^2\boldsymbol{\rho}\boldsymbol{v} = \text{const.}$$

- $\dot{M}$ is the wind mass-loss rate

Wind driven by thick lines

- momentum equation

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0 \Rightarrow \frac{\mathrm{d}\rho}{\mathrm{d}r} = -\frac{\rho}{v}\frac{\mathrm{d}v}{\mathrm{d}r} - \frac{2\rho}{r}$$

- momentum equation:

$$\left(v^2 - a^2\right)\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} + \frac{f_{\mathrm{rad}}}{\rho} - \frac{GM(1-\Gamma)}{r^2}$$

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{f_{\mathrm{rad}}}{\rho} - \frac{GM(1-\Gamma)}{r^2}$$

- neglect of the gas-pressure term $a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} \ll f_{\mathrm{rad}}$ (possible in the supersonic part of the wind)

Wind driven by thick lines

- momentum equation

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 F(r)}{\rho c^2}\frac{\mathrm{d}v}{\mathrm{d}r} - \frac{GM(1-\Gamma)}{r^2}$$

- inclusion of the expression for the optically thick line force for $r \gg R_*$
- $F(r) = \frac{L_\nu}{4\pi r^2}$, where L_ν is the monochromatic stellar luminosity (constant)

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\right] \frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi \rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\right] \frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi \rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

- has a critical point

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\right] \frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi \rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

- has a critical point

- consequently

$$\dot{M} \equiv 4\pi r^2 \rho v(r) = \frac{\nu_0 L_\nu}{c^2} \approx \frac{L}{c^2}$$

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\right] \frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi\rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

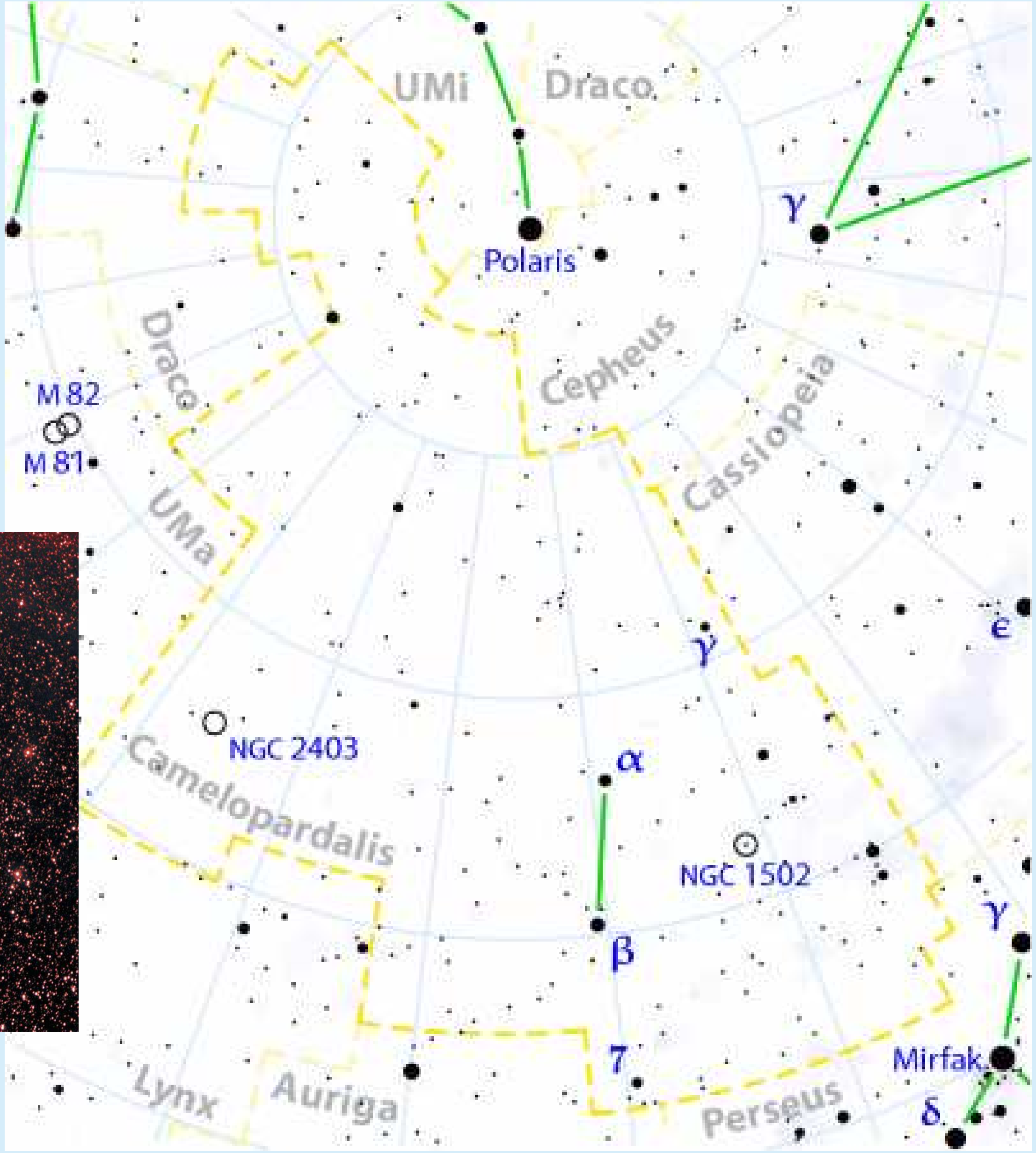
- has a critical point

- consequently

$$\dot{M} \equiv 4\pi r^2 \rho v(r) = \frac{\nu_0 L_\nu}{c^2} \approx \frac{L}{c^2}$$

$\Rightarrow$ mass-loss rate due to one optically thick line approximatively equal to the "photon mass-loss rate" (L is stellar luminosity)

Example: α Cam



Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\text{R}_\odot$
mass M	$43\,\text{M}_\odot$

(Lamers et al. 1995)

Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\mathrm{R}_\odot$
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- mass-loss rate due to one optically thick line
  $\dot{M} \approx L/c^2$

Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\mathrm{R}_\odot$
mass M	$43\,\mathrm{M}_\odot$

- mass-loss rate due to one optically thick line
  $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines
  $\dot{M} \approx N_{\mathrm{thick}} L/c^2$

Example: α Cam

temperature T_{eff}	30 900 K
radius R_*	27.6 $R_\odot$
mass M	43 $M_\odot$

- mass-loss rate due to one optically thick line
  $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines
  $\dot{M} \approx N_{\text{thick}} L/c^2$
- NLTE calculations: $N_{\text{thick}} \approx 1000$

Example: α Cam

temperature T_{eff}	30 900 K
radius R_*	27.6 $R_\odot$
mass M	43 $M_\odot$

- mass-loss rate due to one optically thick line
  $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines
  $\dot{M} \approx N_{thick} L/c^2$
- NLTE calculations: $N_{thick} \approx 1000$
- $L = 4\pi\sigma R_*^2 T_{eff}^4$, $L = 620\,000\,L_\odot$

Example: α Cam

temperature T_{eff}	30 900 K
radius R_*	27.6 $R_\odot$
mass M	43 $M_\odot$

- mass-loss rate due to one optically thick line $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines $\dot{M} \approx N_{\mathrm{thick}} L/c^2$
- NLTE calculations: $N_{\mathrm{thick}} \approx 1000$
- $L = 4\pi\sigma R_*^2 T_{\mathrm{eff}}^4$, $L = 620\,000\,L_\odot$
- $\dot{M} \approx 4 \times 10^{-5}\,M_\odot\,\mathrm{yr}^{-1}$, more precise estimate: $1.5 \times 10^{-6}\,M_\odot\,\mathrm{yr}^{-1}$ (Krtička & Kubát 2008)

CAK theory

- in reality the wind is driven by a mixture of optically thick and thin lines
  - optically thin line force

$$f_{\mathrm{rad}} = \frac{1}{c}\chi_{\mathrm{L}}(r)F(r)$$

  - optically thick line force

$$f_{\mathrm{rad}} = \frac{\nu_0 F(r)}{c^2}\frac{\mathrm{d}v}{\mathrm{d}r}$$

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  - optically thick line force

$$f_{\mathrm{rad}} = \frac{\nu_0 F(r)}{c^2}\frac{\mathrm{d}v}{\mathrm{d}r}$$

- Sobolev optical depth $\tau_{\mathrm{S}} = \frac{\chi_{\mathrm{L}}(r)c}{\nu_0 \frac{\mathrm{d}v}{\mathrm{d}r}}$

$$f_{\mathrm{rad}} = \frac{1}{c}\chi_{\mathrm{L}}(r)F(r)\left(\tau_{\mathrm{S}}^{-1}\right)^{\alpha}$$

where $\alpha = 0$ (thin) or $\alpha = 1$ (thick)

CAK theory

- in reality the wind is driven by a mixture of optically thick and thin lines

$$\Rightarrow \quad 0 < \boldsymbol{\alpha} < 1$$

CAK theory

- in reality the wind is driven by a mixture of optically thick and thin lines
- the radiative force in the CAK approximation (Castor, Abbott & Klein 1975)

$$f_{\rm rad} = k \frac{\sigma_{\rm Th} n_{\rm e} L}{4\pi r^2 c} \left(\frac{1}{\sigma_{\rm Th} n_{\rm e} v_{\rm th}} \frac{{\rm d}v}{{\rm d}r} \right)^{\alpha}$$

- where
  - k, α are constants (force multipliers)
  - $\sigma_{\rm Th}$ is the Thomson scattering cross-section
  - $n_{\rm e}$ is the electron number density
  - $v_{\rm th}$ is hydrogen thermal speed (for $T = T_{\rm eff}$)

(Abbott 1982)

CAK theory

- in reality the wind is driven by a mixture of optically thick and thin lines
- the radiative force in the CAK approximation (Castor, Abbott & Klein 1975)

$$f_{\mathrm{rad}} = k \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}} L}{4\pi r^2 c} \left(\frac{1}{\sigma_{\mathrm{Th}} n_{\mathrm{e}} v_{\mathrm{th}}} \frac{\mathrm{d}v}{\mathrm{d}r} \right)^{\alpha}$$

- nondimensional parameters k and α describe the line-strength distribution function (CAK, Puls et al. 2000)
- in general NLTE calculations necessary to obtain k and α (Abbott 1982)

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = f_{\mathrm{rad}} - \frac{\rho G M (1-\Gamma)}{r^2}$$

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$\rho v \frac{dv}{dr} = k \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}} L}{4\pi r^2 c} \left(\frac{1}{\sigma_{\mathrm{Th}} n_{\mathrm{e}} v_{\mathrm{th}}} \frac{dv}{dr} \right)^{\alpha} - \frac{\rho G M (1 - \Gamma)}{r^2}$$

CAK theory

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$$r^2 v \frac{\mathrm{d}v}{\mathrm{d}r} = k \frac{\sigma_{\mathrm{Th}} L}{4\pi c} \frac{n_{\mathrm{e}}}{\rho} \left(\frac{\rho}{n_{\mathrm{e}}} \frac{4\pi r^2 v}{\sigma_{\mathrm{Th}} \dot{M} v_{\mathrm{th}}} \frac{\mathrm{d}v}{\mathrm{d}r} \right)^{\alpha} - GM(1 - \Gamma)$$

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$$r^2 v \frac{dv}{dr} = k \frac{\sigma_{Th} L}{4\pi c} \frac{n_e}{\rho} \left(\frac{\rho}{n_e} \frac{4\pi r^2 v}{\sigma_{Th} \dot{M} v_{th}} \frac{dv}{dr} \right)^{\alpha} - GM(1-\Gamma)$$

- velocity in terms of the escape speed

$$w \equiv \frac{v^2}{v_{esc}^2}, \text{ where } v_{esc}^2 = \frac{2GM(1-\Gamma)}{R_*}$$

- new radial variable

$$x \equiv 1 - \frac{R_*}{r}$$

(Owocki 2004)

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C\,(w')^{\alpha}$$

- where
  - $w' \equiv \frac{\mathrm{d}w}{\mathrm{d}x}$
  - $C \equiv \frac{k\sigma_{\mathrm{Th}}L}{4\pi c G M(1-\Gamma)}\frac{n_{\mathrm{e}}}{\rho}\left(\frac{\rho}{n_{\mathrm{e}}}\frac{4\pi G M(1-\Gamma)}{\sigma_{\mathrm{Th}}\dot{M}v_{\mathrm{th}}}\right)^{\alpha}$
  - $\frac{\rho}{n_{\mathrm{e}}} \approx m_{\mathrm{H}}$
  - algebraic equation

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

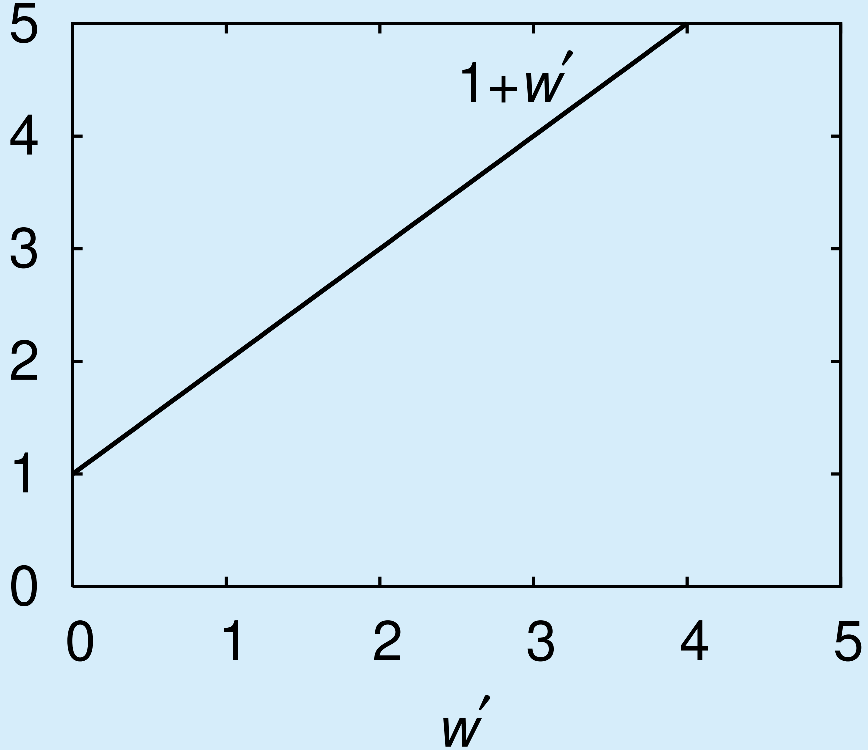
$$1 + w' = C\left(w'\right)^{\alpha}$$

- different solutions for different values of C (or mass-loss rate $\dot{M}$)

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C\,(w')^{\alpha}$$



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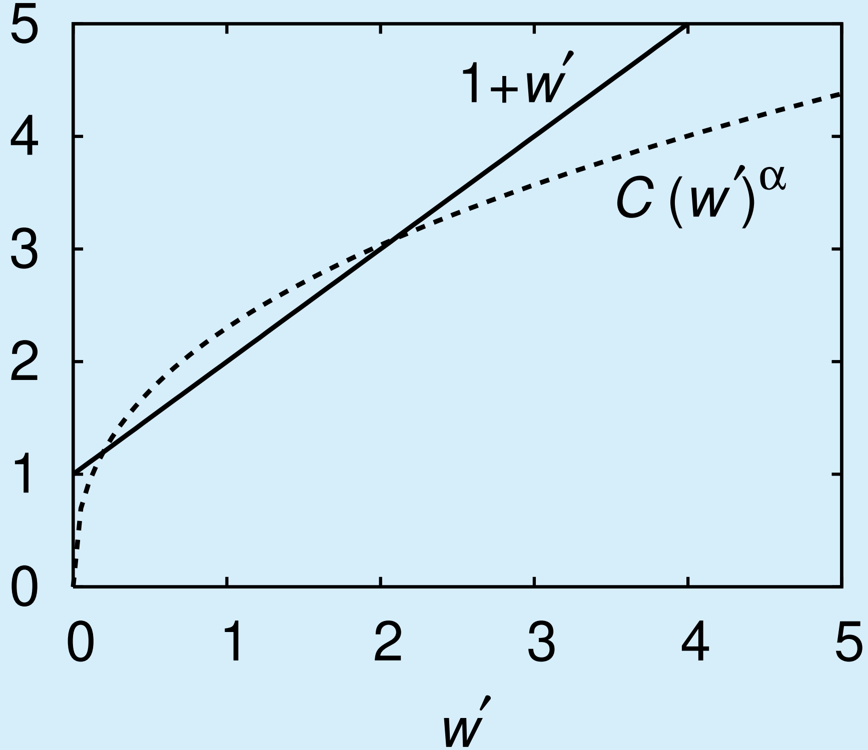

- large C (small $\dot{M}$): two solutions

CAK theory

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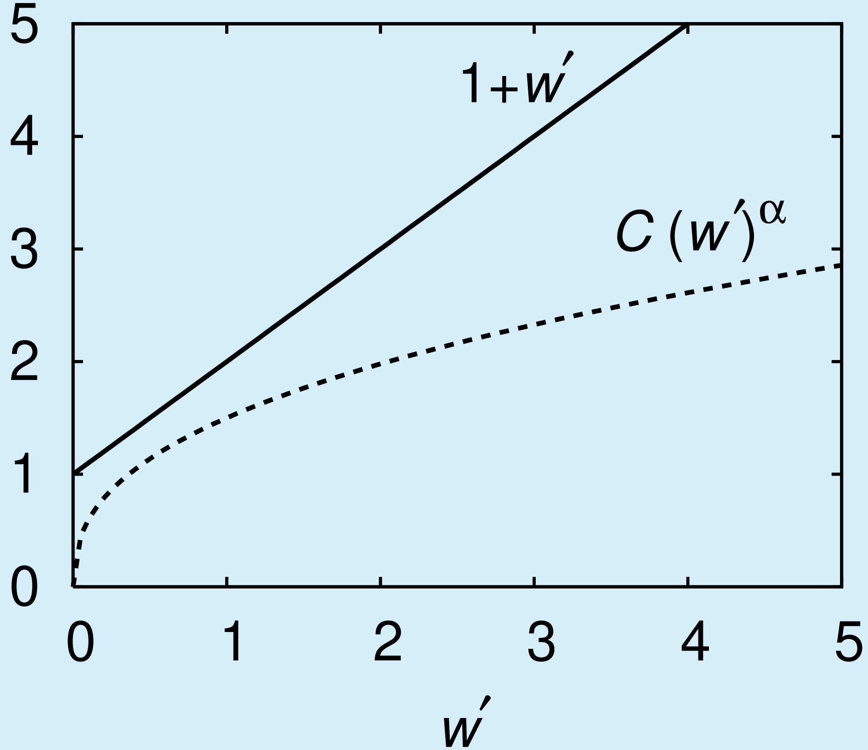

- small C (large $\dot{M}$): no solution

CAK theory

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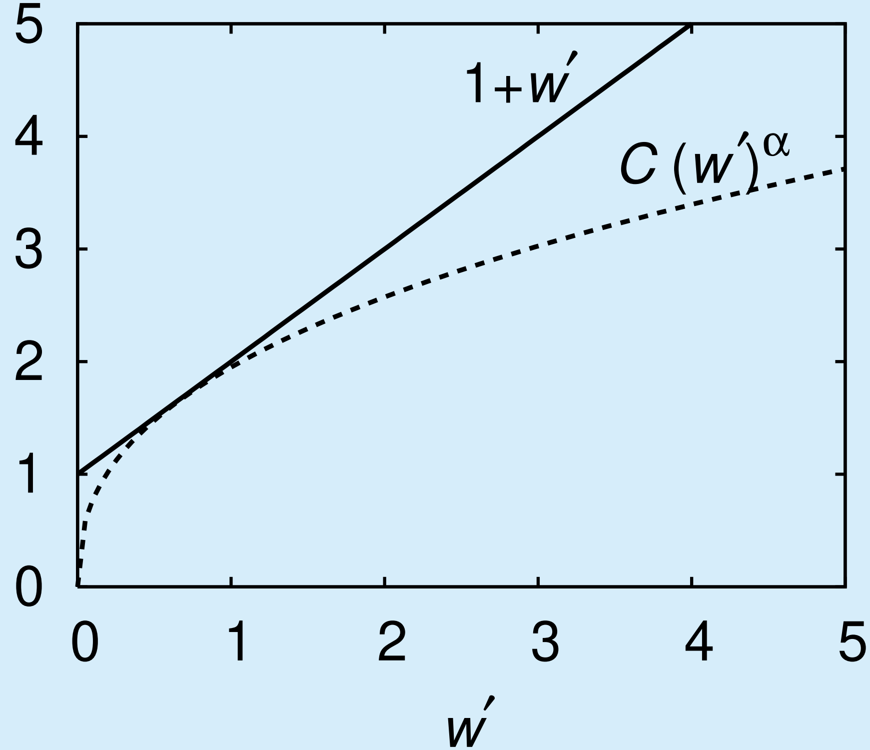
$$1 + w' = C\,(w')^{\alpha}$$



