Machine Learning with Time Series
Forecasting, Recurrent neural networks

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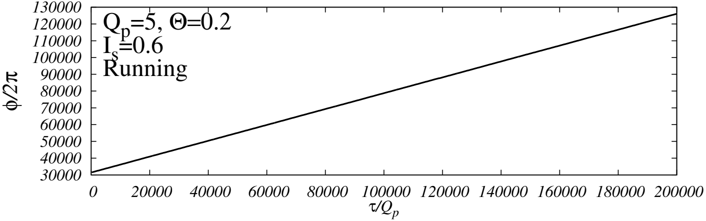
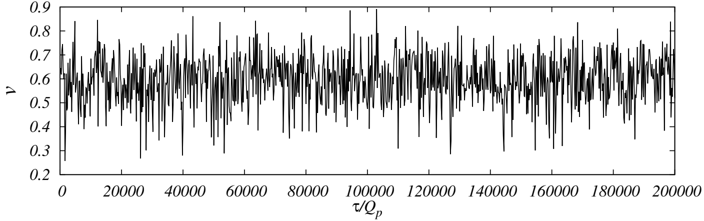
November 2022

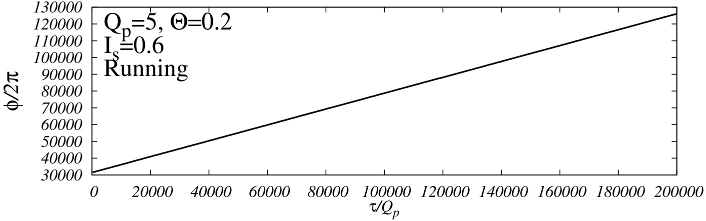
The world is changing in time

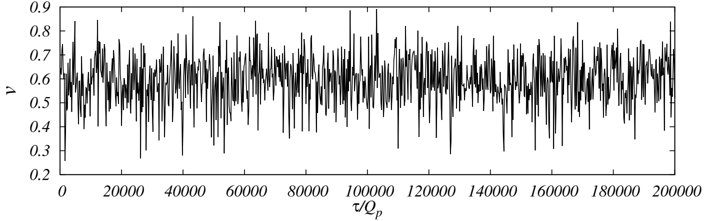




- Time series are ubiquitous in science and industry
- Time dependent data aren't statistically independent
- Forecasting is important, but it is not the only interesting task for ML

The world is changing in time





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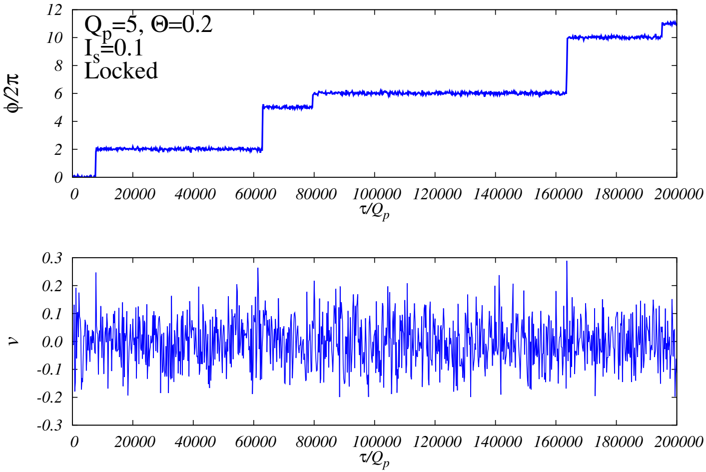

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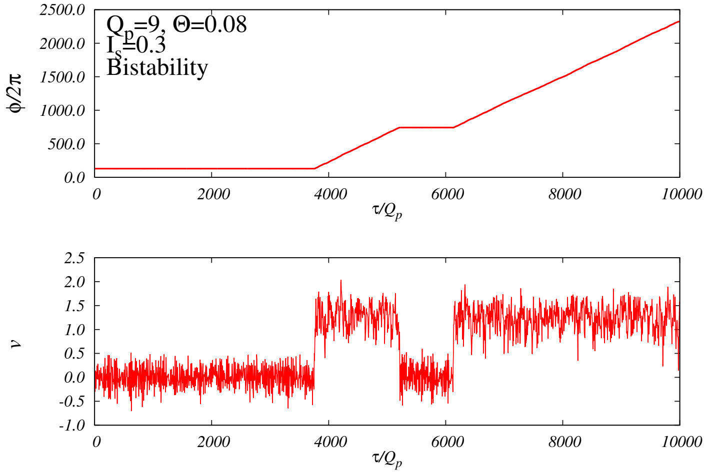


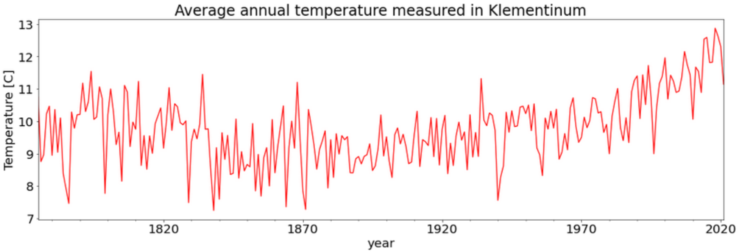
Outline

- **About Data**
- **Common predictor based techniques**
- **Recurrent Neurons and Layers**
  - Memory cells
  - Input and output
- **Simple RNN**
- **Deep RNN**

Time dependent data

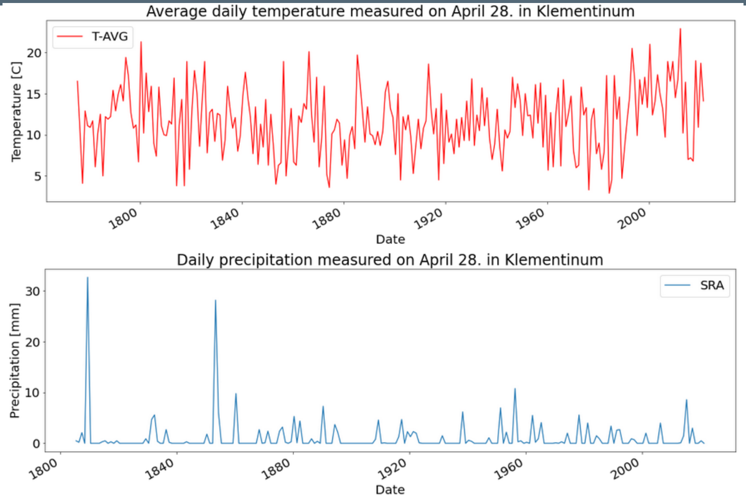
Data types

- Single series, single variable



Data types

- Single series, single variable
- Single series, multivariate



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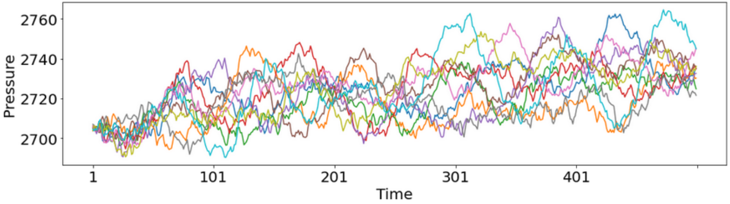