- critical value of C ($\dot{M}$): one solution

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C \left(w'\right)^{\alpha}$$

- critical (CAK) solution for a specific value of $\dot{M}$: the only smooth solution of detailed momentum equation from the stellar surface to infinity
- CAK solution: the largest $\dot{M}$ possible

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C\,(w')^{\alpha}$$

- critical (CAK) solution for a specific value of $\dot{M}$: the only smooth solution of detailed momentum equation from the stellar surface to infinity

$\Rightarrow$ possible to derive the wind mass-loss rate and velocity profile

$$w'_{\mathrm{c}} = \frac{\alpha}{1-\alpha}$$

$$C_{\mathrm{c}} = \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}}$$

CAK theory

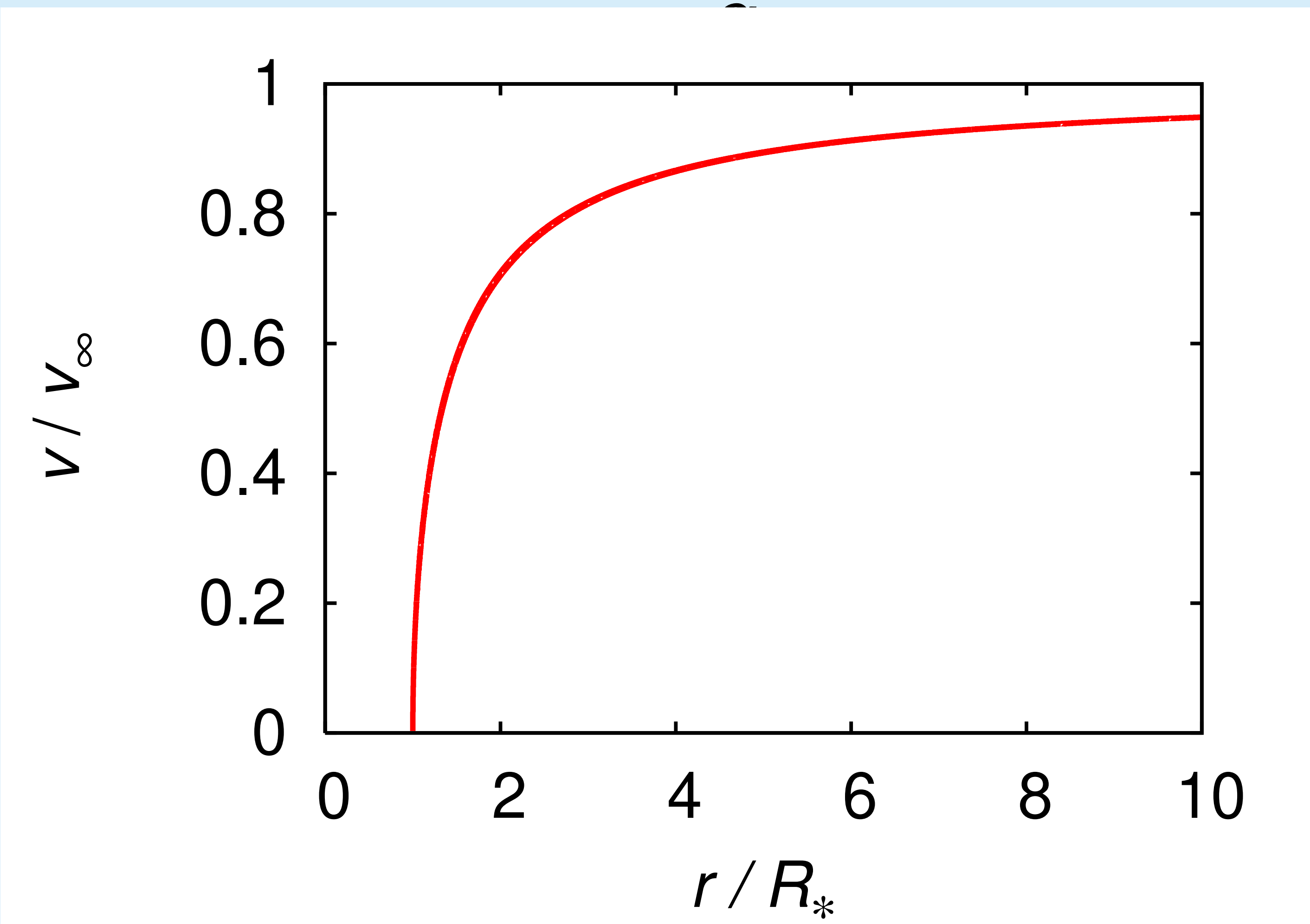
$$w'_{\rm c} = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow w = \frac{\alpha}{1-\alpha} x \Rightarrow v = v_\infty \left(1 - \frac{R_*}{r}\right)^{1/2}$$

- where the terminal velocity

$$v_\infty = v_{\rm esc} \sqrt{\frac{\alpha}{1-\alpha}}$$

CAK theory



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- v_∞ scales with v_{esc}!
- as v_∞ of order of $100\,\mathrm{km\,s^{-1}}$, hot star winds are strongly supersonic!

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- example: α Cam, $v_{\rm esc} = 620\,{\rm km\,s^{-1}}$, $\alpha = 0.61$

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- v_∞ scales with v_{esc}!
- example: α Cam, $v_{\mathrm{esc}} = 620\,\mathrm{km\,s^{-1}}$, $\alpha = 0.61$
  $\Rightarrow$ prediction: $v_\infty = 780\,\mathrm{km\,s^{-1}}$

CAK theory

$$C_c = \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}}$$

$$\Rightarrow \quad \dot{M} = \left[\frac{4\pi m_H G M (1-\Gamma)}{\sigma_{Th}}\right]^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{v_{th}\,(1-\alpha)^{\frac{\alpha-1}{\alpha}}} \left(\frac{kL}{c}\right)^{\frac{1}{\alpha}}$$

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- example: α Cam: $\dot{M} \approx 9 \times 10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$

Beyond the classical CAK theory

- inclusion of the dependence of k on the ionisation equilibrium – δ parameter (Abbott 1982)

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Beyond the classical CAK theory

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- dropping of the Sobolev approximation (Gräfener & Hamann 2008, Sander et al. 2017, Krtička & Kubát 2017, Sundqvist et al. 2019)

Comparison with observations

- nice wind theory $\Rightarrow$ compare it with observations!

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no coffee time yet...

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- problem: it is not possible to “measure” the wind parameters directly from observations

⇒ we have to work more to understand the wind spectral characteristics

Comparison with observations

- nice wind theory $\Rightarrow$ compare it with observations!
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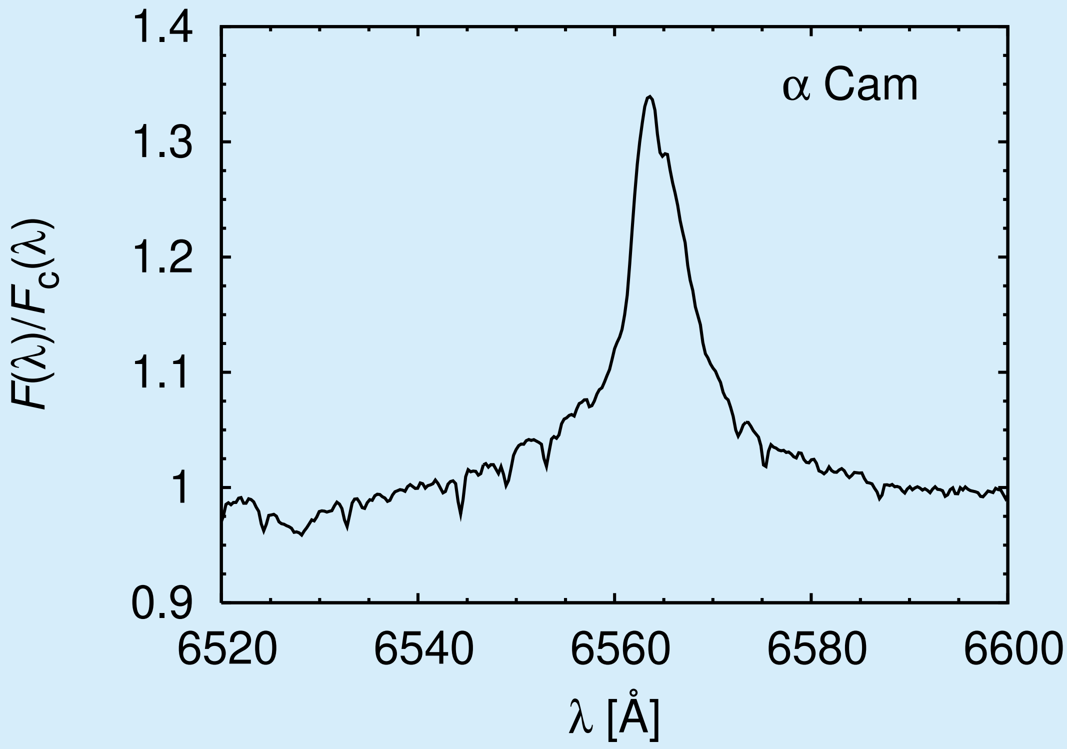
- problem: it is not possible to “measure” the wind parameters directly from observations

$\Rightarrow$ we have to work more to understand the wind spectral characteristics

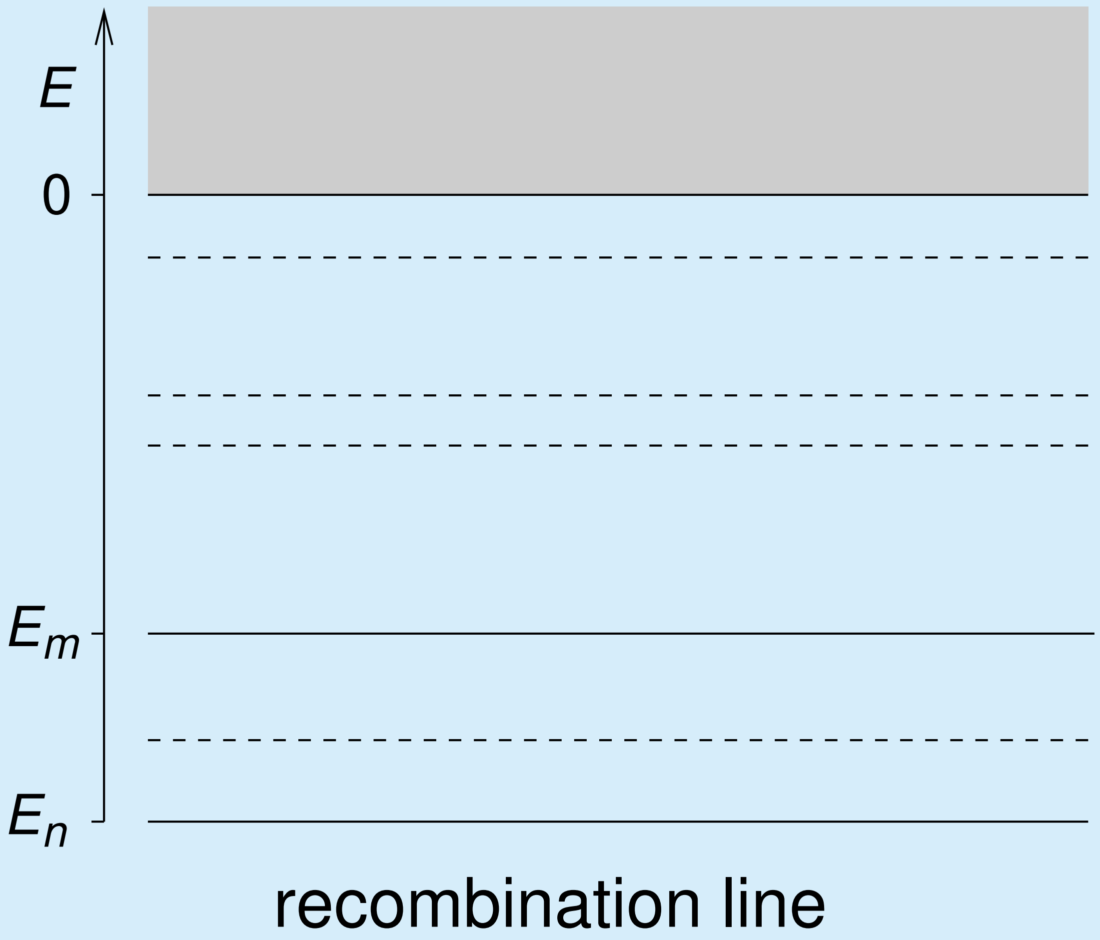
- more theory, please!

Observations: Hα line profiles

- Hα emission line of α Cam



Observations: Hα line profiles



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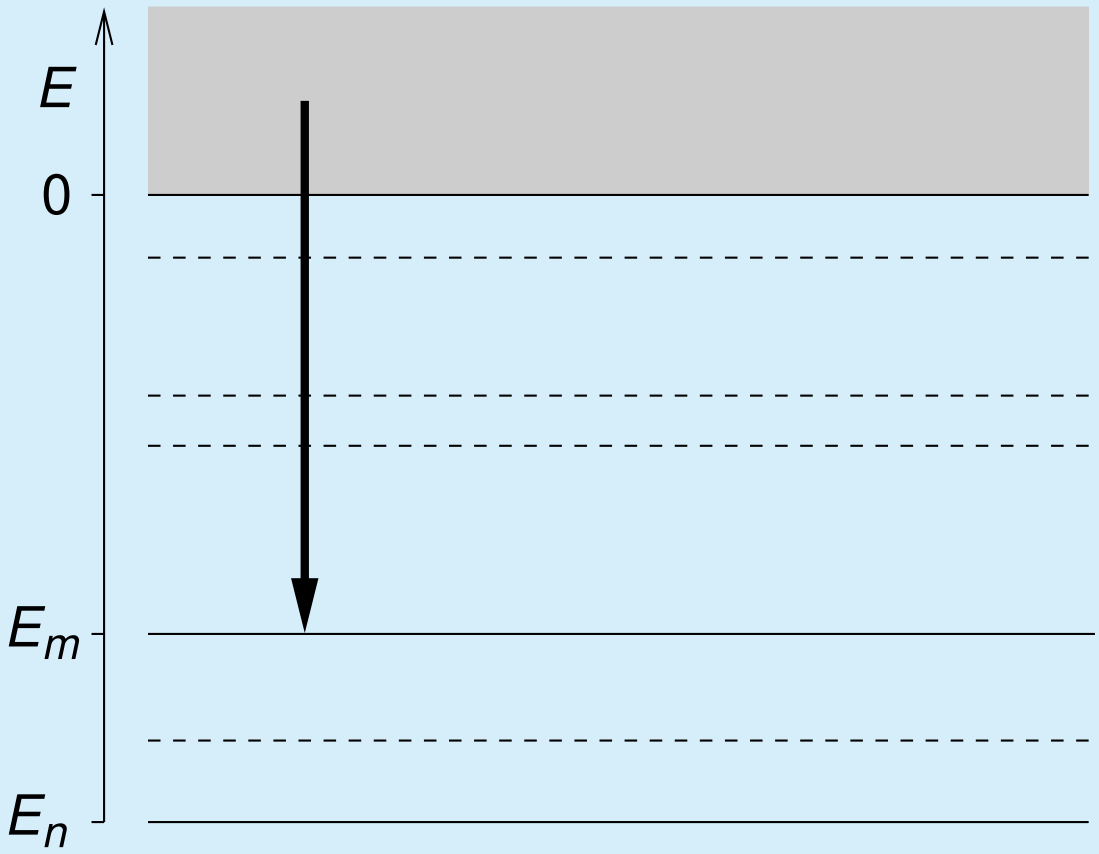
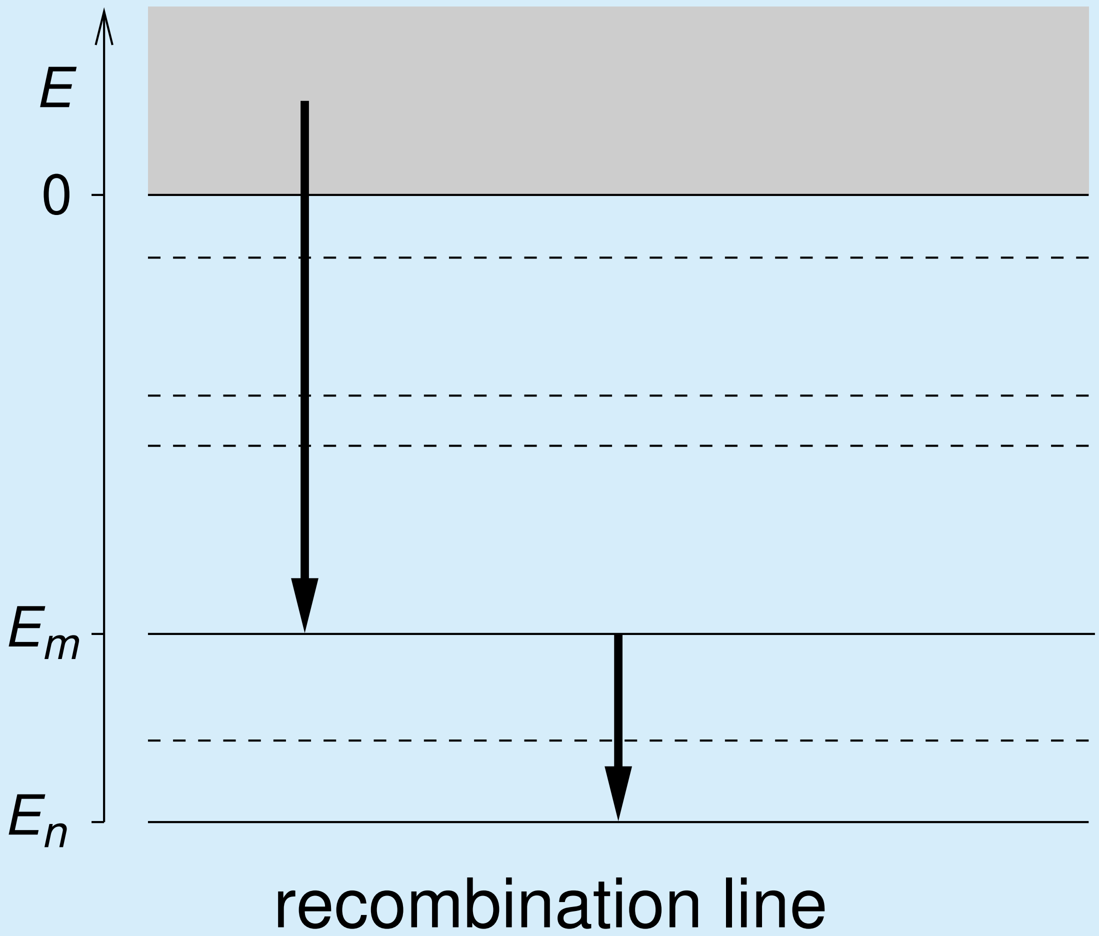
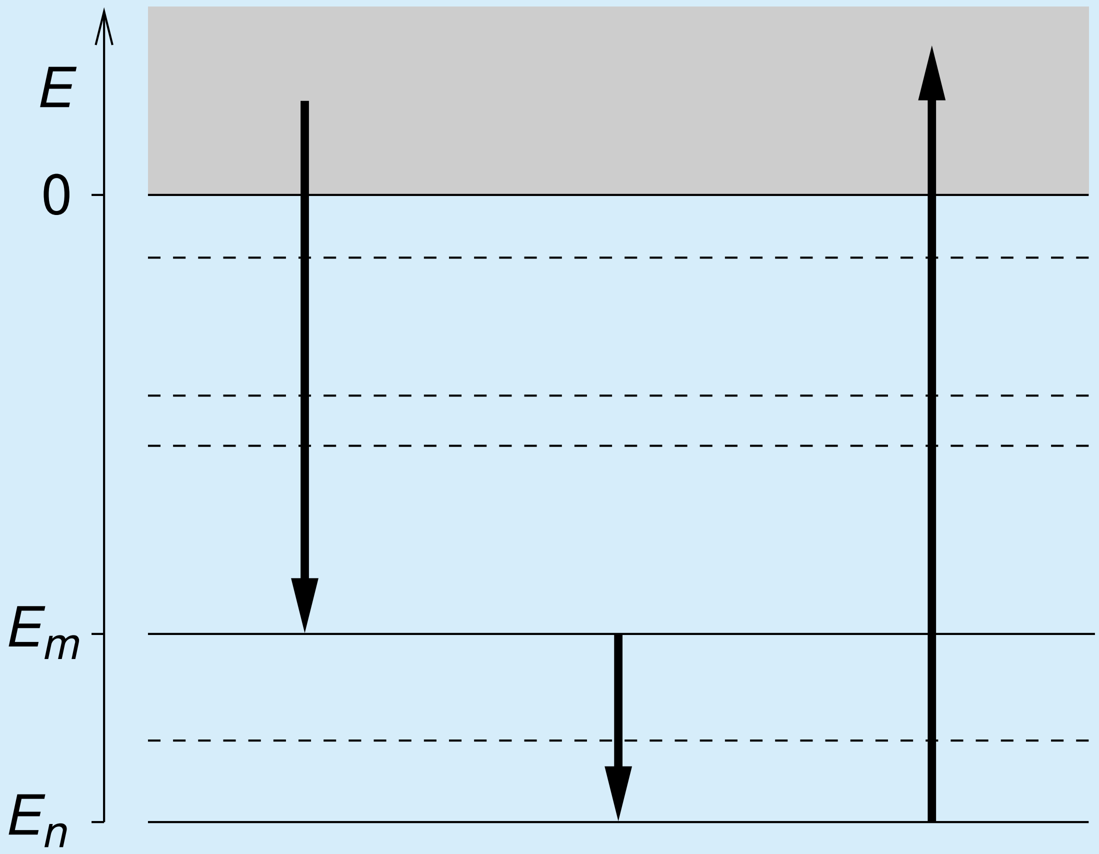


recombination line

Observations: Hα line profiles



Observations: Hα line profiles



recombination line

Observations: Hα line profiles

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- number of Hα photons emitted per unit of time

$$N_{\mathrm{H}\alpha} \sim n_{\mathrm{p}} n_{\mathrm{e}}$$

- where
  - n_{p} is the number density of H^{+}
  - n_{e} is the number density of free electrons

Observations: Hα line profiles

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- number of Hα photons emitted per unit of time

$$N_{H\alpha} \sim n_p n_e$$

- as $n_p \sim \dot{M}$ and $n_e \sim \dot{M} \Rightarrow N_{H\alpha} \sim \dot{M}^2$

Observations: Hα line profiles

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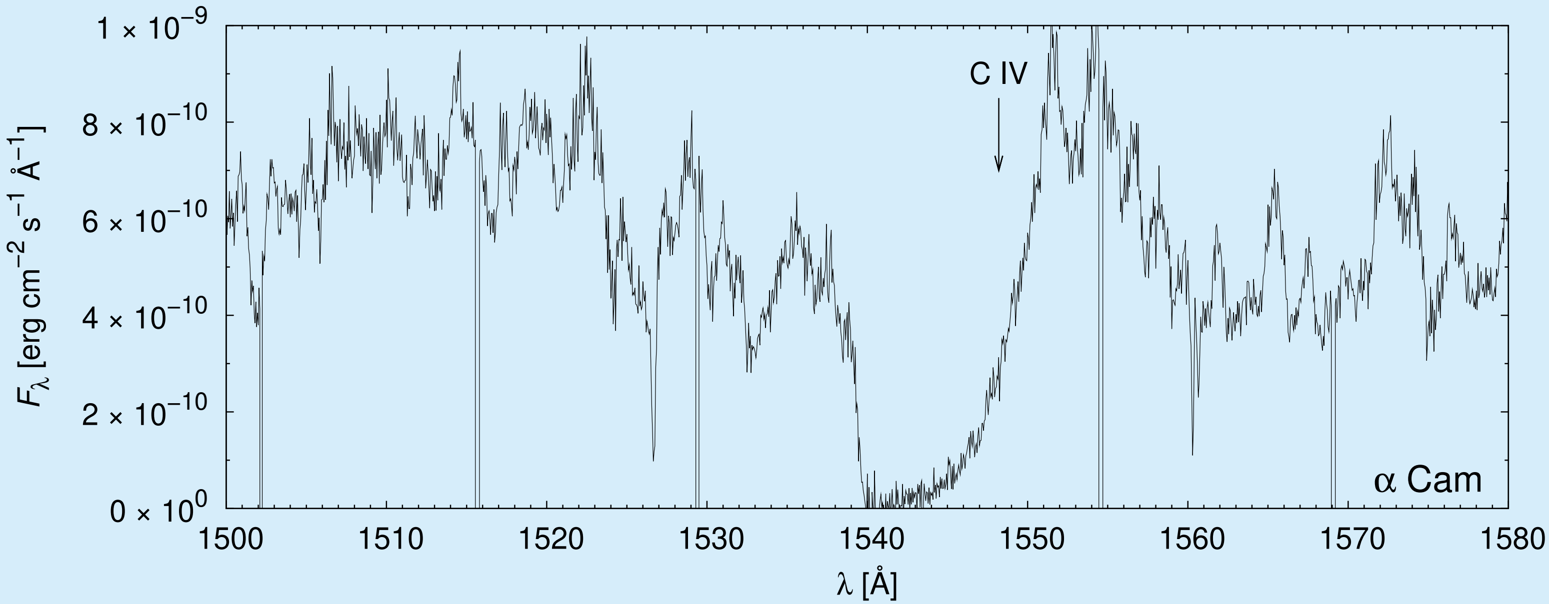
$\Rightarrow$ possibility to derive $\dot{M}$ using NLTE models

- example: α Cam
  - our estimate: $9 \times 10^{-6}\,M_\odot\,yr^{-1}$
  - Hα line observation: $1.5 \times 10^{-6}\,M_\odot\,yr^{-1}$ (Puls et al. 2006)

Observations: P Cyg lines I.

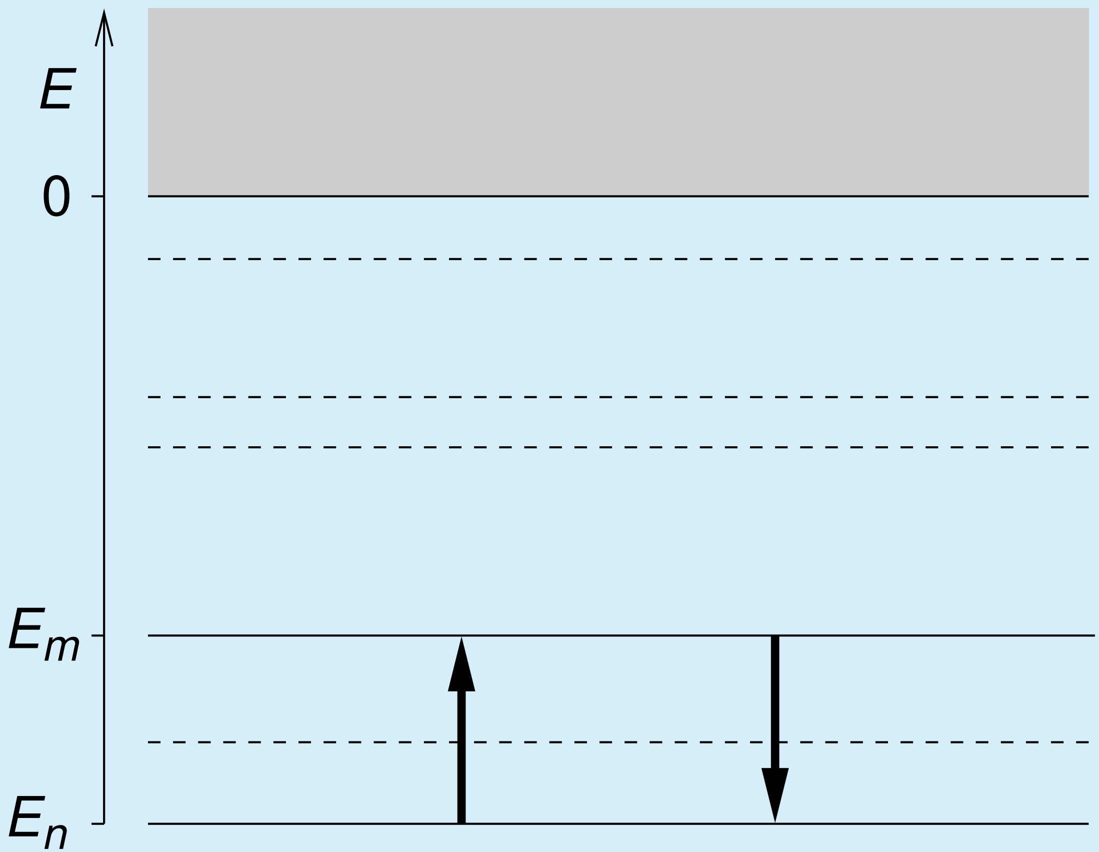
- IUE spectrum of α Cam



- saturated line profile of P Cyg type

Observations: P Cyg lines I.

- lines of the most abundant ion of a given element



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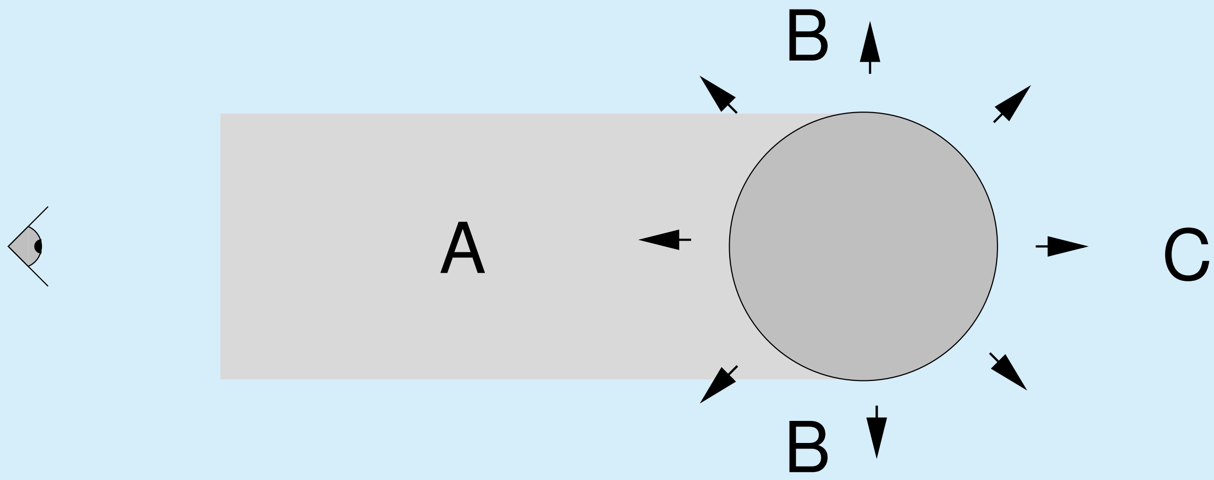

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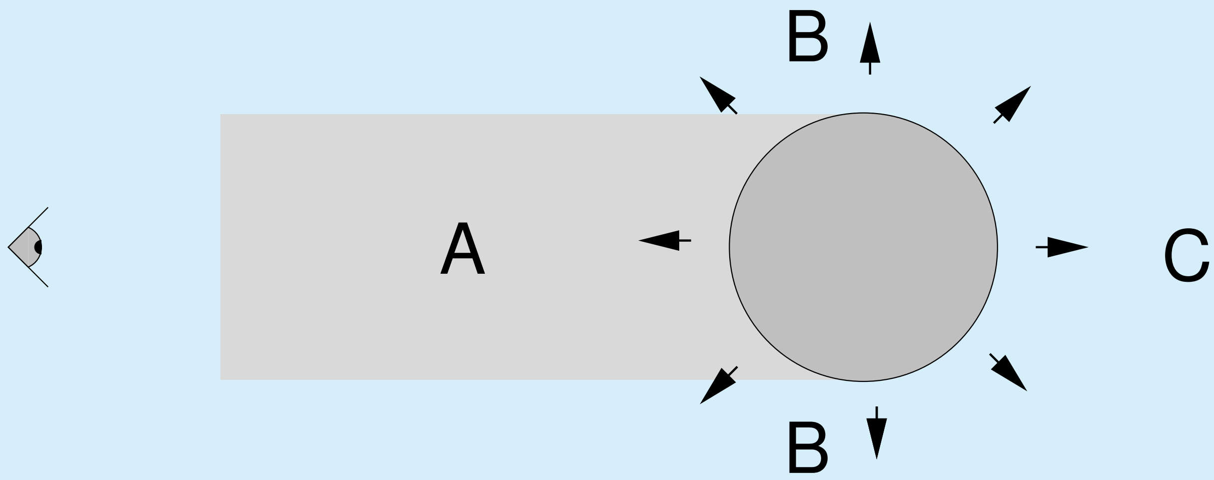
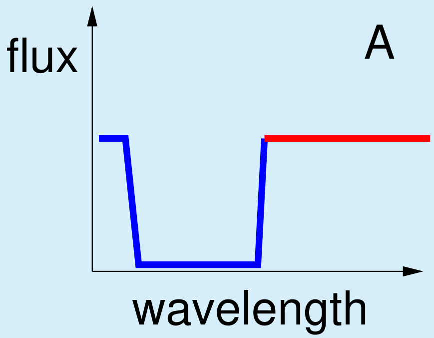
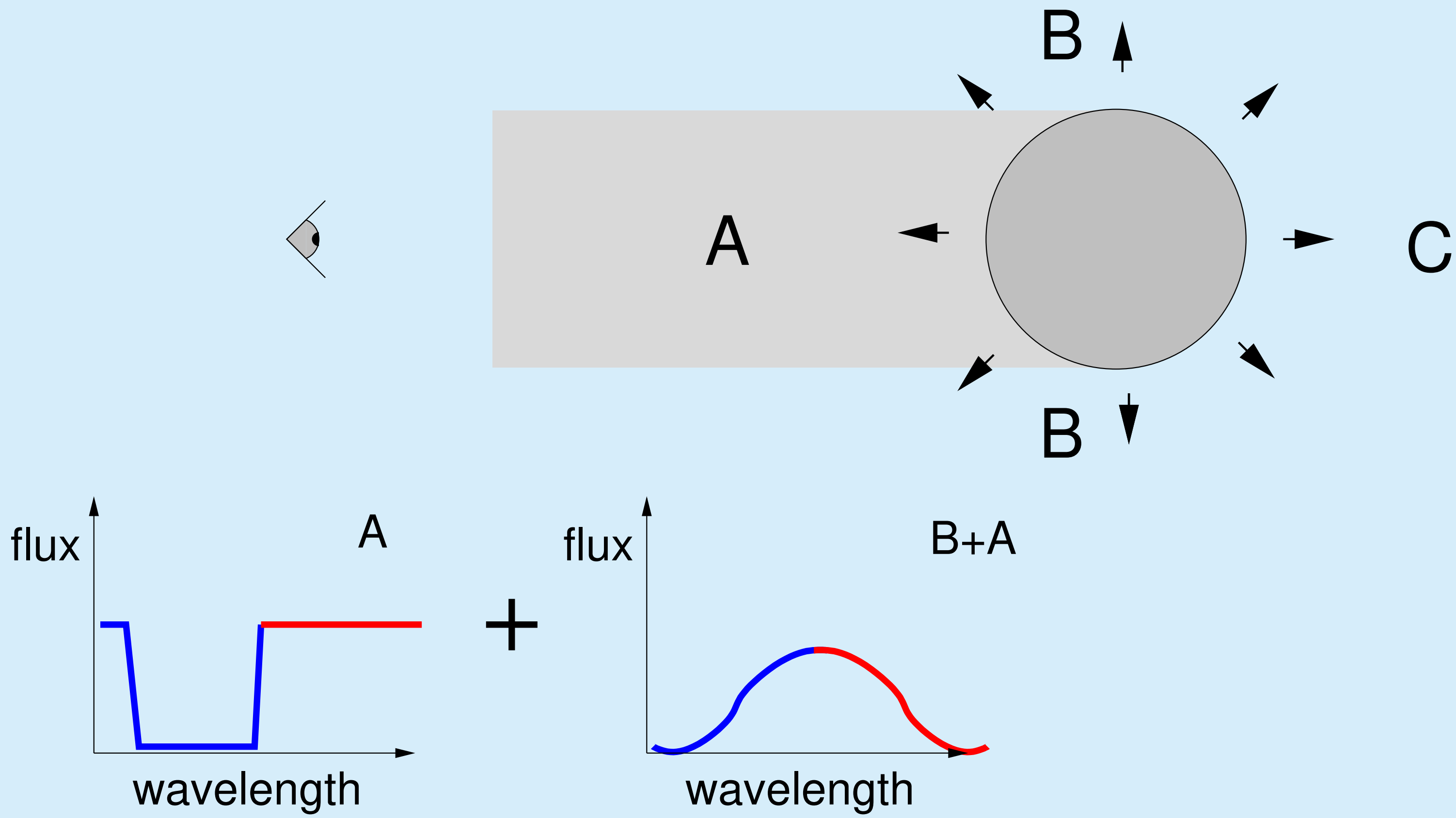




Observations: P Cyg lines I.