Data types

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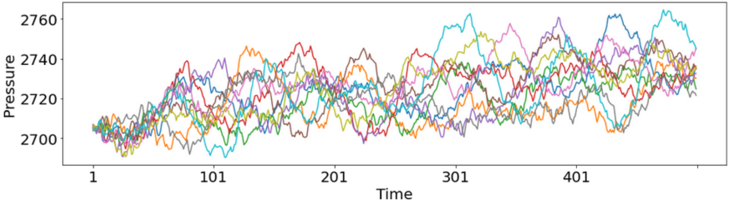
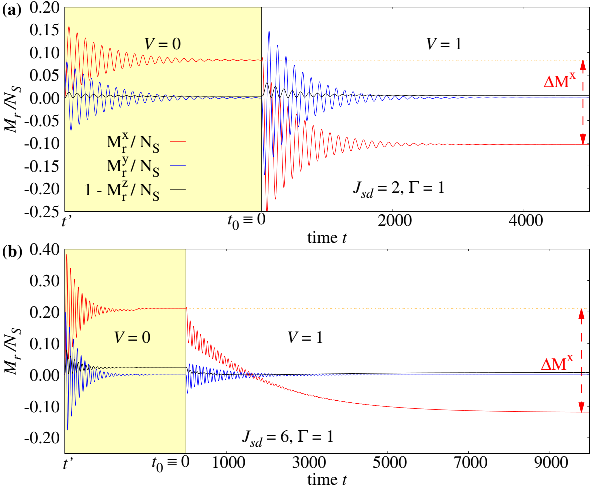
- Single series, multivariate
- Panel data
    - Panel data may have misaligned time points



Time series characteristics

- Trend analysis



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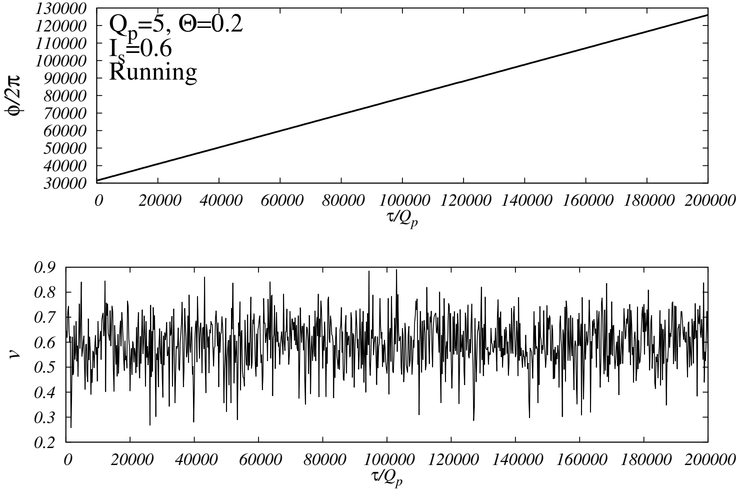
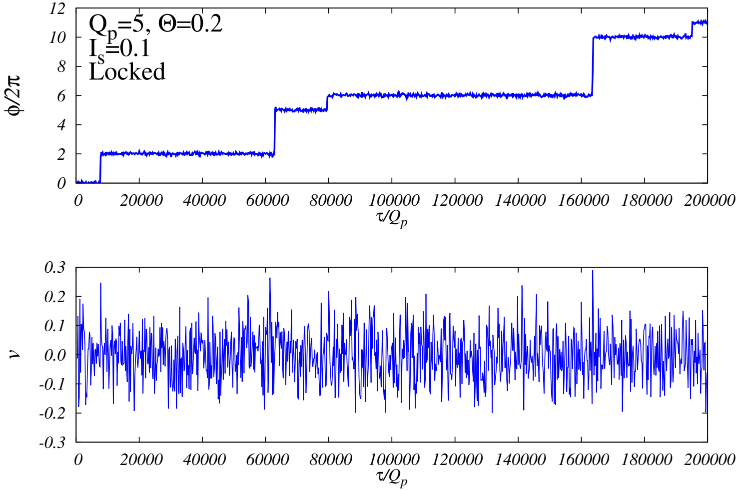


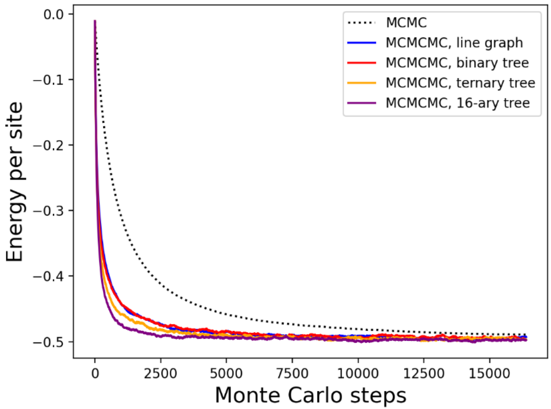
Time series characteristics

- Trend analysis
- Outliers



Time series characteristics

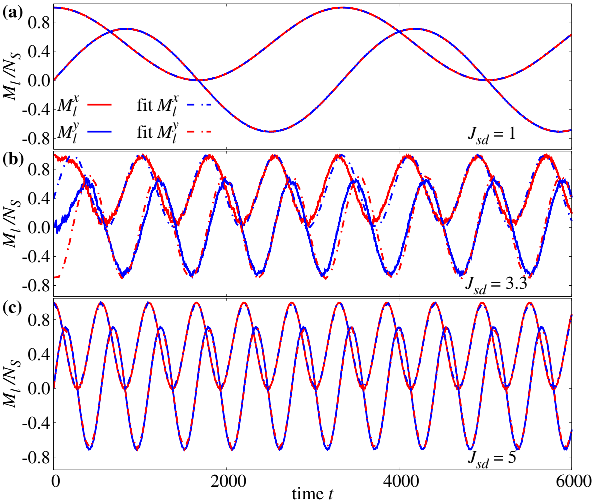
- Trend analysis
- Outliers
- Stationarity



arXiv:2102.05613

Time series characteristics

- Trend analysis
- Outliers
- Stationarity
- Periodicity



Forecasting

Using standard ML techniques

Strategy for forecasting with common ML techniques:

d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	$\dots$	d_N	?	?	?

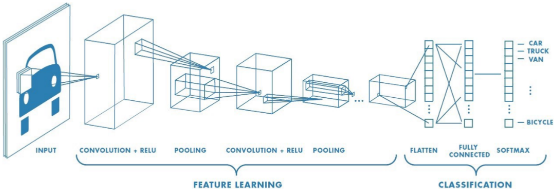
We can make our own training data:

d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	$\dots$	d_N	?	?	?
d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	$\dots$	d_N	?	?	?
d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	$\dots$	d_N	?	?	?
							$\vdots$							
d_1	d_2	d_3	d_4	d_5	d_6	d_7	$\dots$	d_{N-3}	d_{N-2}	d_{N-1}	d_N	?	?	?

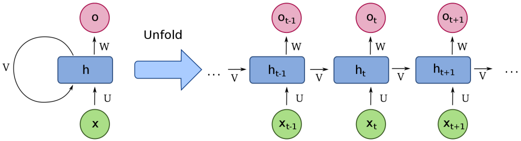
Training data:

d_1	d_2	d_3	d_4	d_5
d_2	d_3	d_4	d_5	d_6
d_3	d_4	d_5	d_6	d_7
		$\vdots$		

RNN and CNN



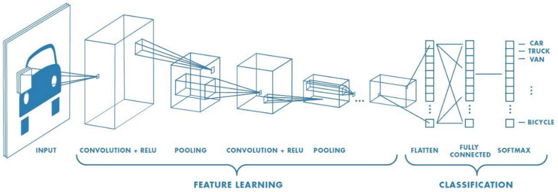
source: towardsdatascience.com



source: wikipedia.com

- CNN and RNN are both part of Deep Learning

RNN and CNN



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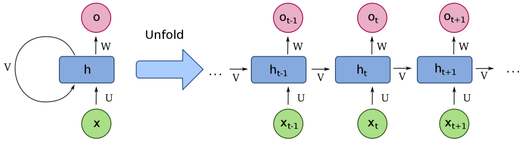

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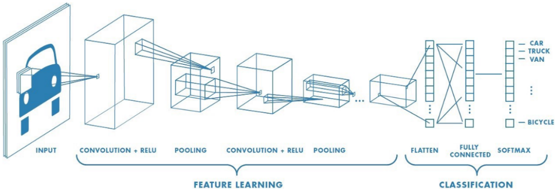
- CNN and RNN follow different architectures

RNN and CNN



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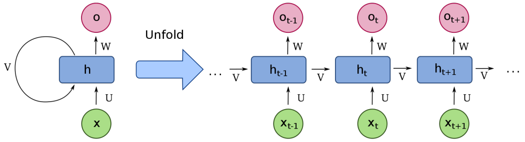

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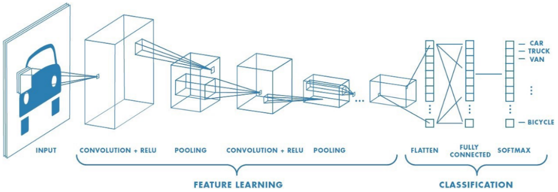

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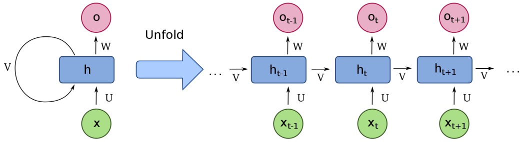

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- Yet, there are CNN capable to deal with large time series

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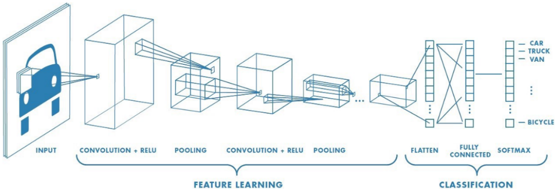

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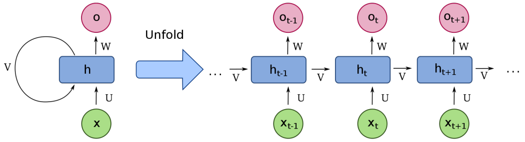

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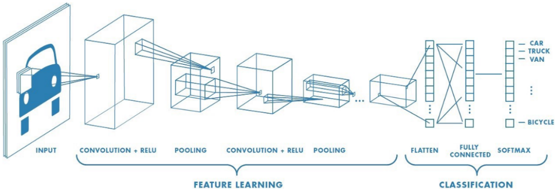
- e.g: WaveNet

RNN and CNN



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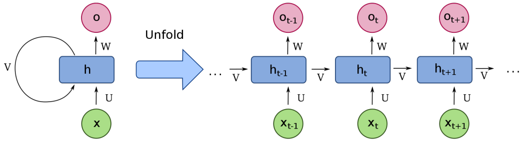

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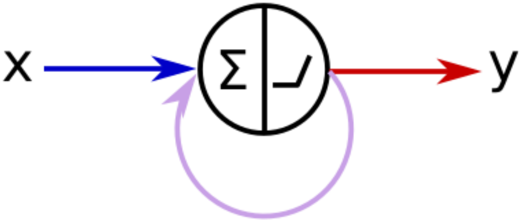
- CNN and RNN follow different architectures
- RNN are specifically build for forecasting
- Yet, there are CNN capable to deal with large time series
- e.g: WaveNet
- Nevertheless, today we will focus on RNN

Recurrent Neurons and Layers

Recurrent Neuron

- RNN looks like a standard feedforward NN



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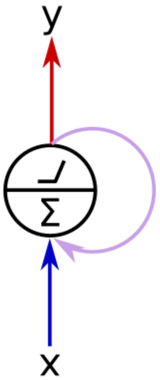

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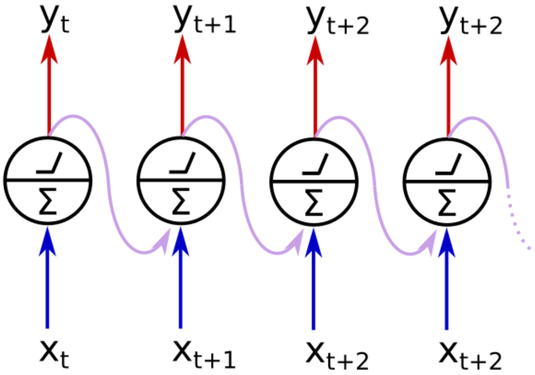