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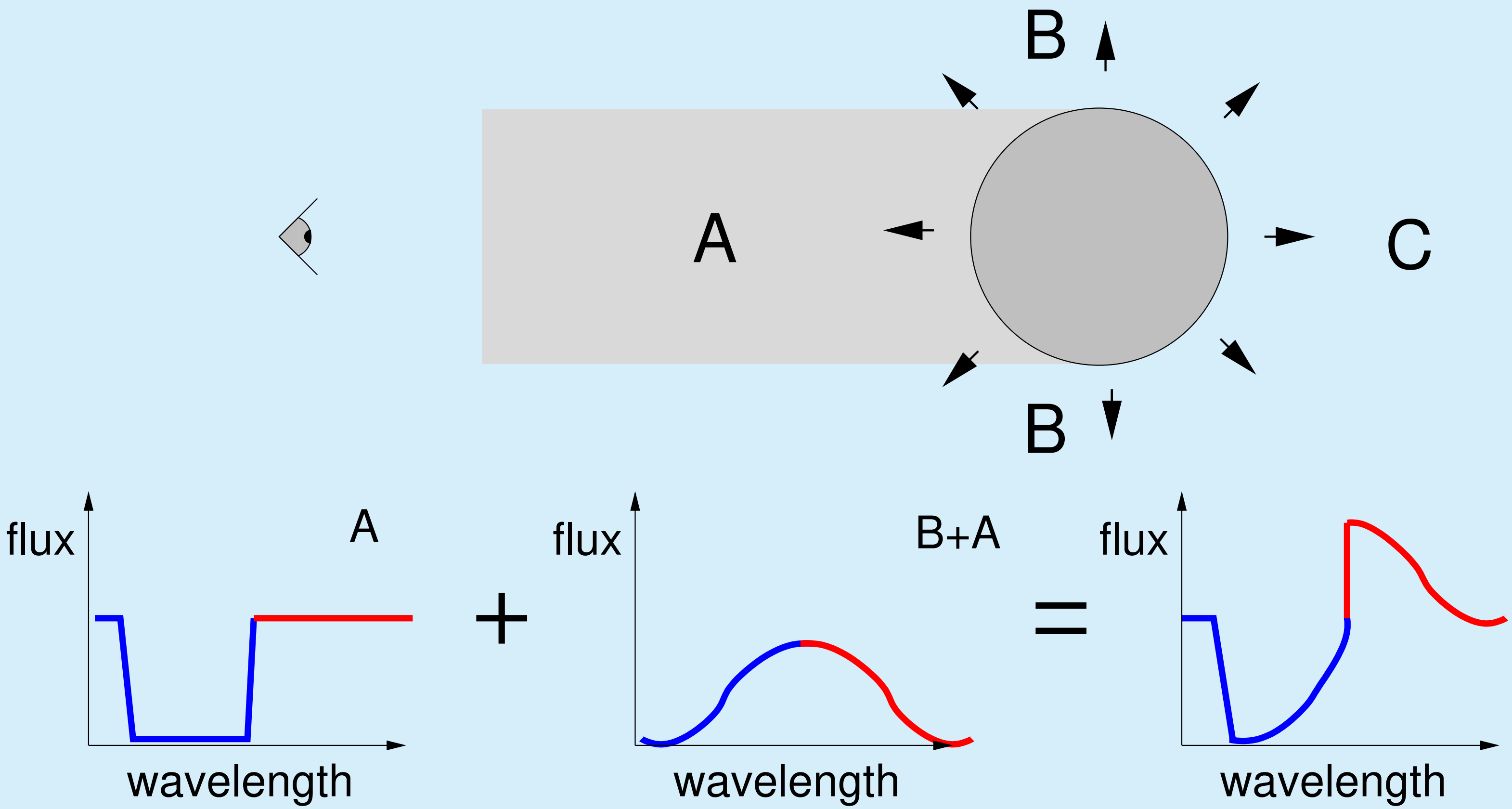
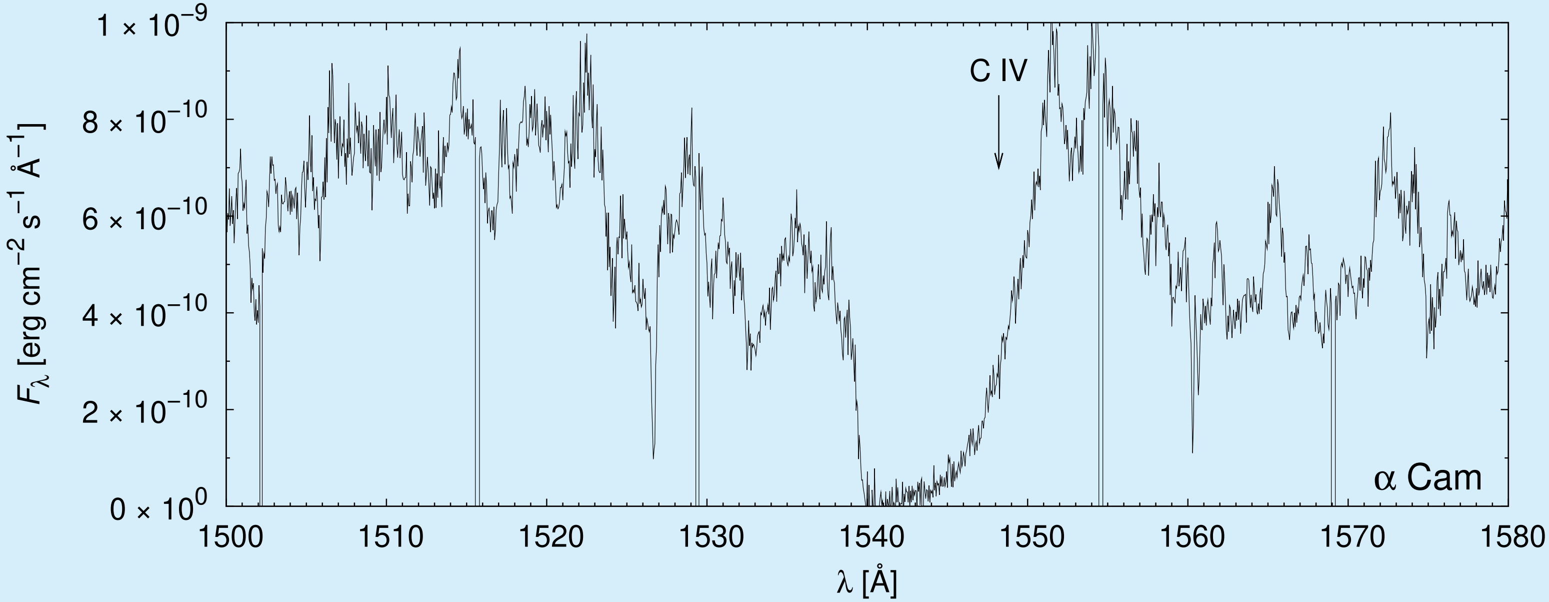


Observations: P Cyg lines I.

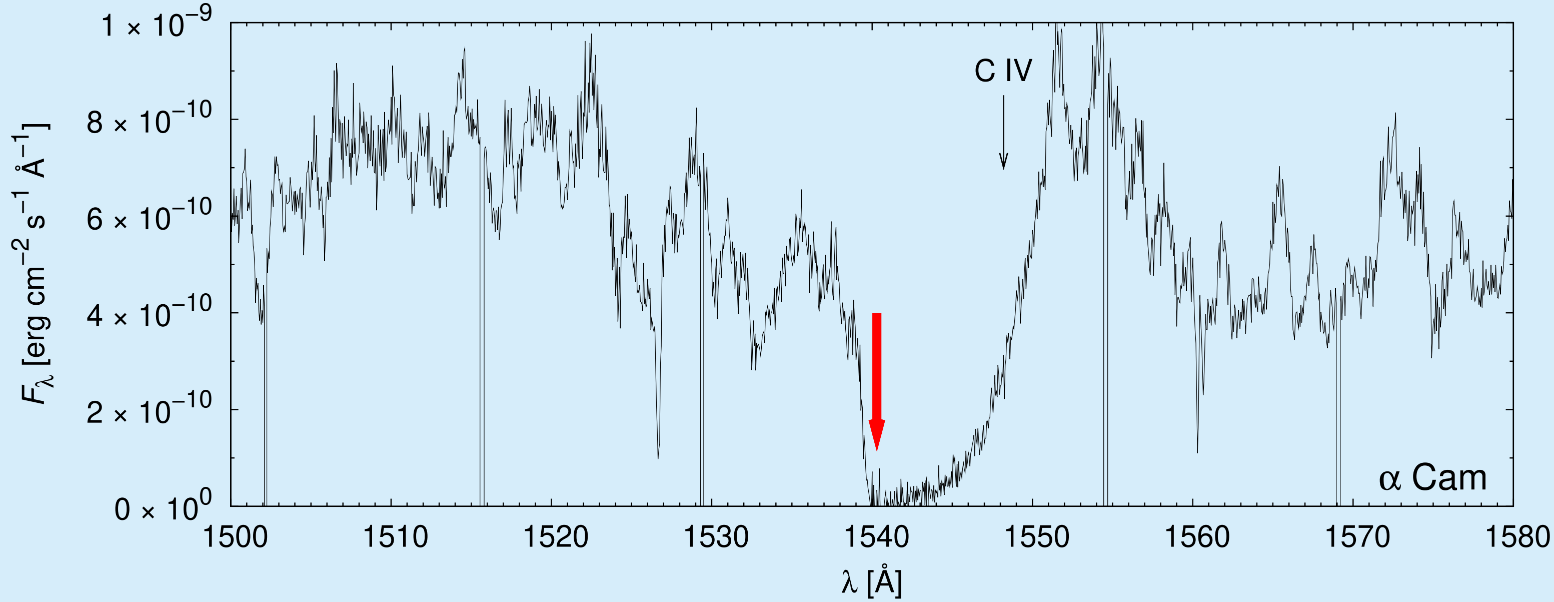
- IUE spectrum of α Cam



- absorption in the wind between star and observer
- emission due to the wind around the star

Observations: P Cyg lines I.

- IUE spectrum of α Cam



- the absorption edge originates in the wind with the highest velocity in the direction of observer

Observations: P Cyg lines I.

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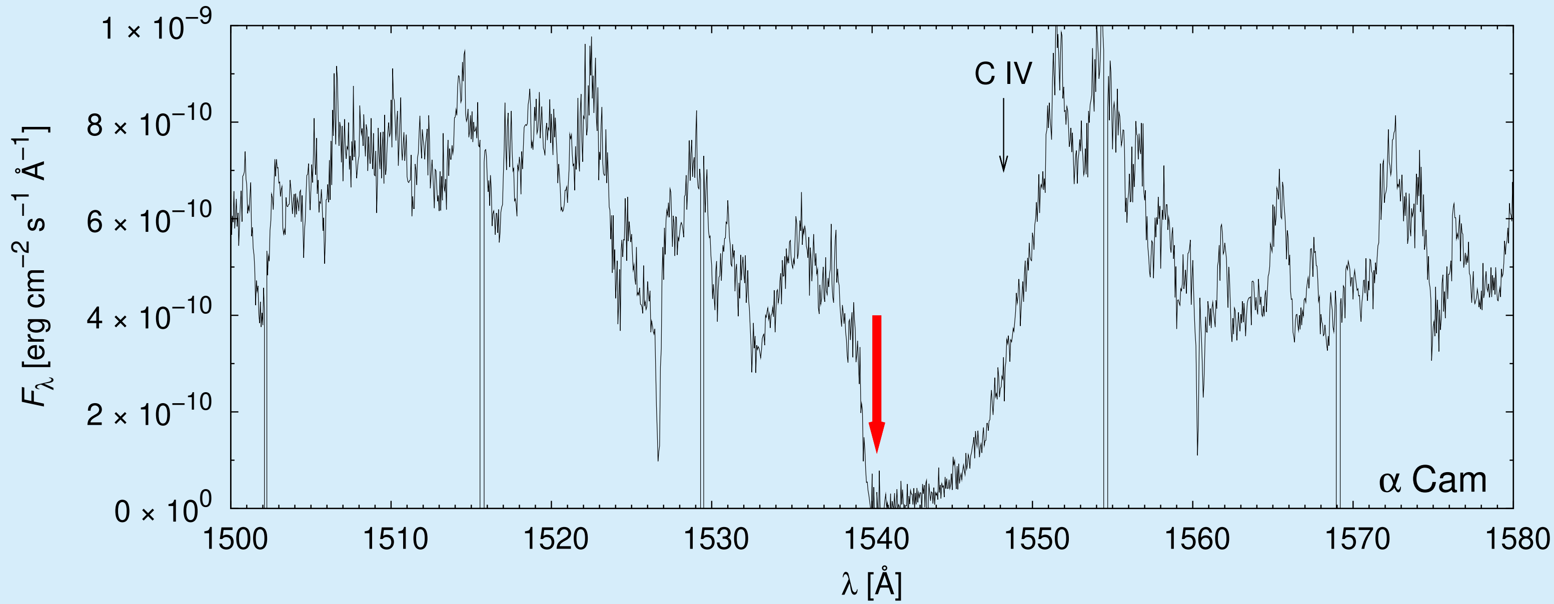
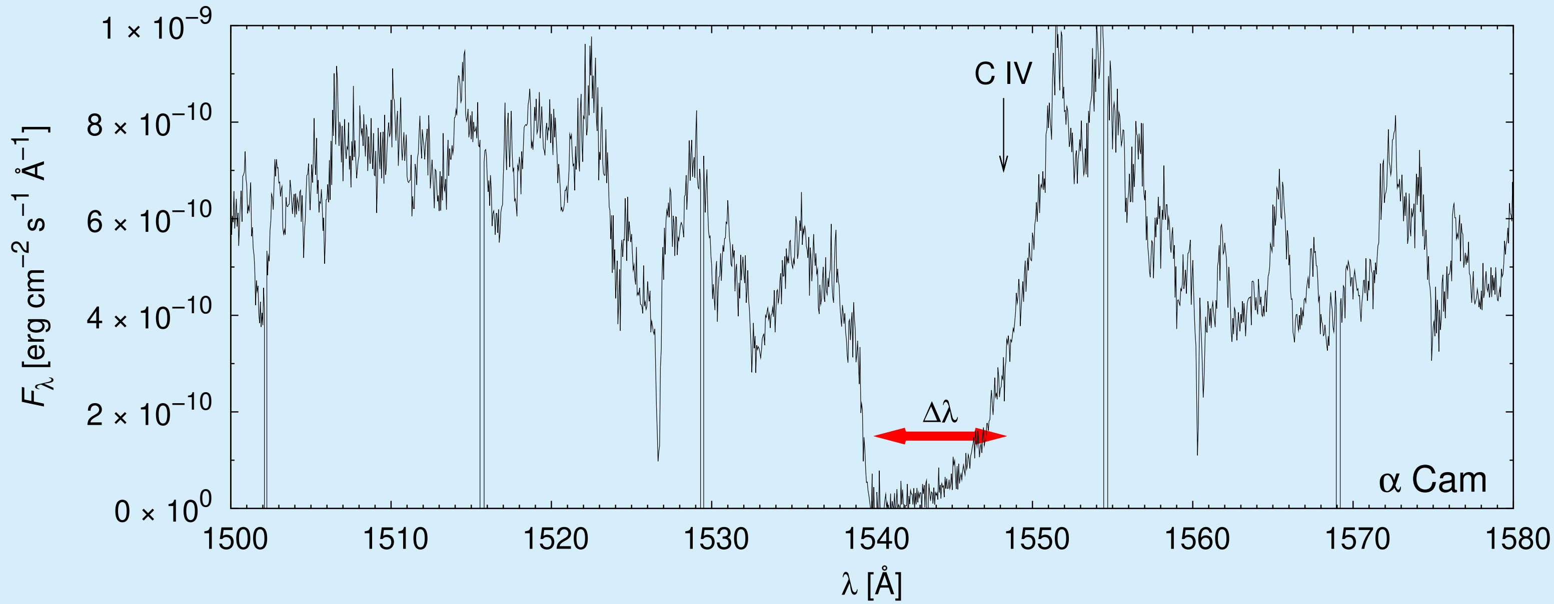


- the absorption edge originates in the wind with the highest velocity in the direction of observer
- possibility to derive the terminal velocity v_∞

Observations: P Cyg lines I.

- IUE spectrum of $\boldsymbol{\alpha}$ Cam



Observations: P Cyg lines I.

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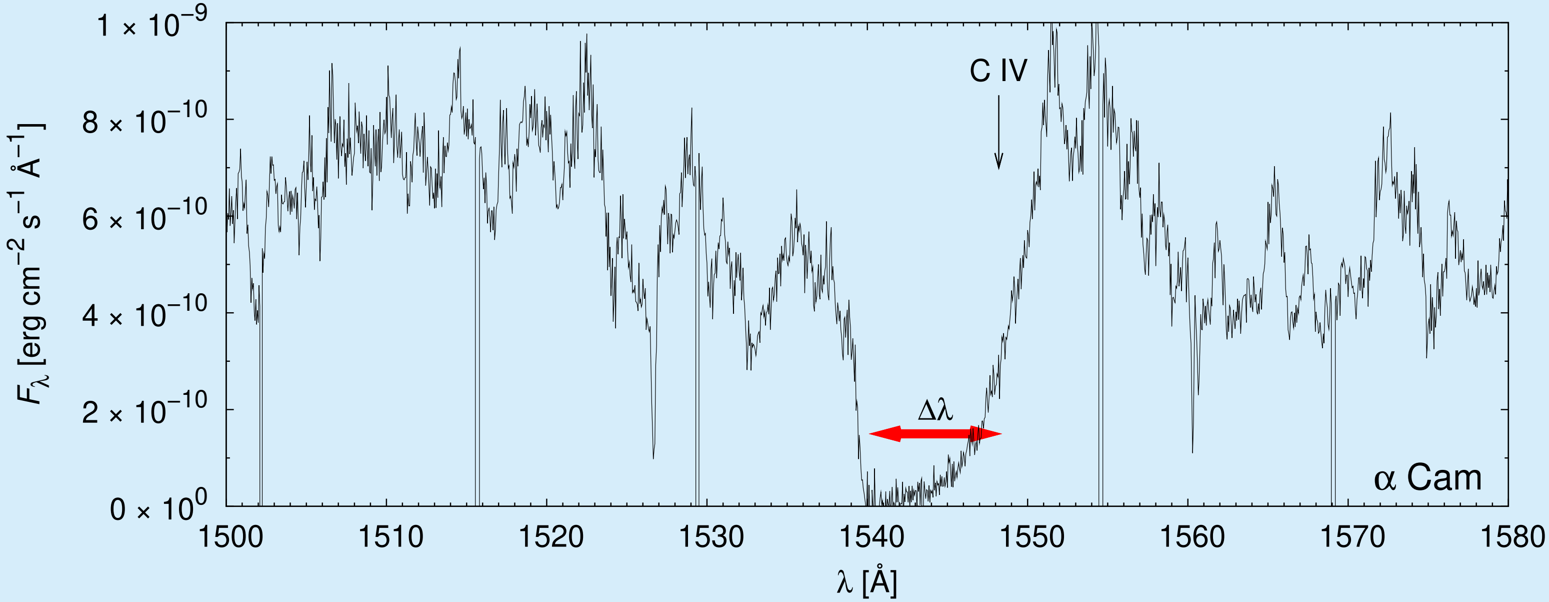
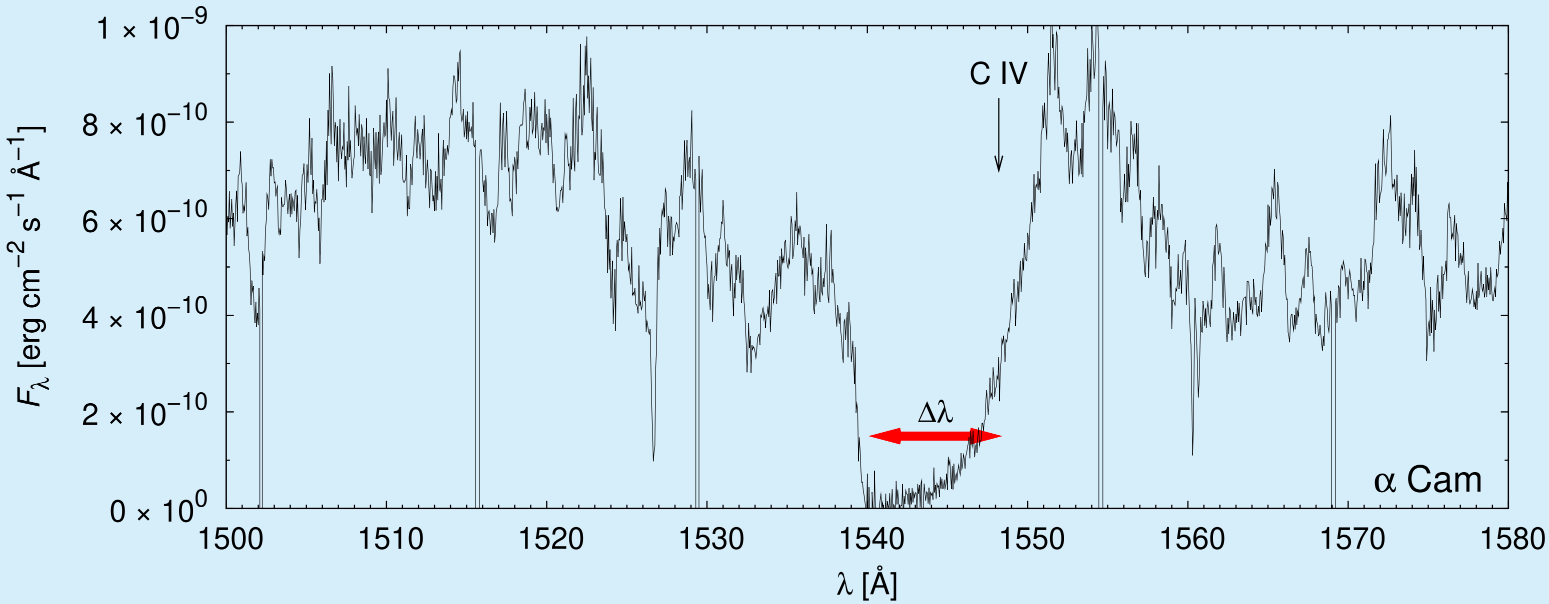


$$v_\infty = \frac{\Delta\lambda}{\lambda_0} c$$

- where λ_0 is the laboratory wavelength of a given line

Observations: P Cyg lines I.