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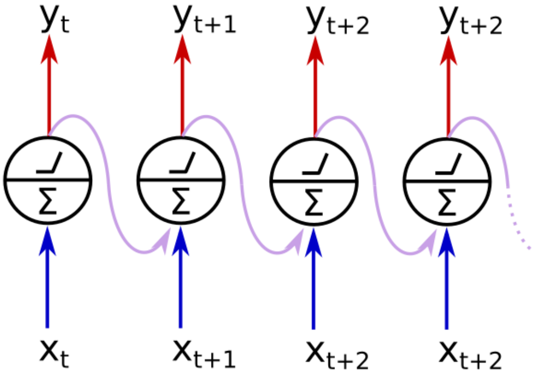

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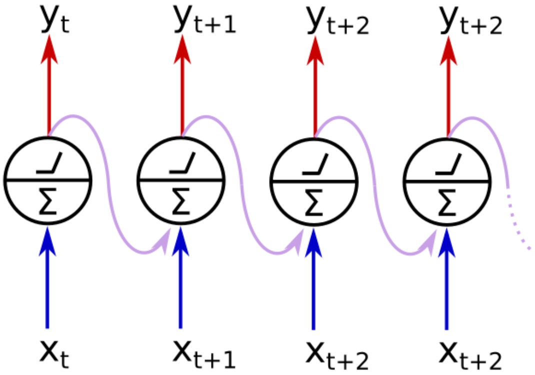
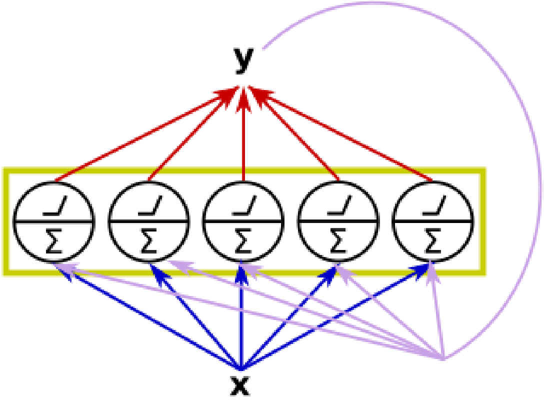
- We can build a layer



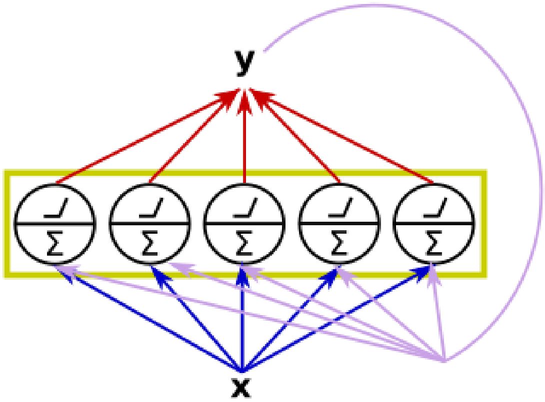
Recurrent Layer

- Each recurrent neuron has two sets of weights $\mathbf{w}_x$ and $\mathbf{w}_y$



- Each recurrent neuron has two sets of weights $\mathbf{w}_x$ and $\mathbf{w}_y$
- Output of RNN at time t:

$$\mathbf{y}_t = \Phi(\mathbf{W}_x \mathbf{x}_t^T + \mathbf{W}_y \mathbf{y}_{t-1}^T + \mathbf{b}) \quad (1)$$



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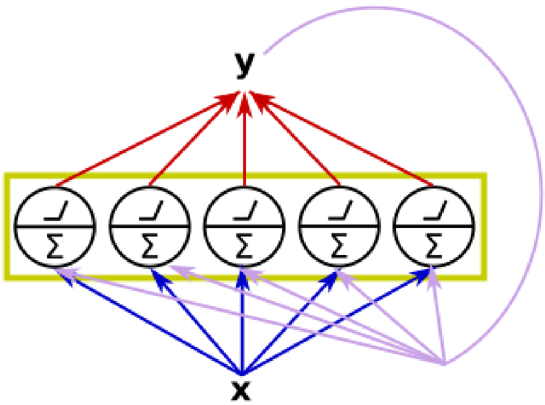

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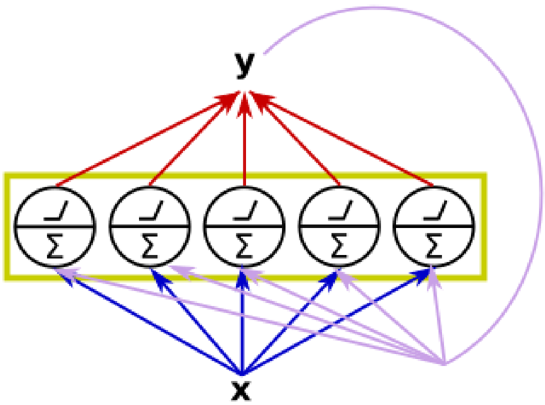
- $\mathbf{y}_t$ is a function of $\mathbf{x}_t$ and $\mathbf{y}_{t-1} \rightarrow$ recurrent behavior



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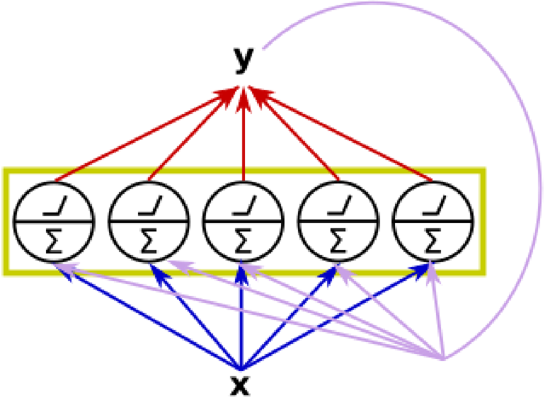
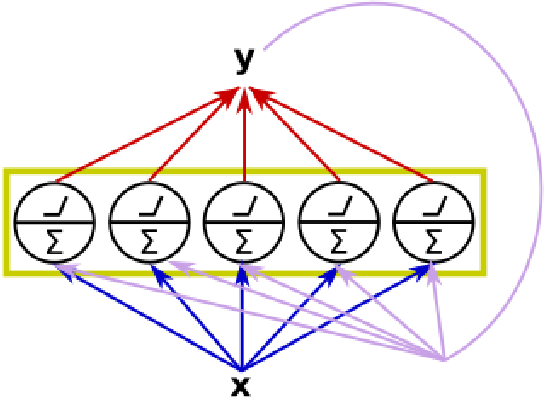
- $\mathbf{y}_t$ is a function of $\mathbf{x}_t$ and $\mathbf{y}_{t-1} \rightarrow$ recurrent behavior
- At $t = 0$ we set $\mathbf{y}_{-1} = 0$