- IUE spectrum of α Cam

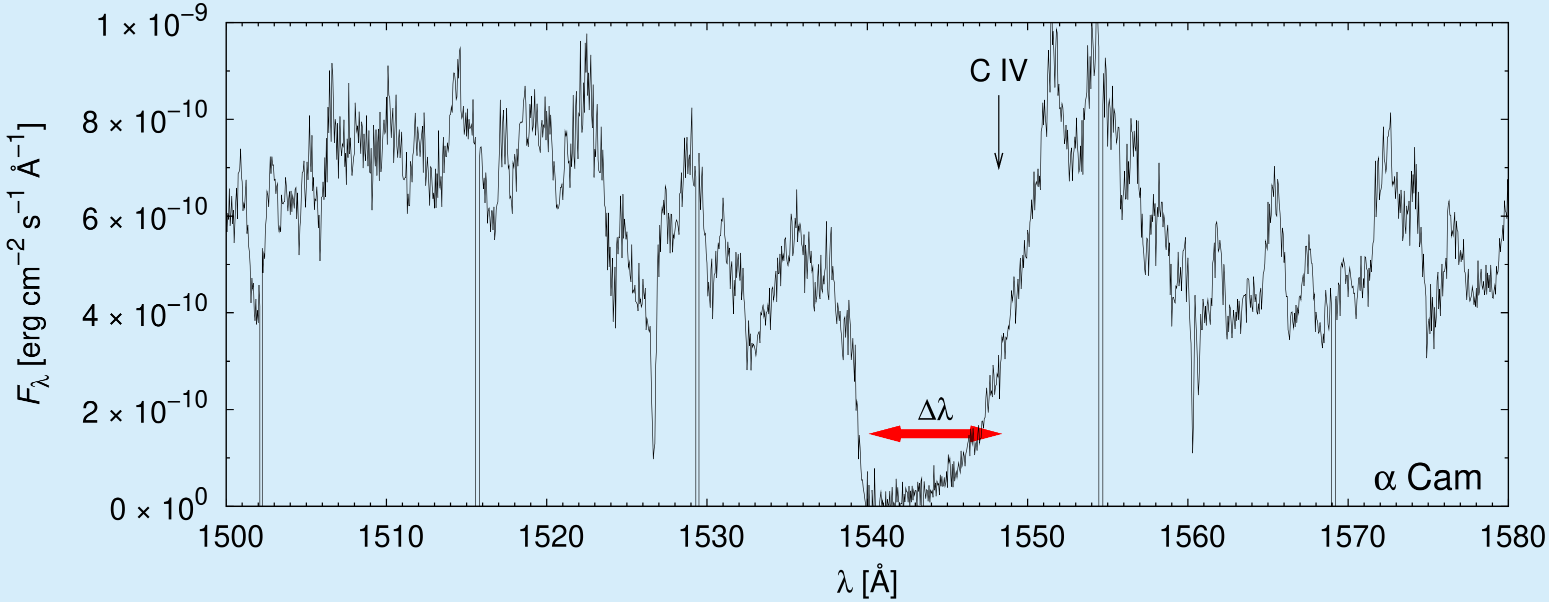


$$v_\infty = \frac{\Delta\lambda}{\lambda_0} c$$

- α Cam: $\Delta\lambda = 7.9\,\text{Å} \Rightarrow v_\infty = 1500\,\mathrm{km\,s^{-1}}$
- our estimate: $780\,\mathrm{km\,s^{-1}}$

Observations: P Cyg lines I.

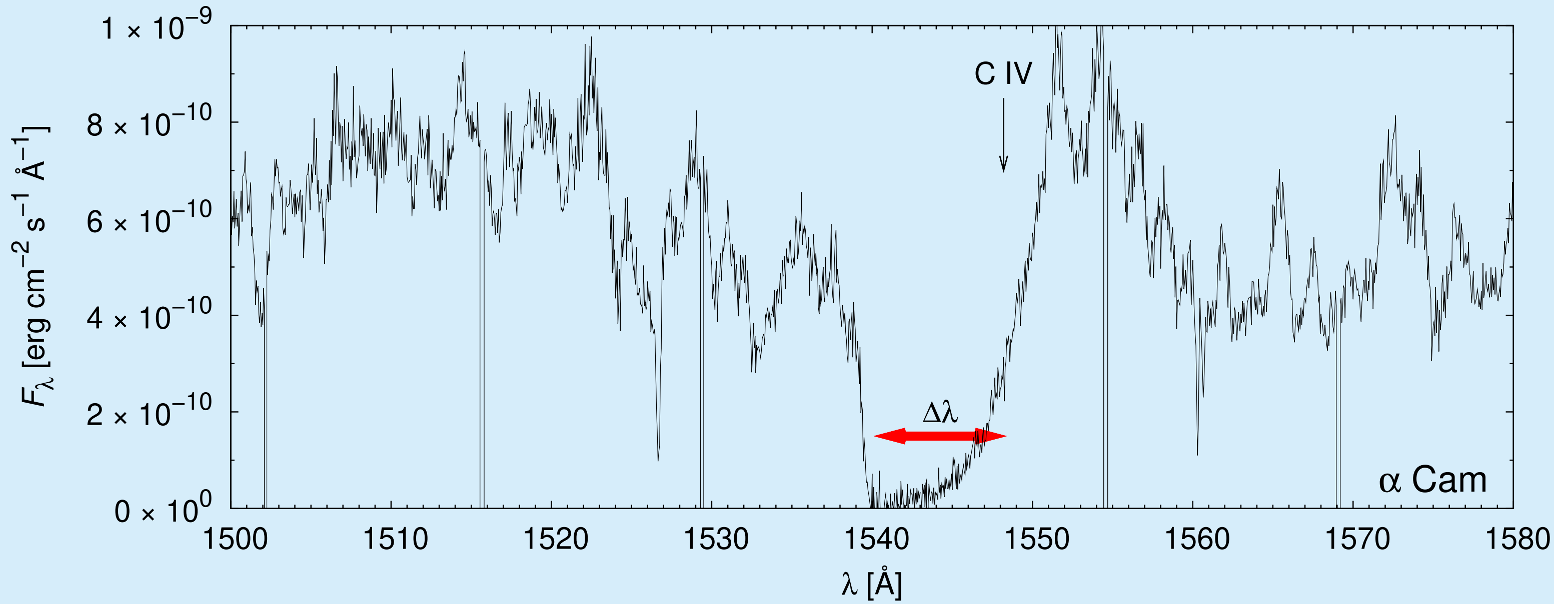
- IUE spectrum of α Cam



- why is the absorption part saturated?

Observations: P Cyg lines I.

- IUE spectrum of $\boldsymbol{\alpha}$ Cam



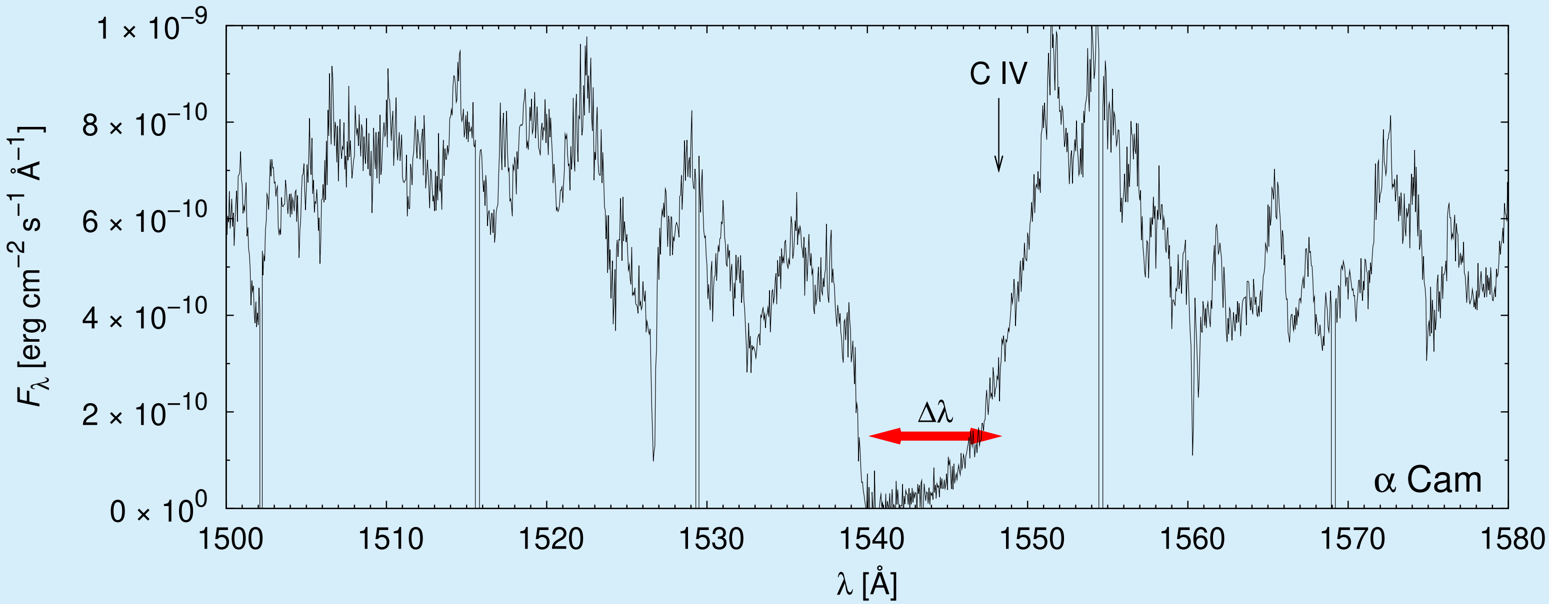
- why is the absorption part saturated?

$$I(y) = I_c(\mu) \exp\left[-\tau(\mu)y\right] + S_L \left\{1 - \exp\left[-\tau(\mu)y\right]\right\}$$

- the emergent intensity: $y \to 1$

Observations: P Cyg lines I.

- IUE spectrum of α Cam



- why is the absorption part saturated?

$$\boldsymbol{I} = \boldsymbol{I}_{\mathrm{c}}(\boldsymbol{\mu}) \exp\left[-\boldsymbol{\tau}(\boldsymbol{\mu})\right] + \boldsymbol{S}_{\mathrm{L}} \left\{1 - \exp\left[-\boldsymbol{\tau}(\boldsymbol{\mu})\right]\right\}$$

- optically thick lines $\boldsymbol{\tau} \gg 1$ with $\boldsymbol{S}_{\mathrm{L}} \ll \boldsymbol{I}_{\mathrm{c}} \Rightarrow \boldsymbol{I} \ll \boldsymbol{I}_{\mathrm{c}}$

Observations: P Cyg lines I.

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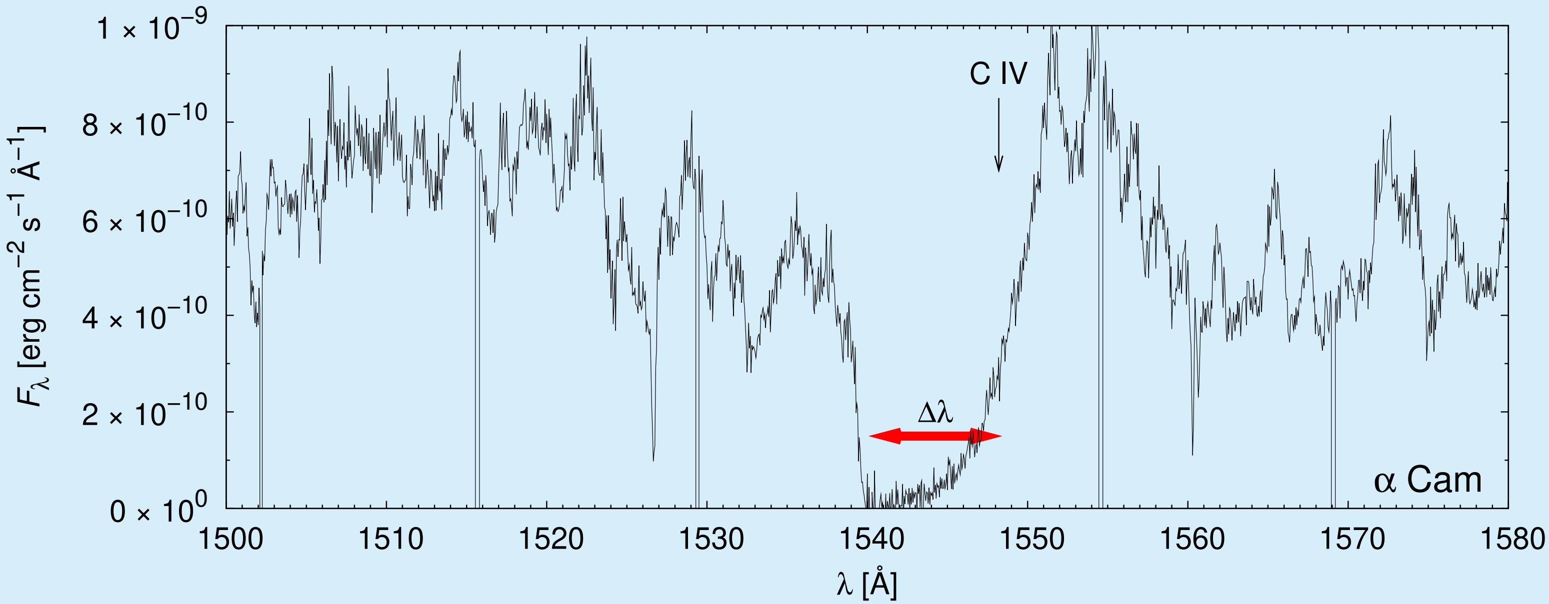


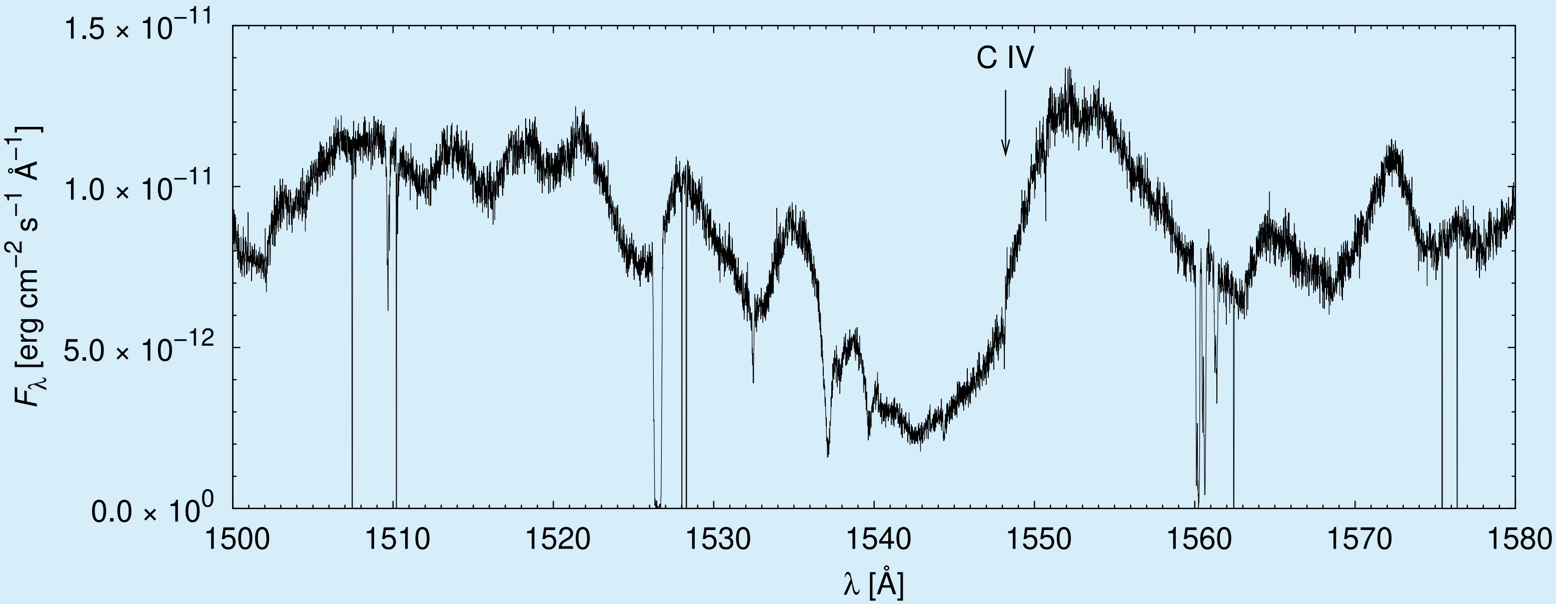
- for saturated lines ($\tau \gg 1$) the absorption part of the P Cyg line profile does not depend on τ

  $\Rightarrow$ determination of v_∞ possible

  $\Rightarrow$ determination of $\dot{M}$ impossible

Observations: P Cyg lines II.

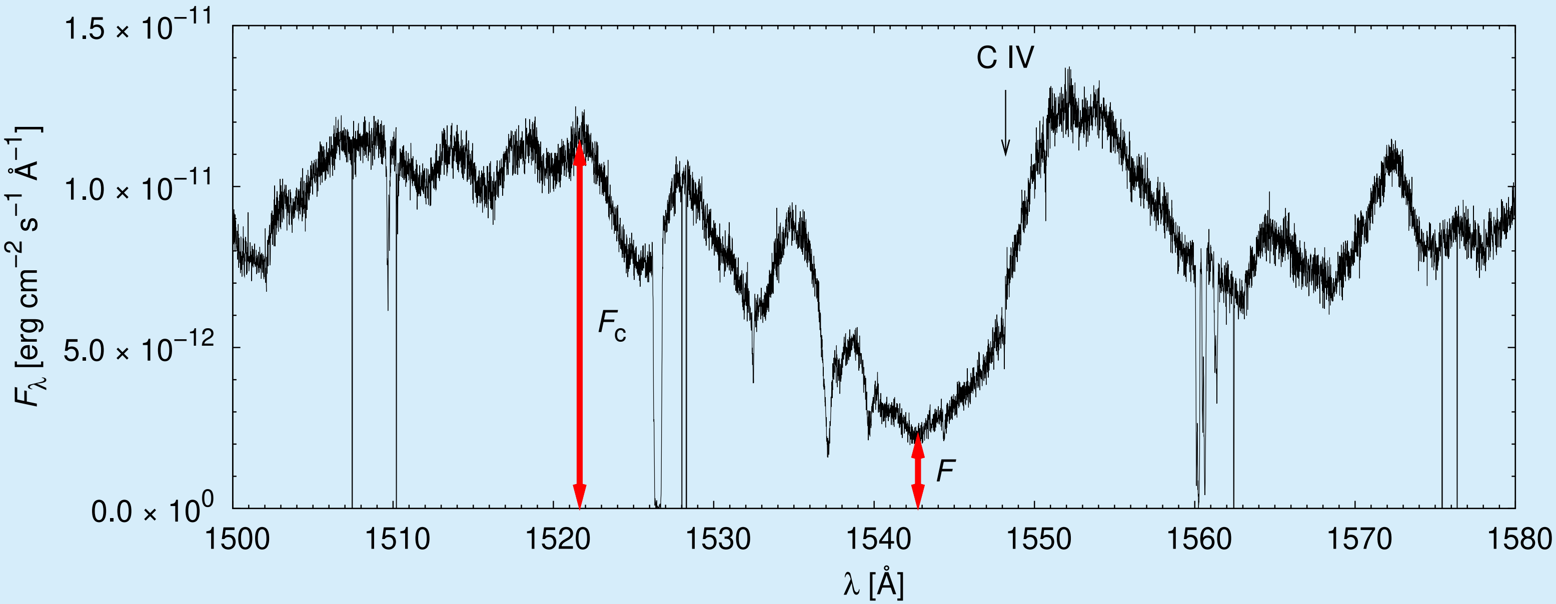
- HST spectrum of HD 13268



- unsaturated line profile of P Cyg type

Observations: P Cyg lines II.

- HST spectrum of HD 13268



$$\frac{F}{F_\mathrm{c}} \approx \exp\left[-\tau(\mu = 1)\right]$$

$$\tau(\mu = 1) = \frac{\chi_\mathrm{L} c}{\nu_0} \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$$

Observations: P Cyg lines II.