Memory cells

- It is a trivial example of a memory cell



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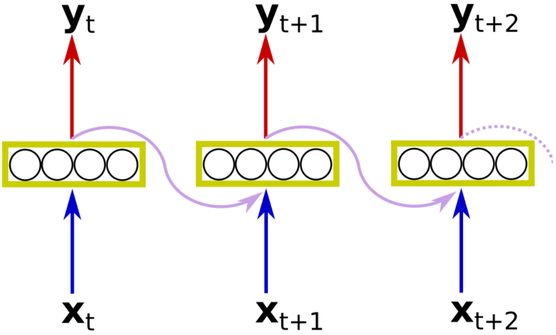

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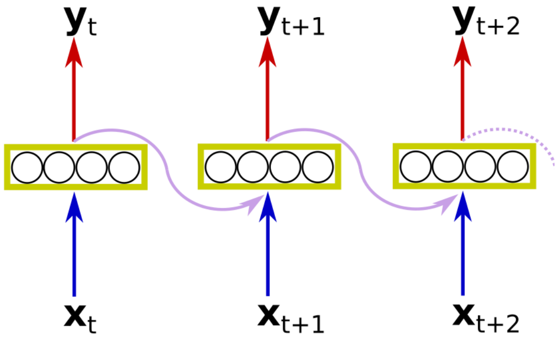
- Neuron computes a weighted sum of the inputs



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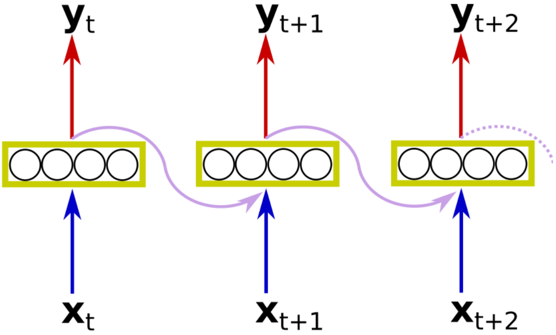
- The memory is a bit short ($\approx$ 10 time steps)



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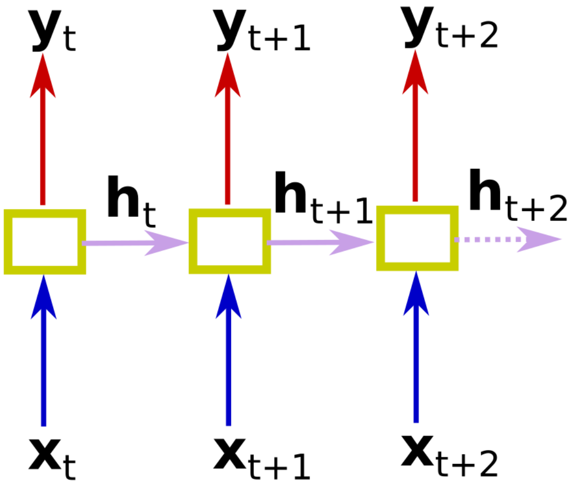
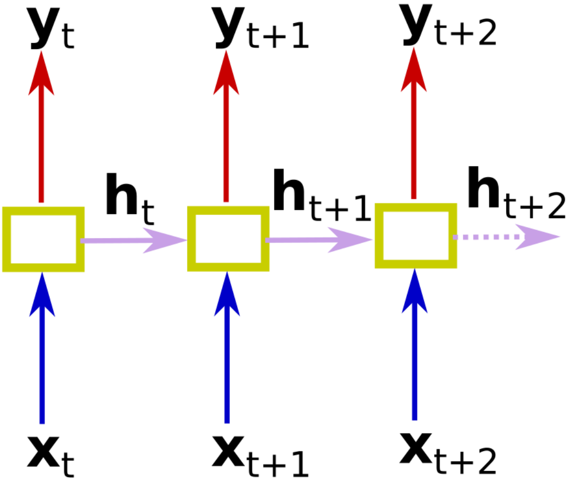


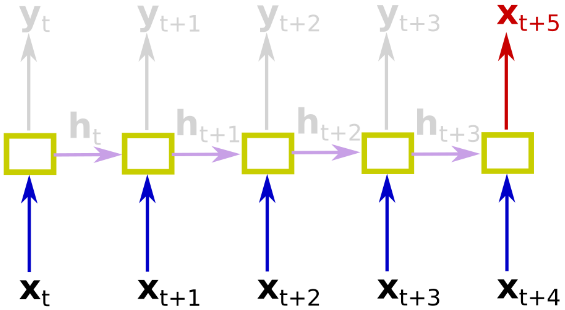
Memory cells

- It is a trivial example of a memory cell
- Output depends on the inputs from previous time steps
- Neuron computes a weighted sum of the inputs
- The memory is a bit short (≈ 10 time steps)
- A more complex cells are needed for longer memories
- Memory function can be also generalized $\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t)$



What is this good for?

- RNN are build for forecasting



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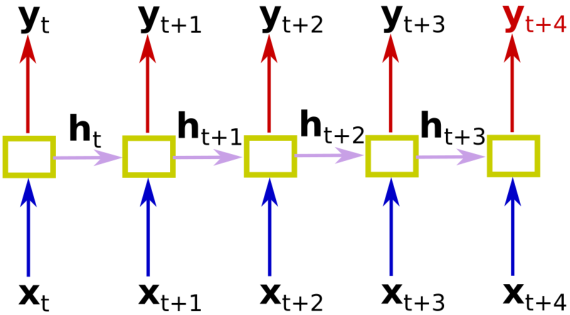

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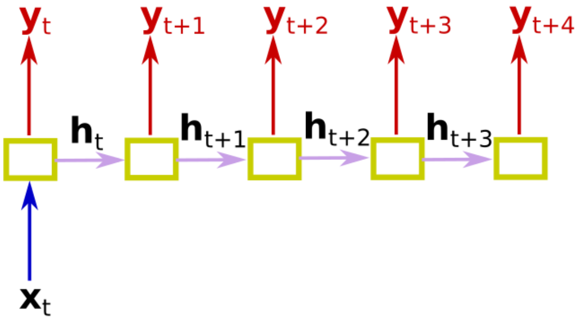
- Sequence generation (similar architecture as CNN)