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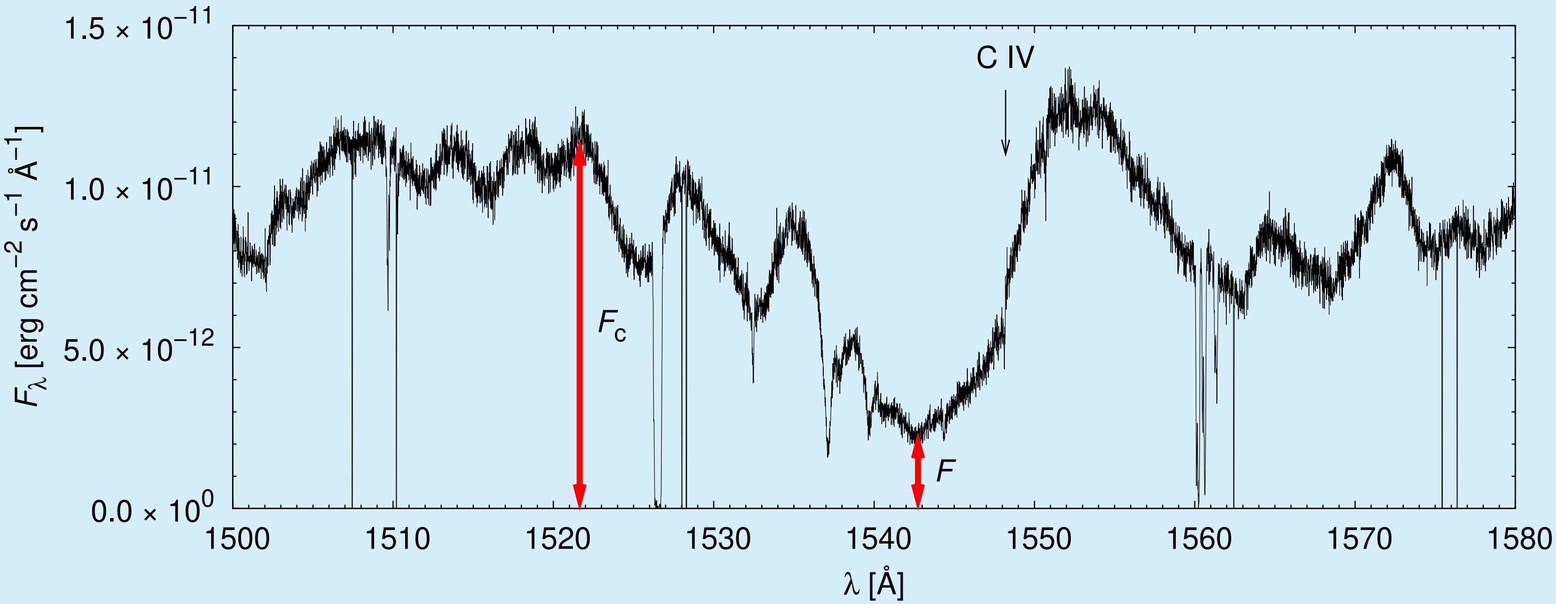

$$\frac{F}{F_\mathrm{c}} \approx \exp\left[-\tau(\mu = 1)\right]$$

$$\tau(\mu = 1) = \frac{\pi e^2}{m_\mathrm{e} c} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j}\right) \frac{c}{\nu_0} \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$$

Observations: P Cyg lines II.

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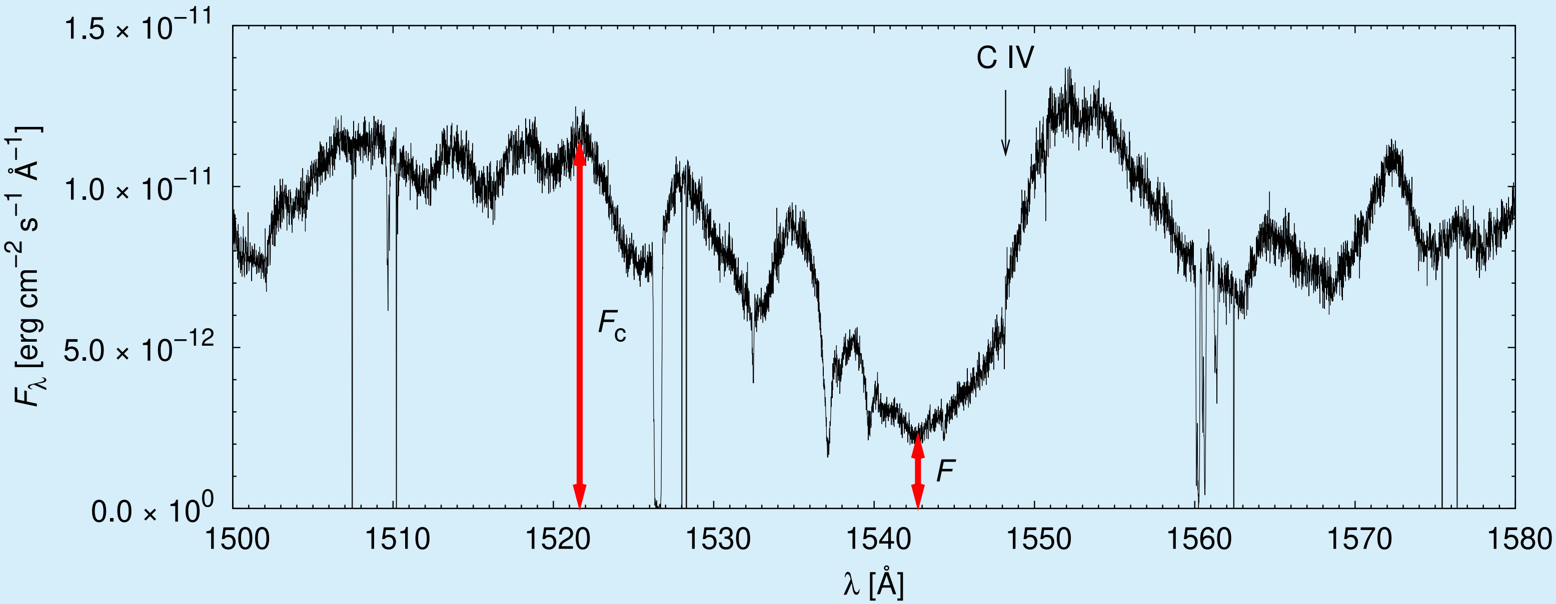

$$\frac{F}{F_\mathrm{c}} \approx \exp\left[-\tau(\mu = 1)\right]$$

$$\tau(\mu = 1) = \frac{\pi e^2}{m_\mathrm{e} c} \lambda_{ij} f_{ij} n_i(r) \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$$

Observations: P Cyg lines II.

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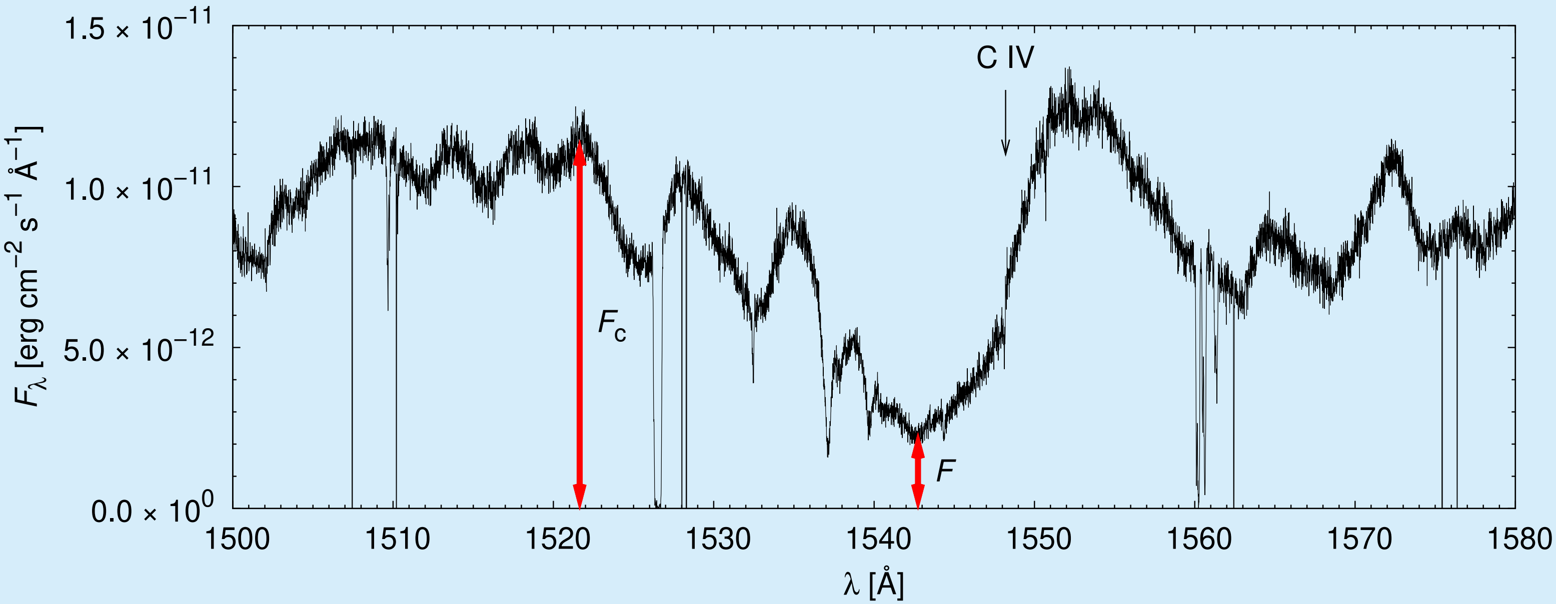
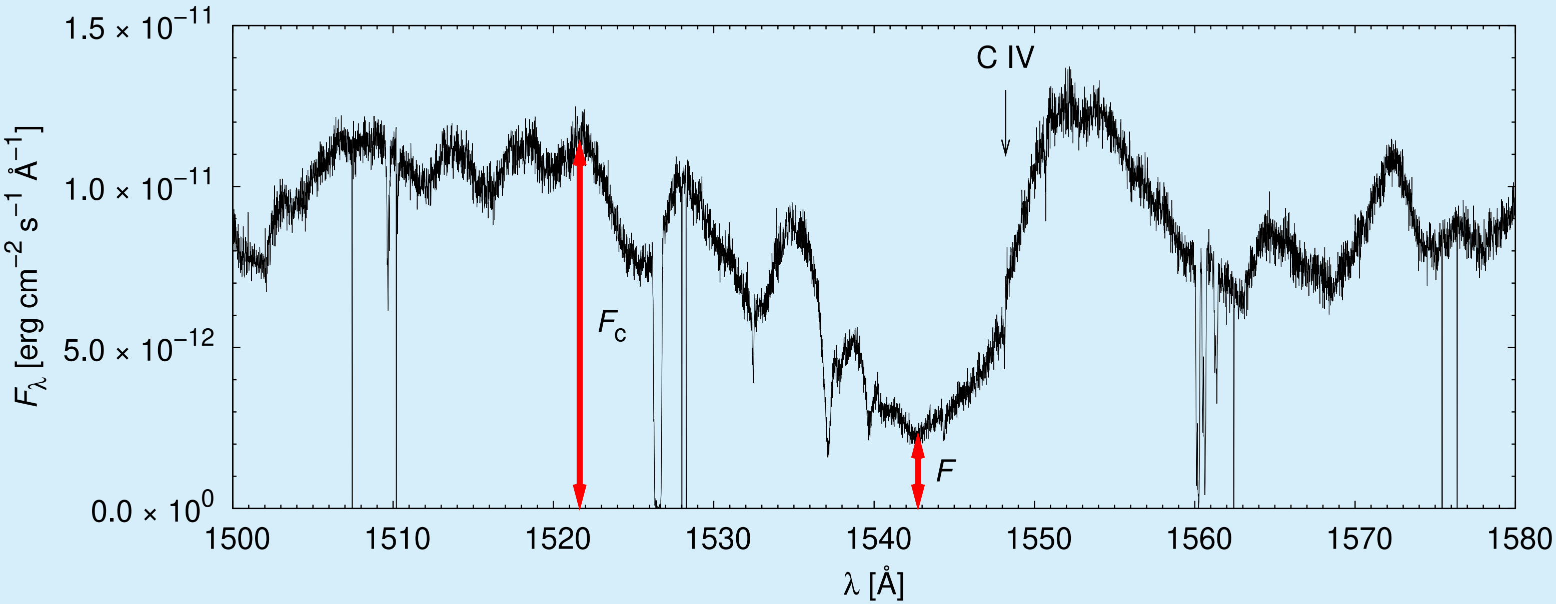


$$\tau(\mu=1)=\frac{\pi e^2}{m_\mathrm{e}c}\lambda_{ij}f_{ij}\frac{q_\mathrm{CIV}Z_\mathrm{C}}{4\pi m_\mathrm{H}}\frac{\dot{M}}{vr^2}\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$$

- Z_C is the carbon number density relatively to H
- q_CIV is the ionisation fraction of CIV

Observations: P Cyg lines II.

- HST spectrum of HD 13268

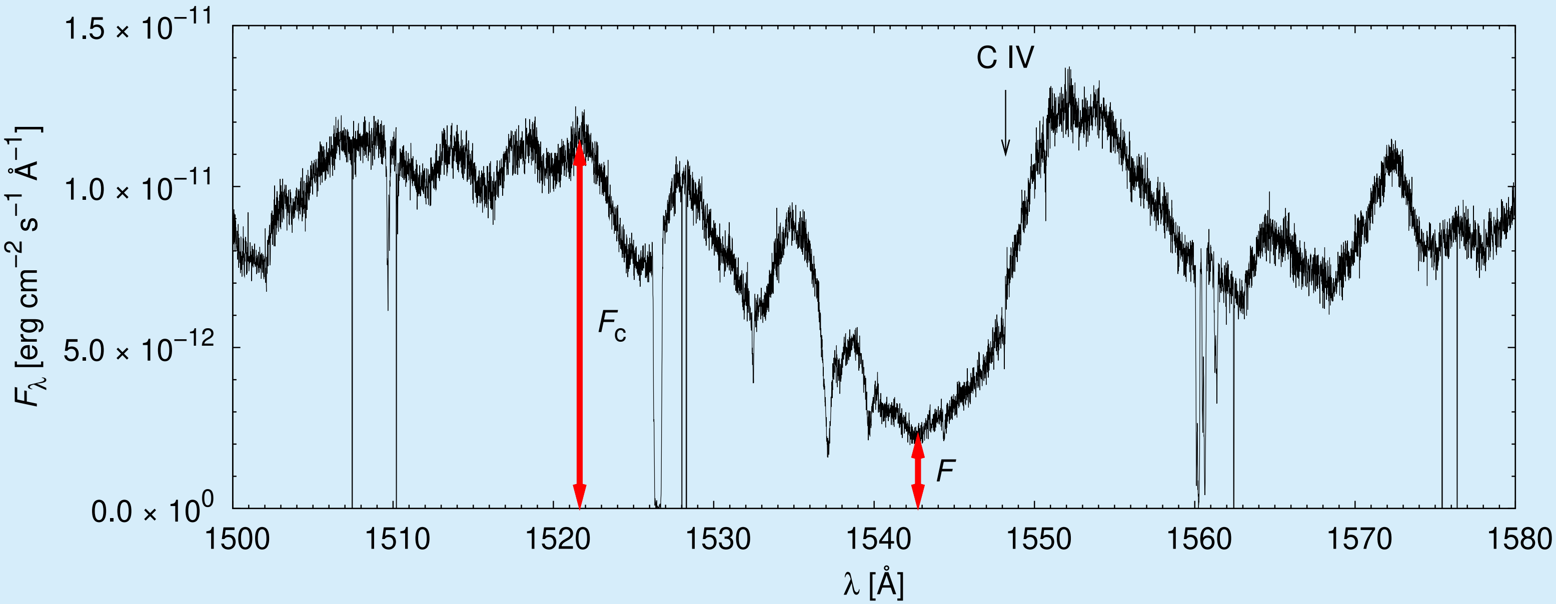


$$\tau(\mu = 1) = \frac{\pi e^2}{m_e c} \lambda_{ij} f_{ij} \frac{Z_C}{4\pi m_H} \frac{1}{v_\infty^2 R_*} q_{CIV} \dot{M}$$

- our order-of-magnitude approximations: $v \to v_\infty$, $r \to R_*$, $dv/dr \to v_\infty / R_*$

Observations: P Cyg lines II.

- HST spectrum of HD 13268

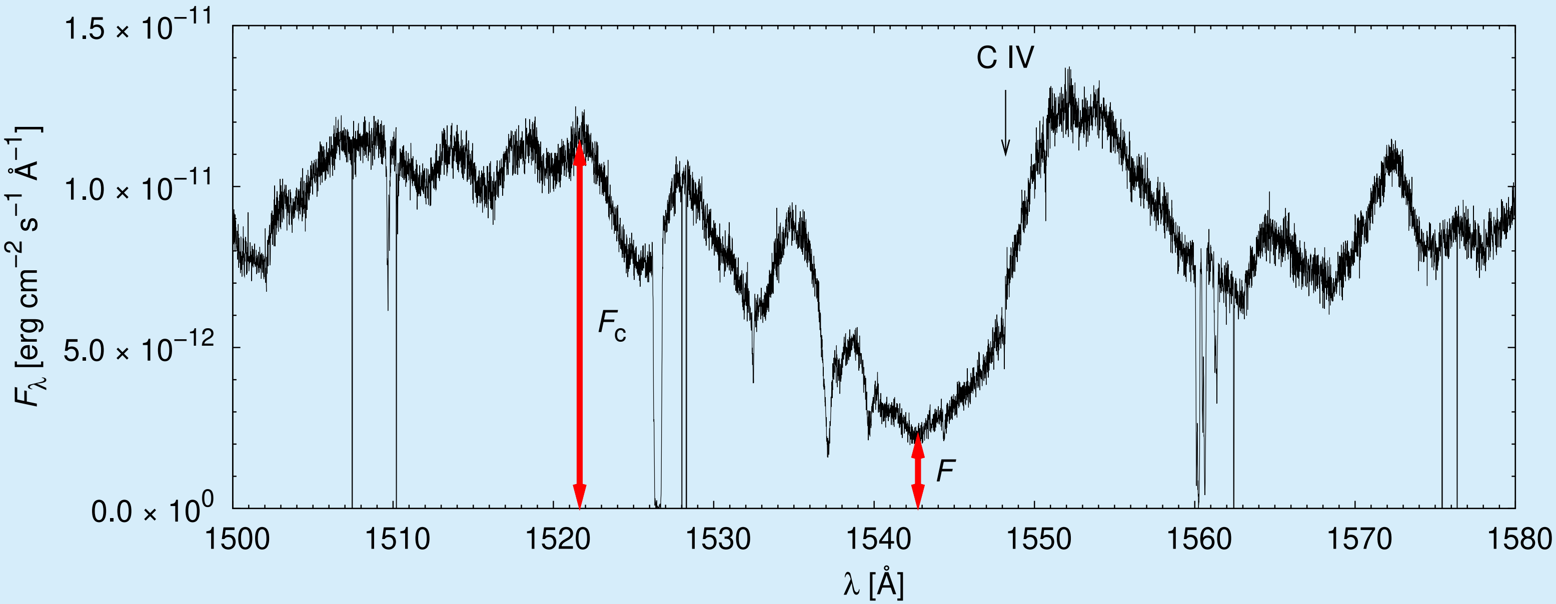


$$\tau(\mu = 1) = \frac{\pi e^2}{m_e c} \lambda_{ij} f_{ij} \frac{Z_C}{4\pi m_H} \frac{1}{v_\infty^2 R_*} q_{CIV} \dot{M}$$

$\Rightarrow$ from unsaturated wind line profiles possible to derive $q_{CIV}\dot{M}$

Observations: P Cyg lines II.