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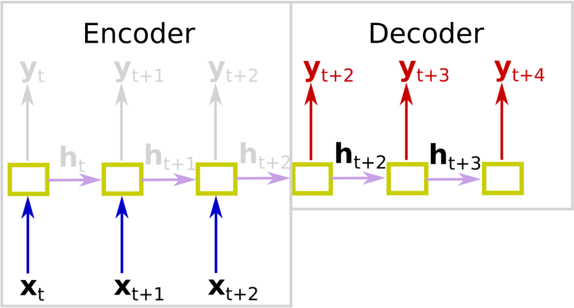
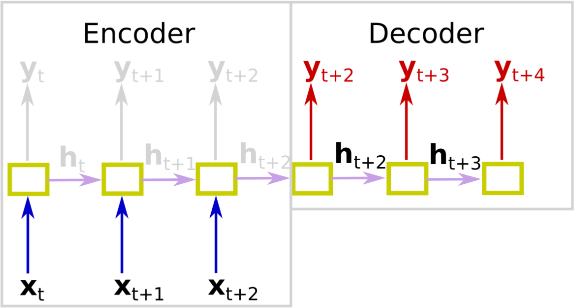
- Sequence generation (similar architecture as CNN)
- And then there are combinations, e.g., Encoder/Decoder



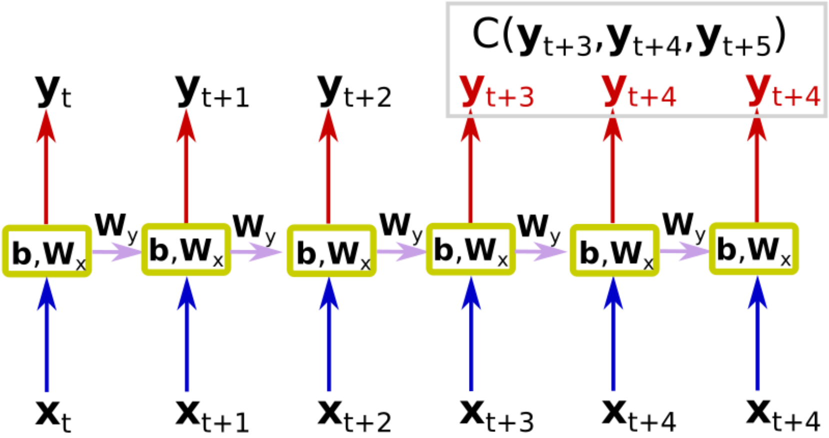
What is this good for?

- RNN are build for forecasting
- But they can do more (e.g., classification)
- Sequence generation (similar architecture as CNN)
- And then there are combinations, e.g., Encoder/Decoder
- Transformation of the data to different basis?



Training

We can use a regular back-propagation, however, it is rolled through time



More than one time-step

- We can simply add the prediction of the last point to the series and predict again
- It is not a good strategy
- Errors accumulate (Dense model is often superior to RNN)
- It is better to train RNN to predict all next values at once

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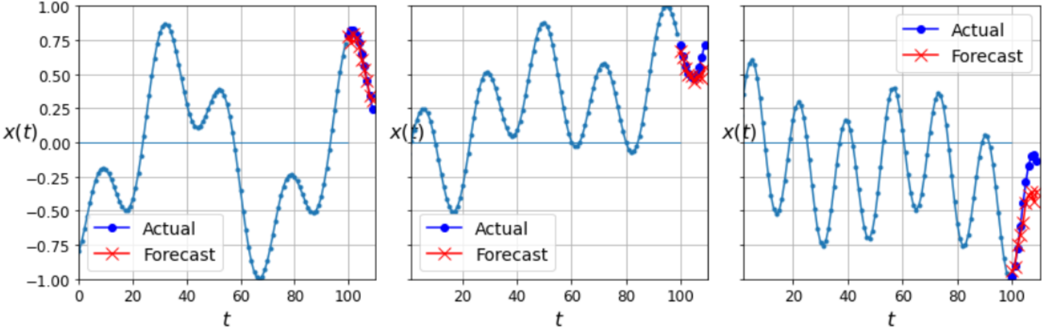
```
model = keras.models.Sequential([
keras.layers.SimpleRNN(20,return_sequences=True,input_shape=[None,1]),
keras.layers.SimpleRNN(20),
keras.layers.Dense(10) ])
```

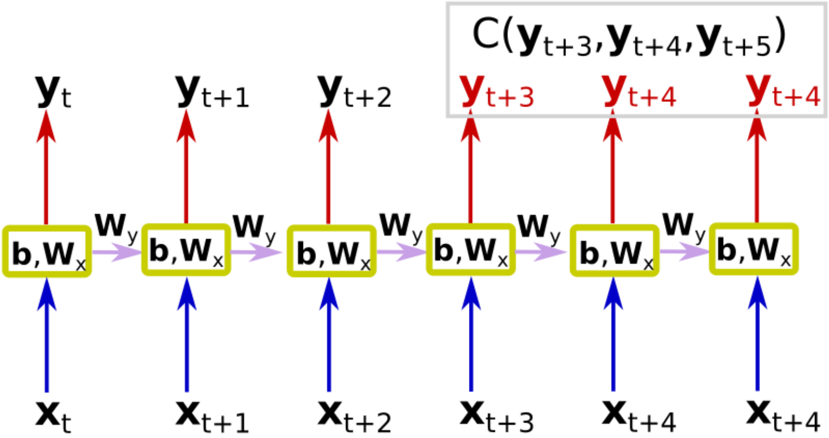
Deep RNN predicting 10 time steps at once:



MSE for the last point was 0.07.

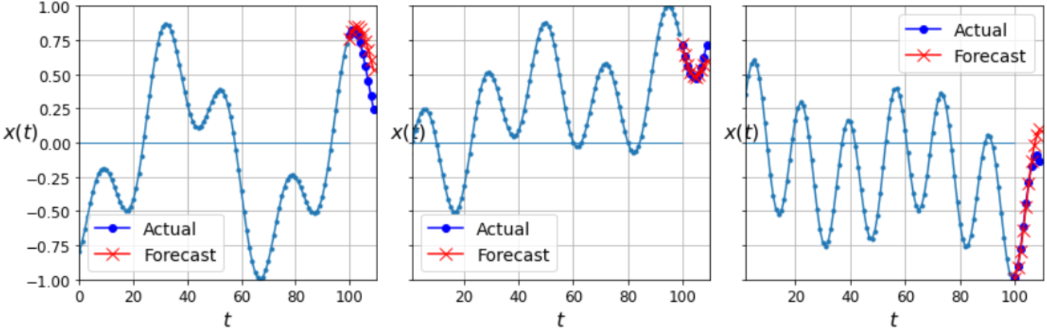
We can still do better

Instead of predicting N time steps at the last time, we can predict and evaluate (calculate the loss function) N time steps at each time step!