- HST spectrum of HD 13268



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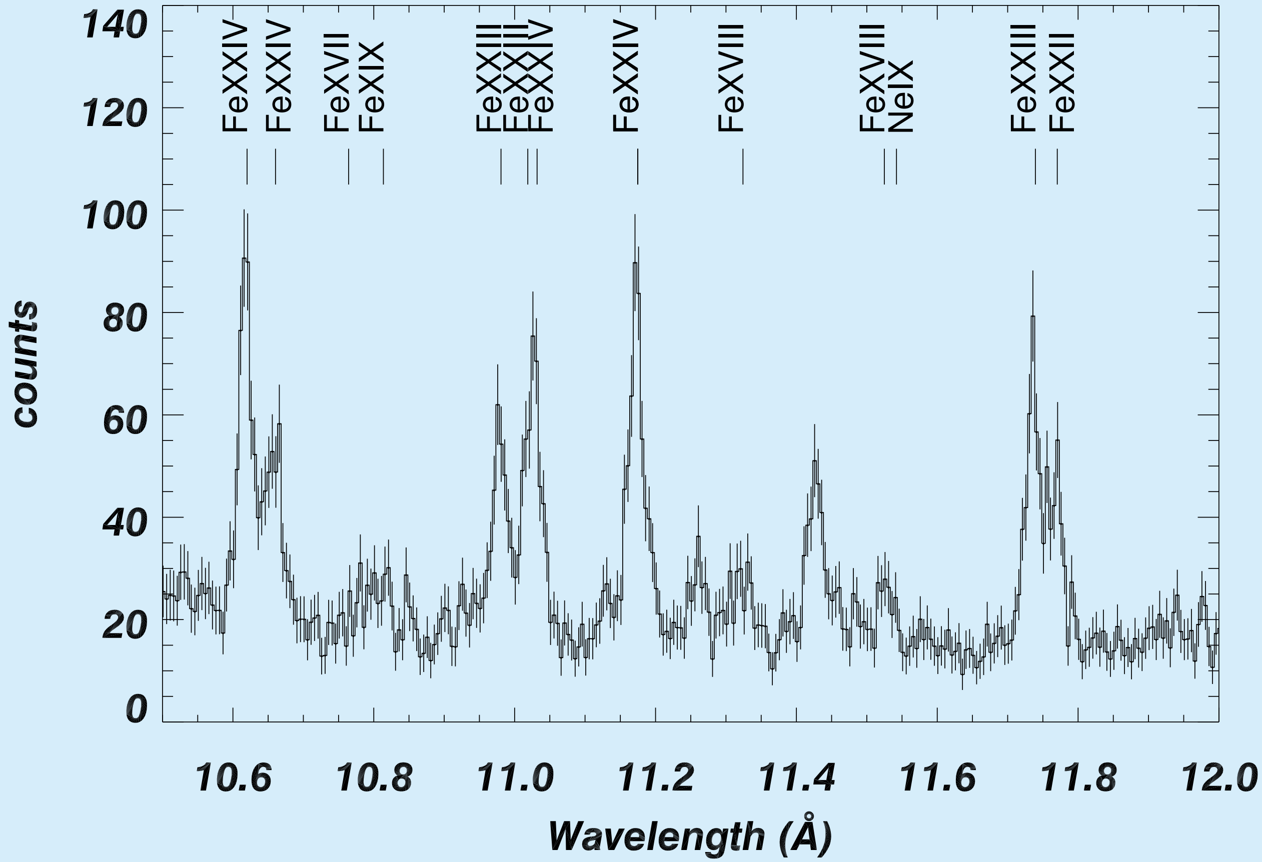
- in our case $q_{CIV}\dot{M} = 4 \times 10^{-10}\,M_\odot\,yr^{-1}$
- $\dot{M}$ can be derived with a knowledge of q_{CIV}

Observation: X-ray emission

- X-ray spectrum θ^1 Ori C



(CHANDRA, Schulz et al. 2003)

Observation: X-ray emission

- X-ray emission of hot stars consists of numerous lines of highly excited elements (N VI, O VII, Fe XXIV, . . .)
- signature of a presence of gas with temperatures of the order 10^6 K
- X-ray emission originates in the wind
  - how?

Observation: X-ray emission

- problem:
  - the wind temperature is of the order of the stellar effective temperature – 10^4 K (as expected from the observed ionisation structure and as derived from NLTE models, e.g., Drew 1989)
  - how can such gas emit X-ray radiation with typical temperatures $\sim 10^6$ K?

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  - most of the wind material is "cool" with temperatures of order of 10^4 K
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  - the "hot" material quickly cools down (radiatively)

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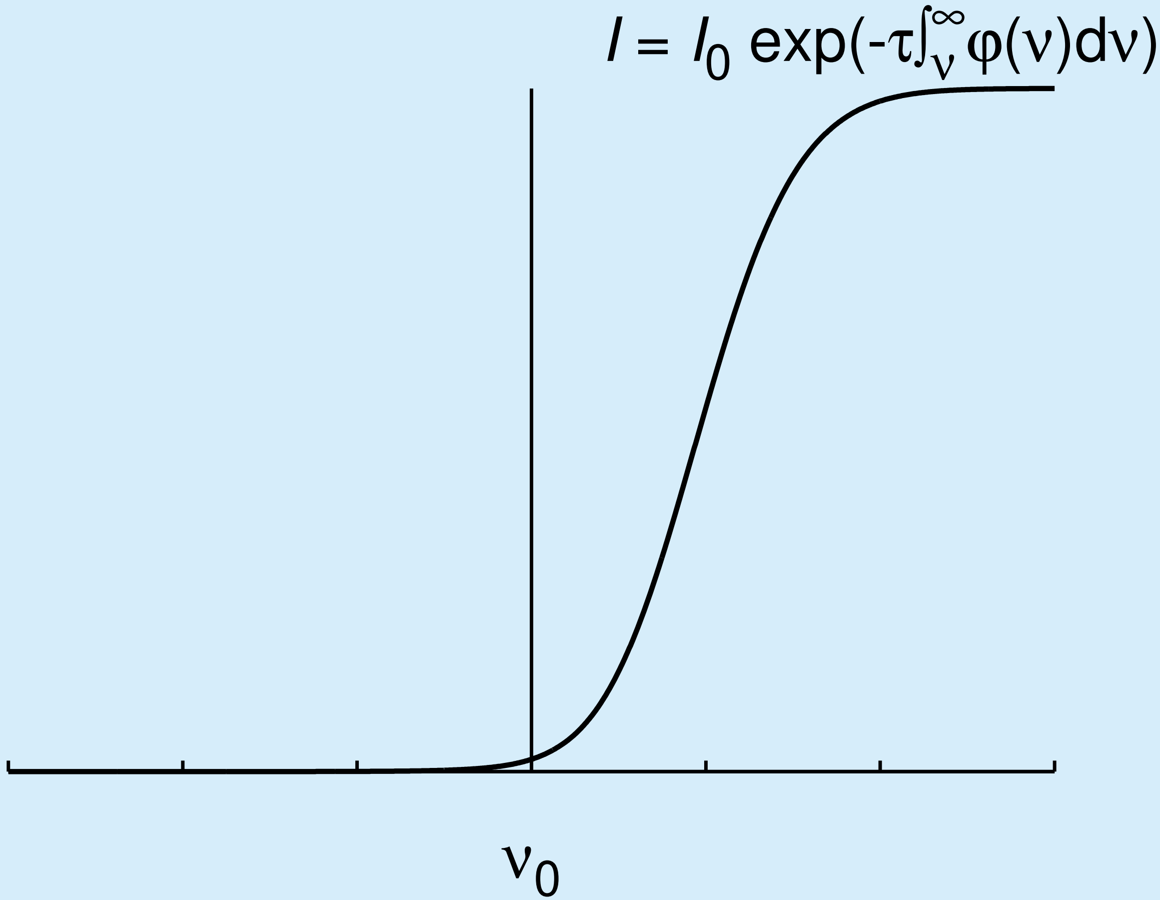
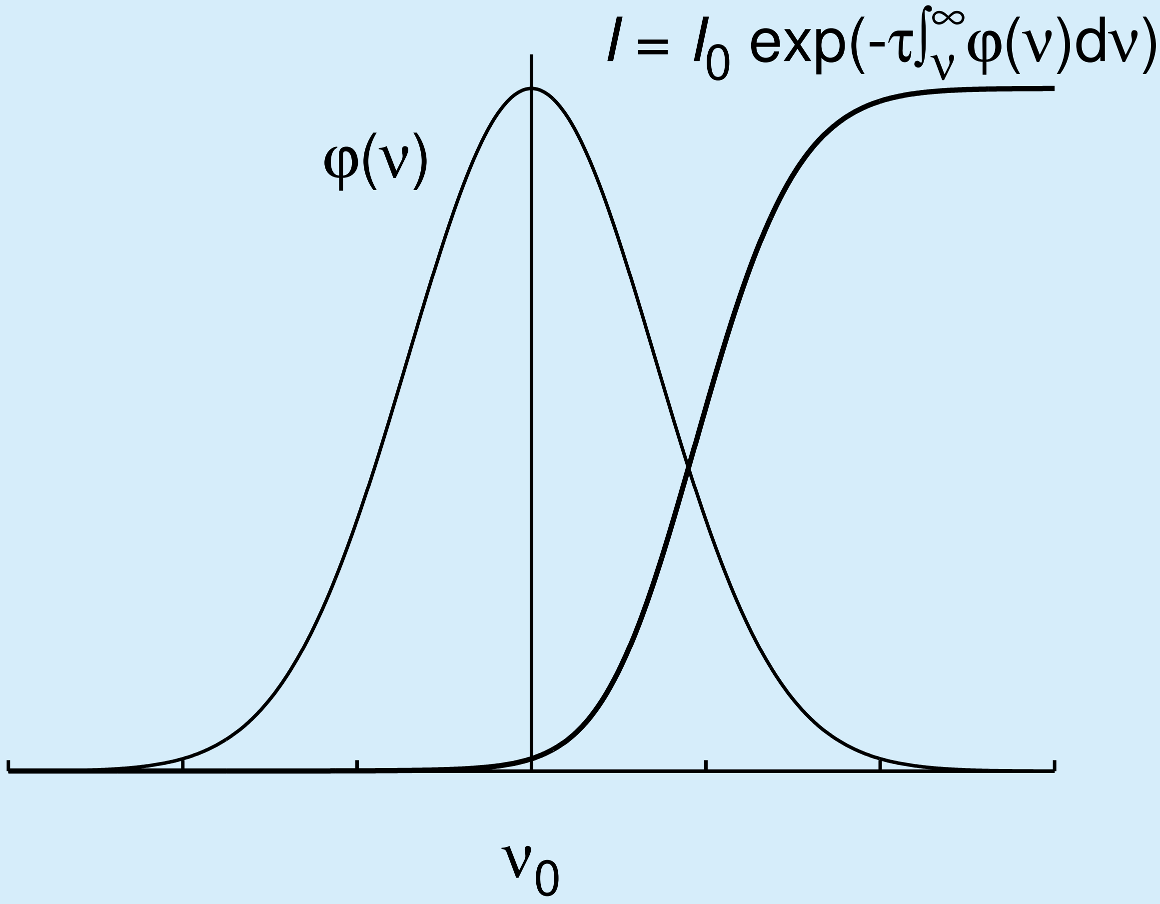
- further problem: how is this possible?

Wind instabilities



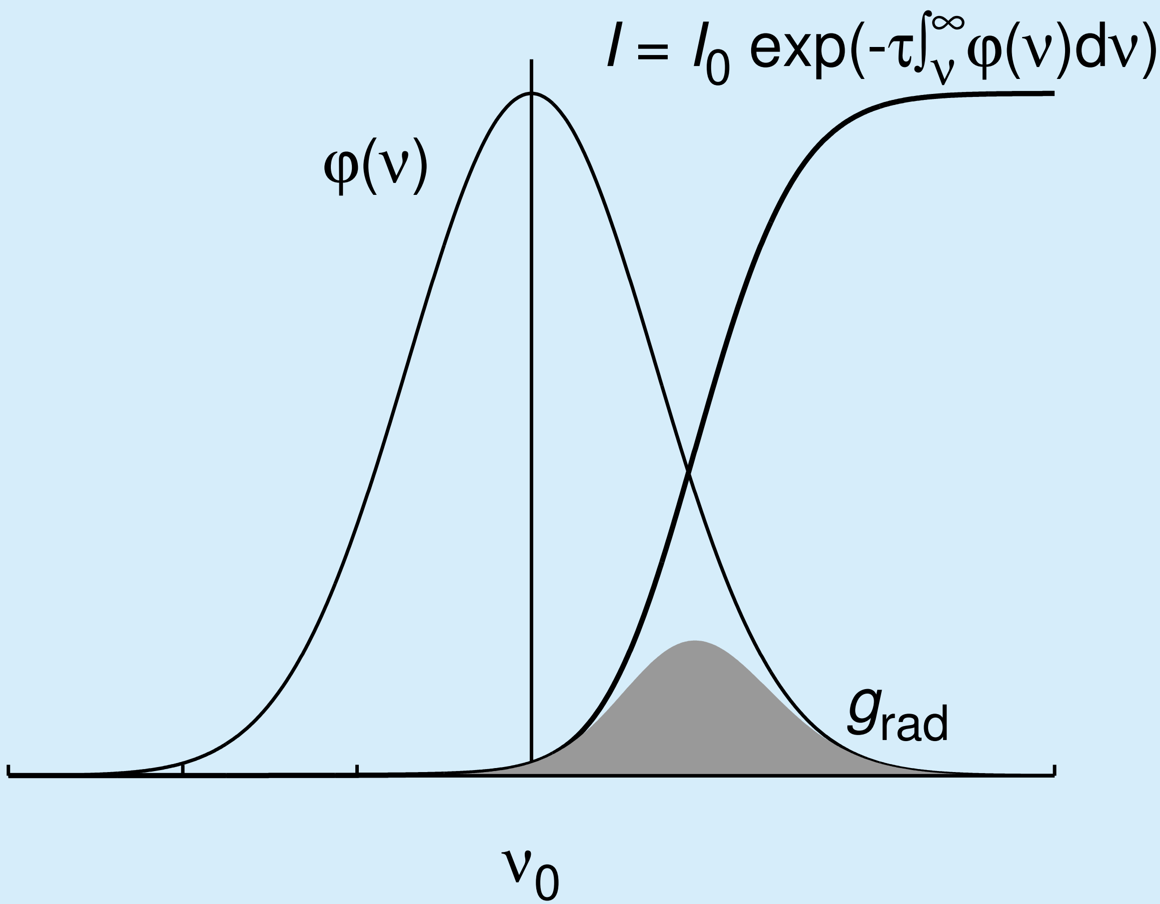
- the radiative transfer in the comoving frame

Wind instabilities



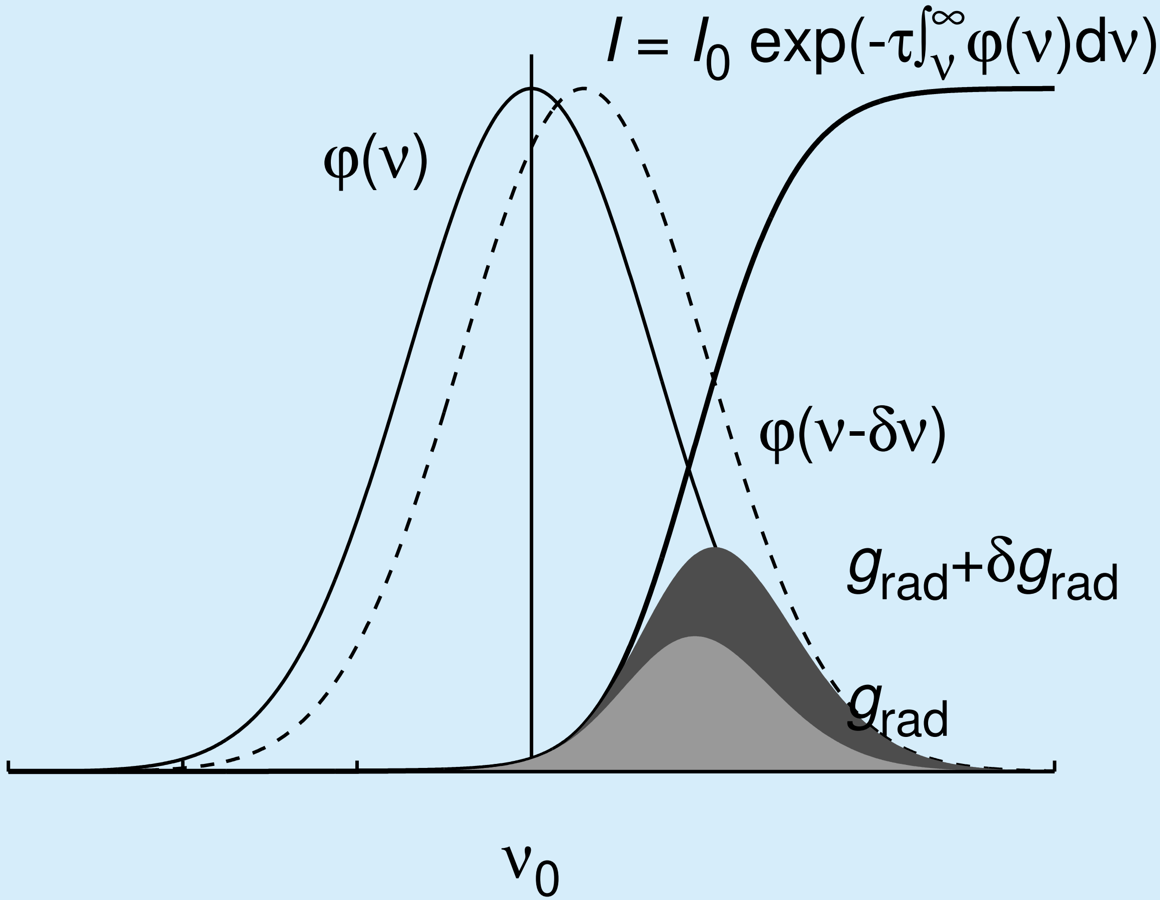
- the absorption profile in the comoving frame

Wind instabilities



- the line force

Wind instabilities



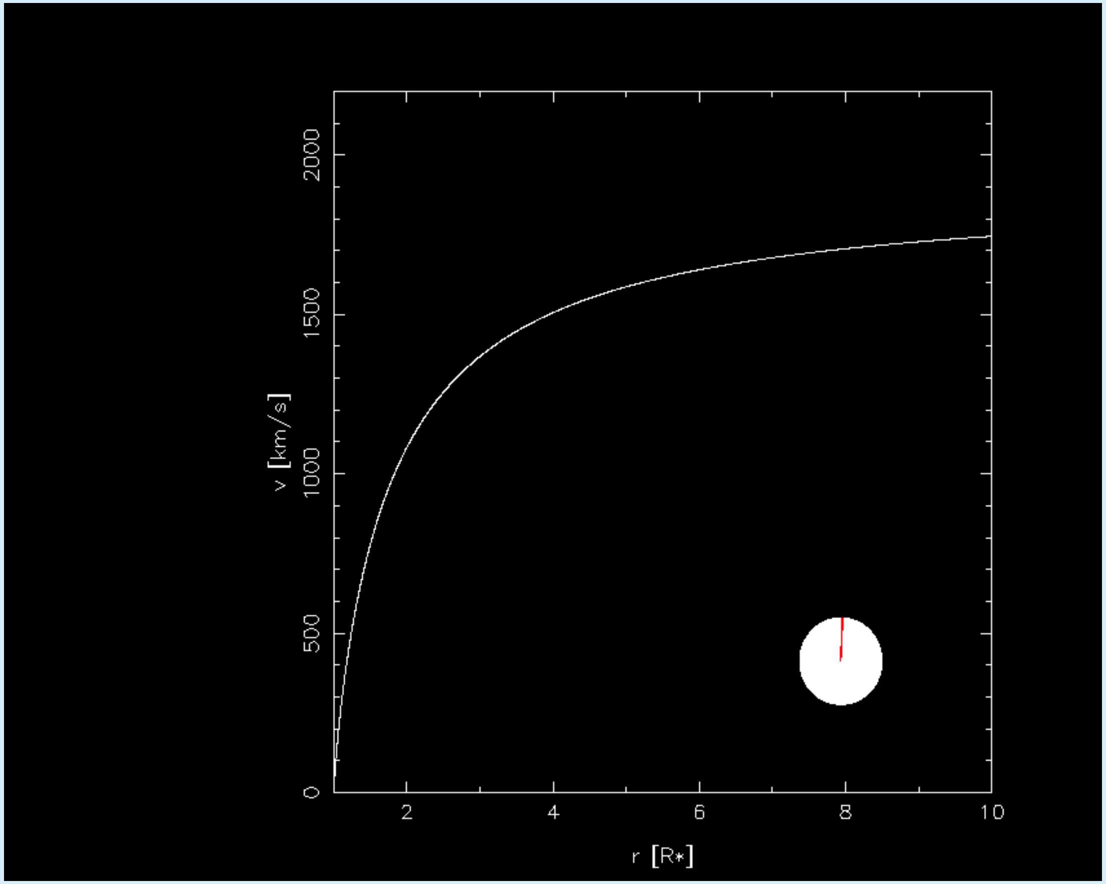
- the line force after a small change of the velocity

Wind instabilities

⇒ hydrodynamical simulations are necessary to describe the instability in detail (Owocki et al. 1988, Feldmeier et al. 1997, Runacres & Owocki 2002)

Wind instabilities

- hydrodynamical simulations (Feldmeier et al. 1997)