We can still do better

Deep RNN sequence to sequence model:

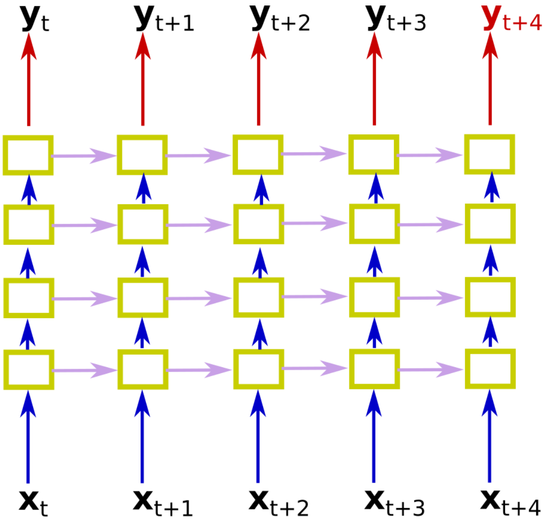


MSE for the last point was 0.015

Common problems and their solutions

Long Sequences

- RNN for long sequence is basically a very deep network



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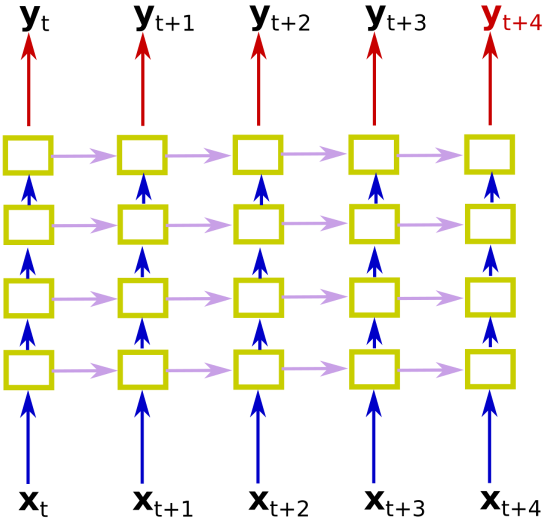

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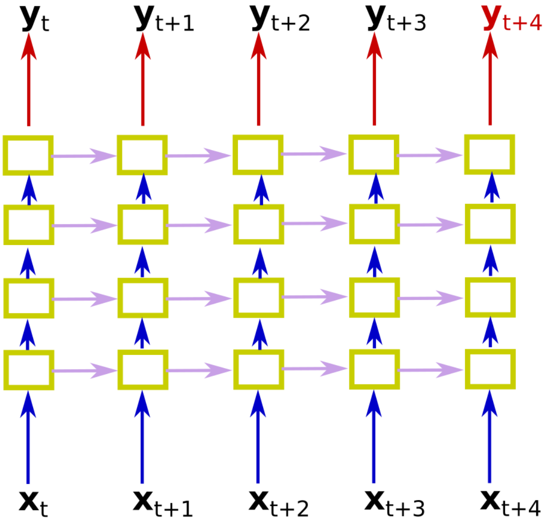

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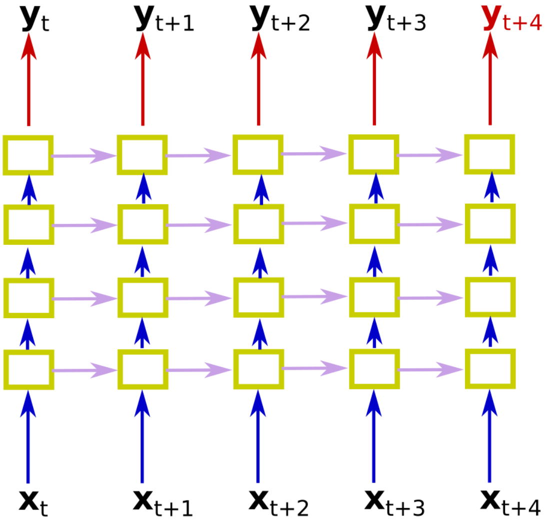
- It suffers from the problem of **unstable gradient**
- RNN has typically **short memory**
- It gradually forgets old inputs



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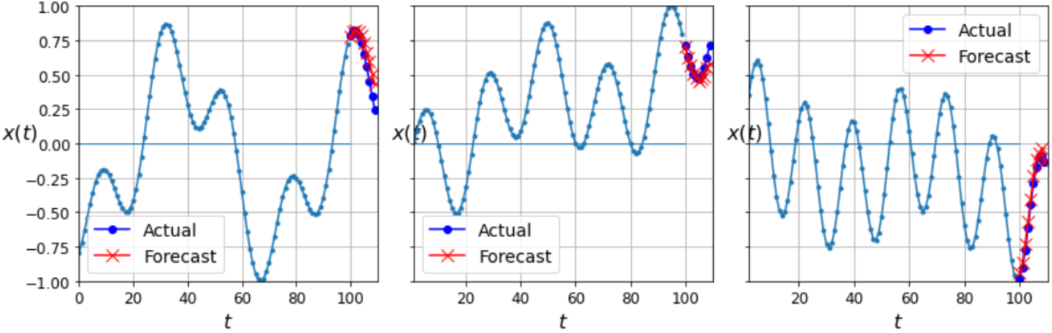
- but there is a layer for that:

```
model = keras.models.Sequential([ keras.layers.LSTM(20,
return_sequences=True, input_shape=[None, 1]),
keras.layers.LSTM(20, return_sequences=True),
keras.layers.TimeDistributed(keras.layers.Dense(10)) ])
```

RNN with LSTM cells



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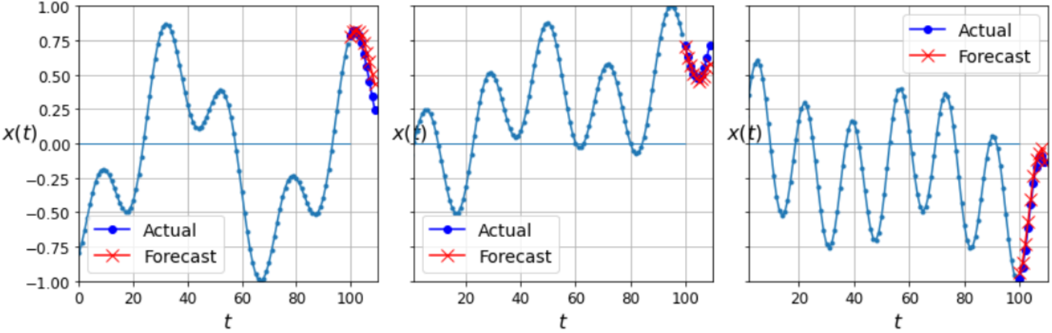