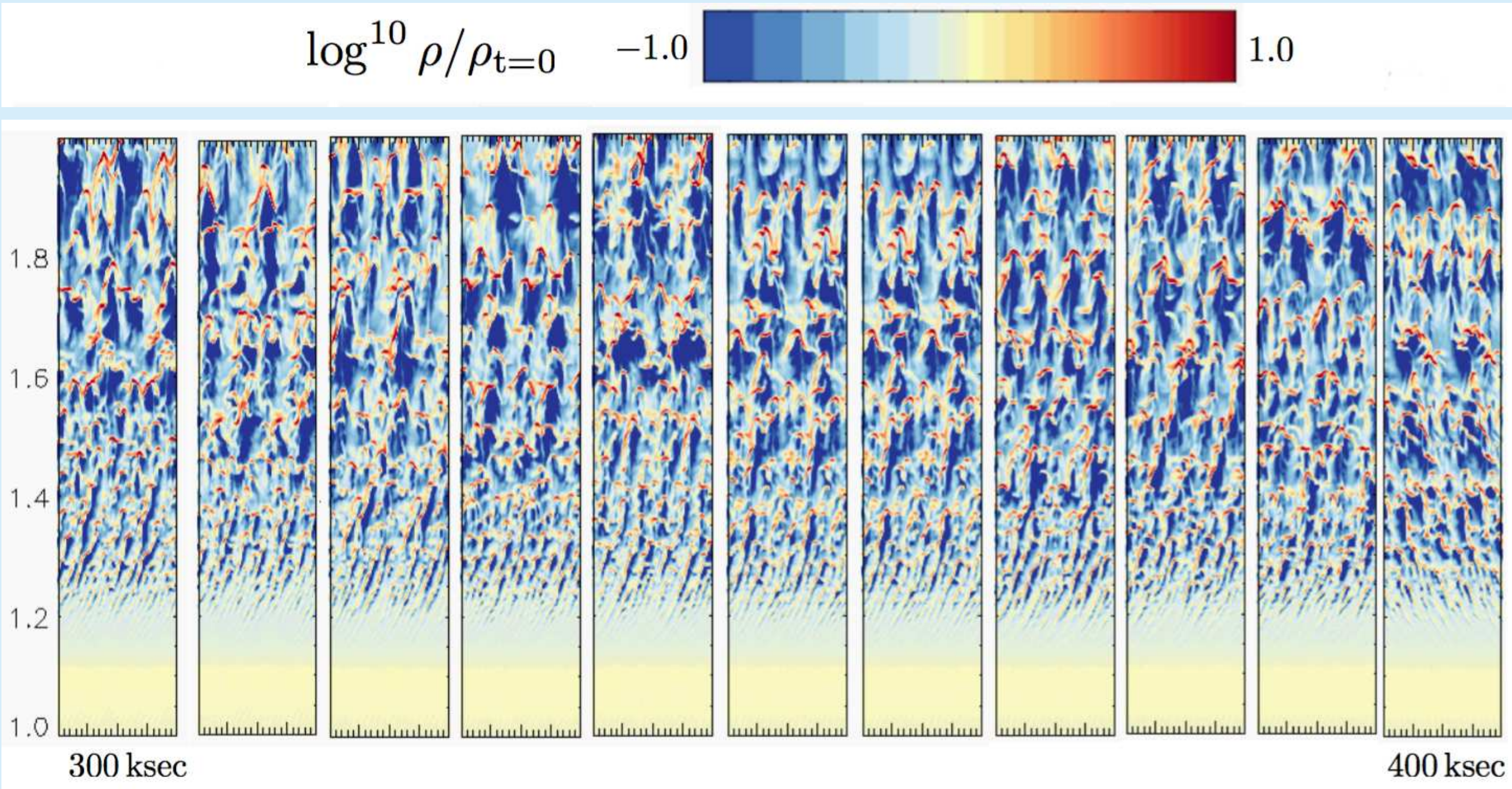


Wind instabilities

- hydrodynamical simulations are able to explain the main properties of X-ray emission of hot stars

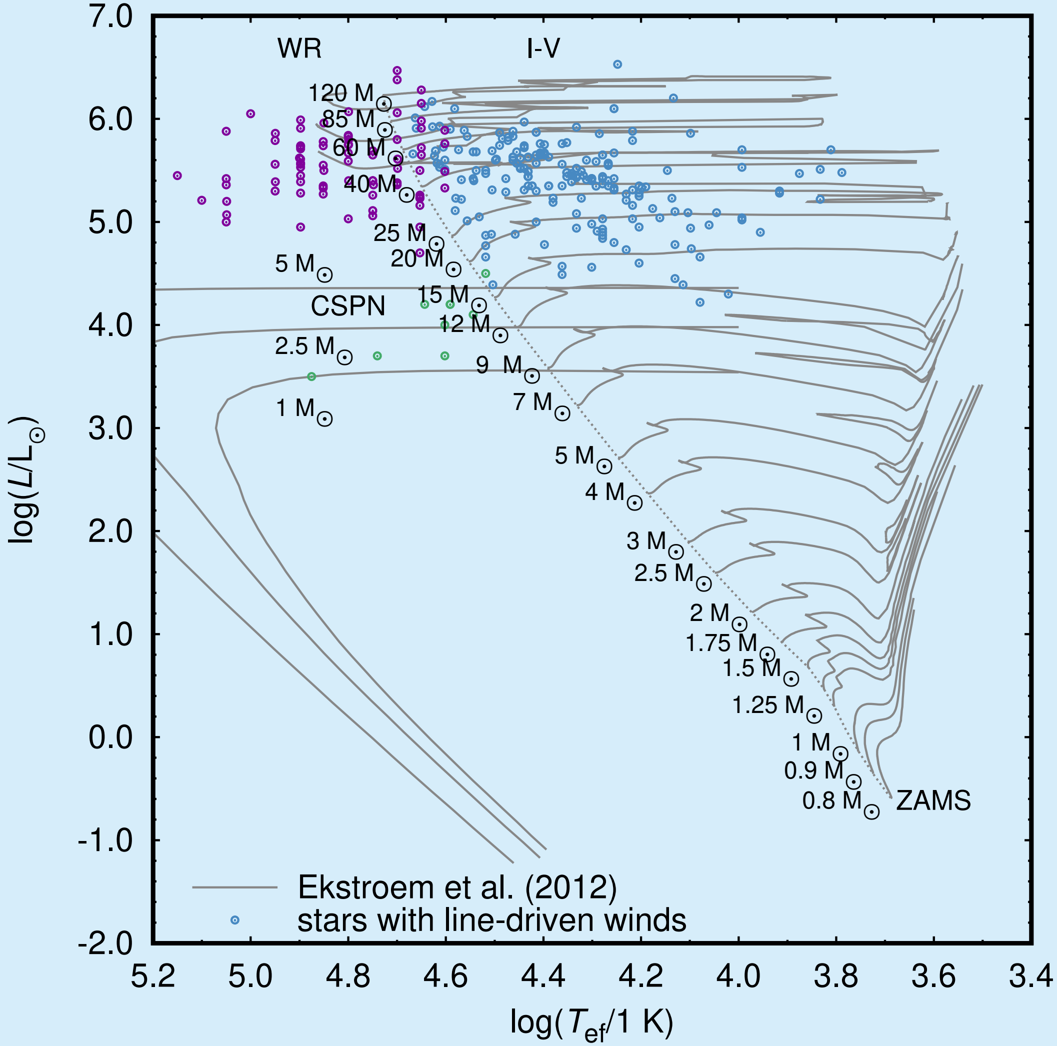
Wind instabilities

- 2D structure of wind due to line-driven wind instability



(Sundqvist et al. 2018)

Stars in HR diagram



- stars with $M \gtrsim 15\,\mathrm{M}_\odot$ have strong winds basically during all evolutionary phases

Importance of hot star winds I.

- stars more massive than $M \gtrsim 20\,\mathrm{M}_\odot$ have strong winds basically during all evolutionary phases
- the duration of the main-sequence phase of massive stars is about 10^6 yr
- during this time massive stars lose mass at the rate of the order of $10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$
- a significant part of stellar mass can be lost due to the winds
- most significant uncertainties of evolution of binary black hole merger progenitors connected with mass-loss (Abbott et al. 2017)

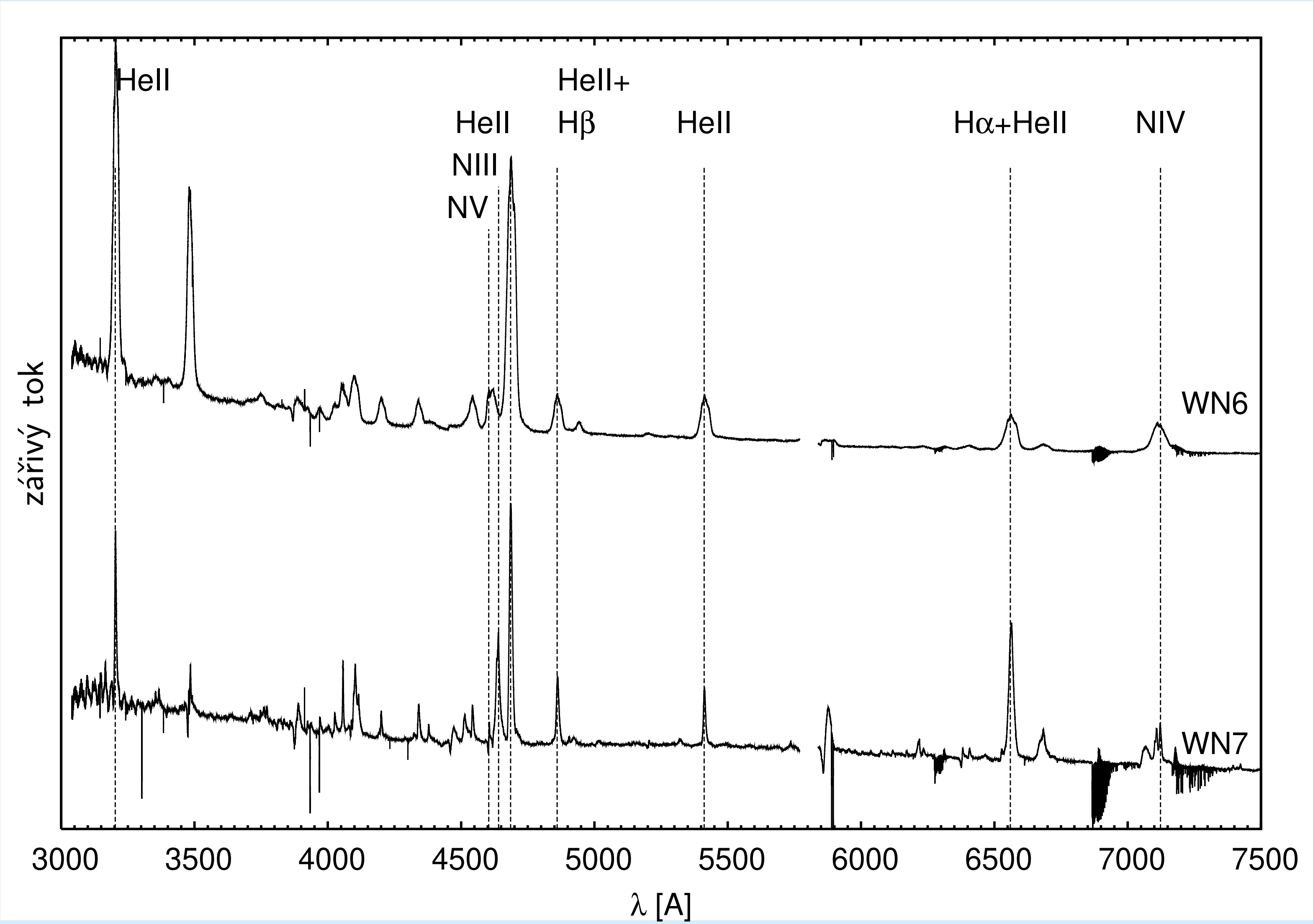
The importance of hot star wind II.

- the evolutionary phases connected with the wind

The importance of hot star wind II.

- the evolutionary phases connected with the wind
- Wolf-Rayett stars
  - hot stars with very strong wind (mass-loss rate could be of the order of $10^{-5}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$)
  - wind starts already in the stellar atmosphere
  - spectrum dominated by emission lines
  - enhanced abundance of nitrogen and/or carbon and oxygen

The importance of hot star wind II.



The importance of hot star wind II.

- the evolutionary phases connected with the wind
- Wolf-Rayett stars
  - how can these stars originate?

The importance of hot star wind II.

- the evolutionary phases connected with the wind
- Wolf-Rayett stars
  - during the stellar evolution the core abundance of nitrogen (hydrogen burning) and carbon+oxygen (helium burning) increases

The importance of hot star wind II.

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  $\Rightarrow$ Wolf-Rayett stars

The importance of hot star wind II.

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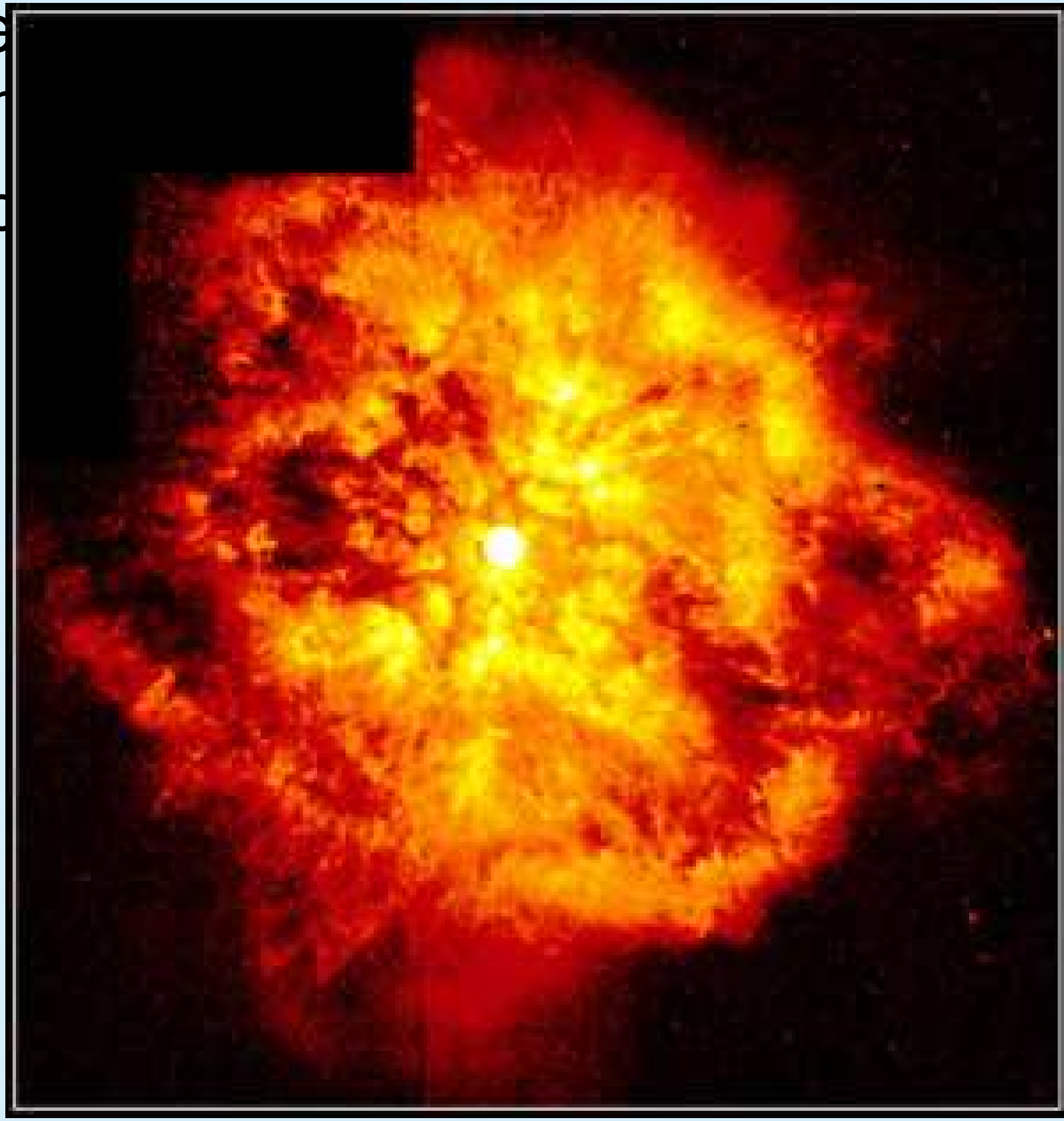


The importance of hot star wind III.

- planetary nebulae
  - during the AGB stage of solar-like stars ($M \approx 1\,\mathrm{M}_\odot$) the star loses a significant part of its mass via slow ($\sim 10\,\mathrm{km\,s^{-1}}$) high-density wind
  - the hot degenerated core is exposed
  - during this stage the star has fast low-density line-driven wind

$\Rightarrow$ planetary nebula: interaction of slow high-density and fast low-density winds

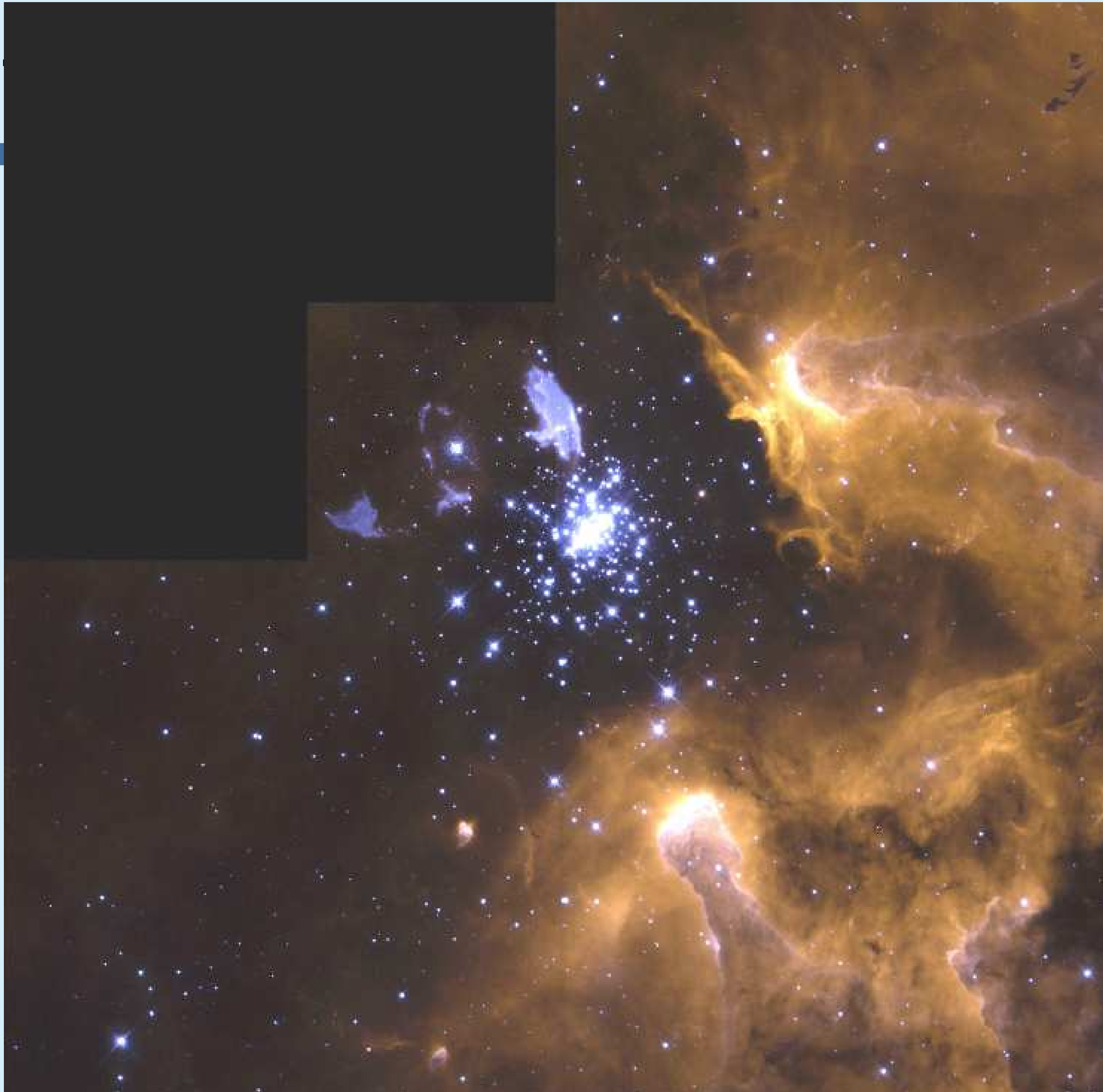
The importance of hot star wind III.

- planetary nebulae

The importance of hot star wind IV.

- hot star wind influence also the interstellar environment
  - enrichment of the interstellar medium
  - momentum input to the interstellar medium
  - is an important source of galactic cosmic ray particles

(e.g., Dale & Bonnell 2008, Aharonian et al. 2018) d IV.



More informations (book, reviews)

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- Puls, J., Vink, J. S., Najarro, F. 2008, Mass loss from hot massive stars, *AA&ARv*, **16**, 209
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