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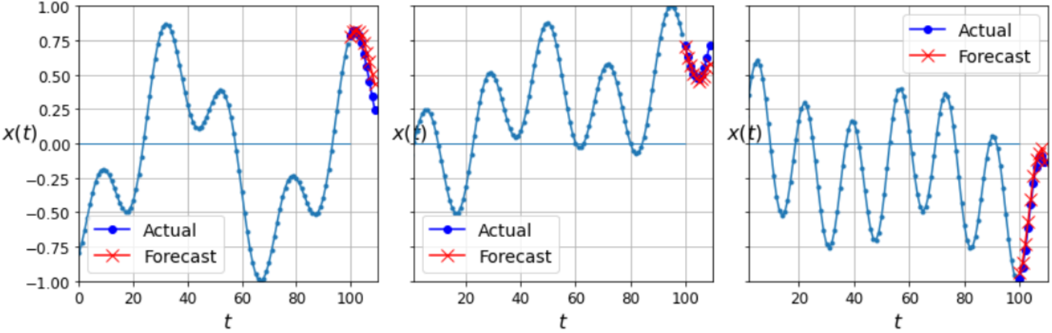

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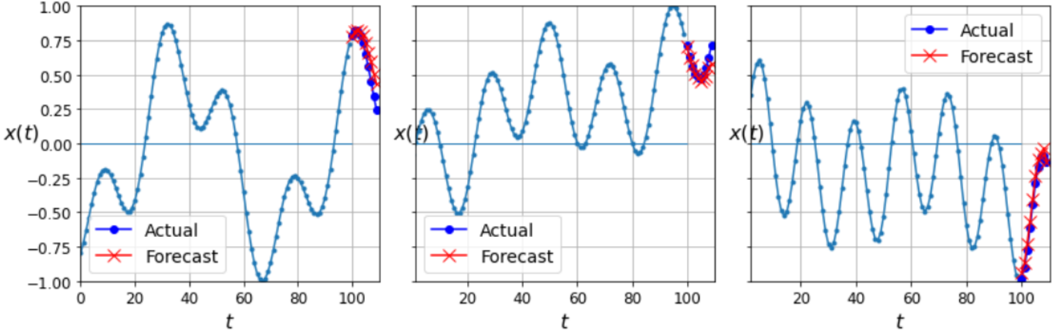

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We can do more if we combine memory cells with CNN.

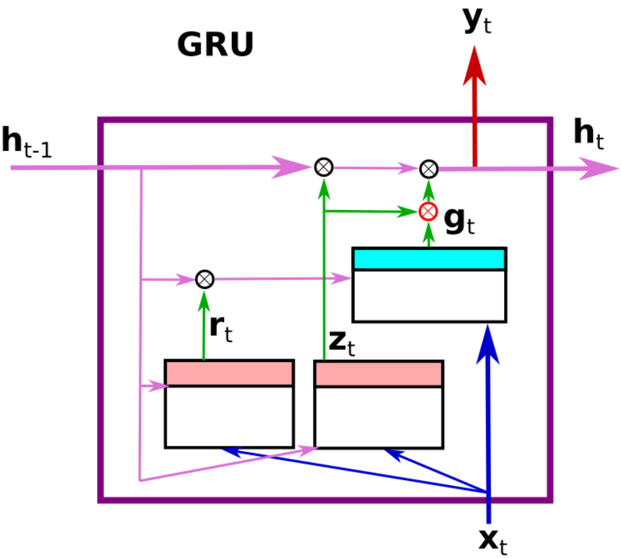
GRU is a simplified version of LSTM

$$\mathbf{z}_t = \sigma(\mathbf{W}_{xz}\mathbf{x}_t + \mathbf{W}_{hz}\mathbf{h}_{t-1} + \mathbf{b}_z)$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_{xr}\mathbf{x}_t + \mathbf{W}_{hr}\mathbf{h}_{t-1} + \mathbf{b}_r)$$

$$\mathbf{g}_t = \tanh\left(\mathbf{W}_{xg}\mathbf{x}_{t-1} + \mathbf{W}_{hg}(\mathbf{r}_t.\mathbf{h}_{t-1}) + \right.$$

$$\mathbf{h}_t = \mathbf{z}_t.\mathbf{h}_{t-1} + (1 - \mathbf{z}_t).\mathbf{g}_t$$



GRU plus 1D CNN

- GRU and LSTM allow to increase the memory to hundreds of time-steps

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model = keras.models.Sequential([
keras.layers.Conv1D(filters=20,kernel_size=4,strides=2,padding="valid",
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- But we might need to work with much longer sequences
- The remedy is to use CNN

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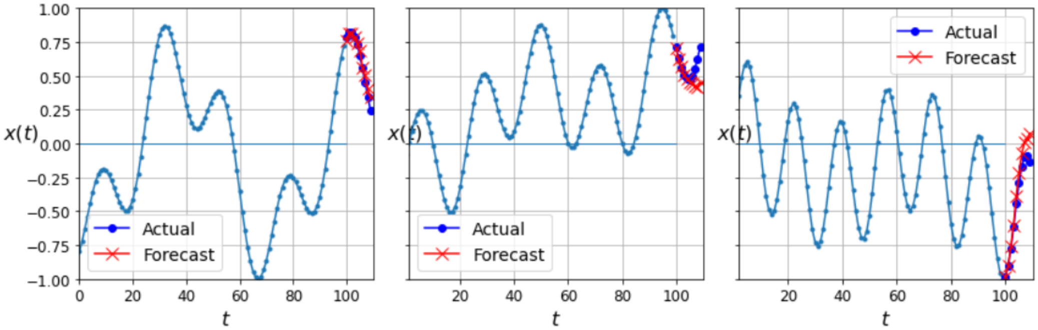

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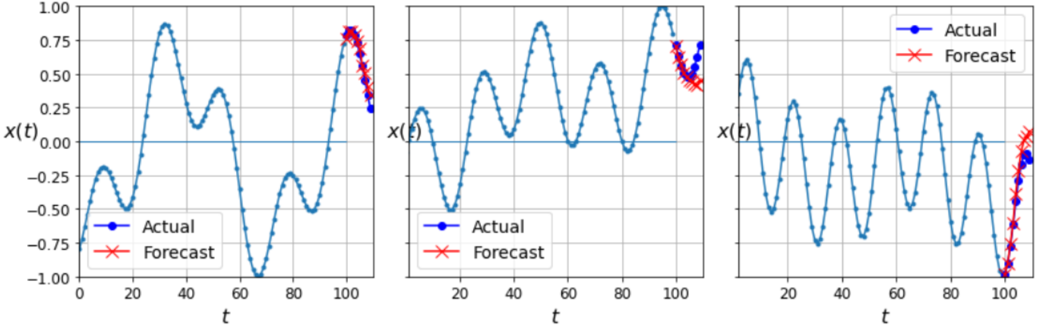

This is so far the best network. Its MSE is for last predicted point is 0.0085

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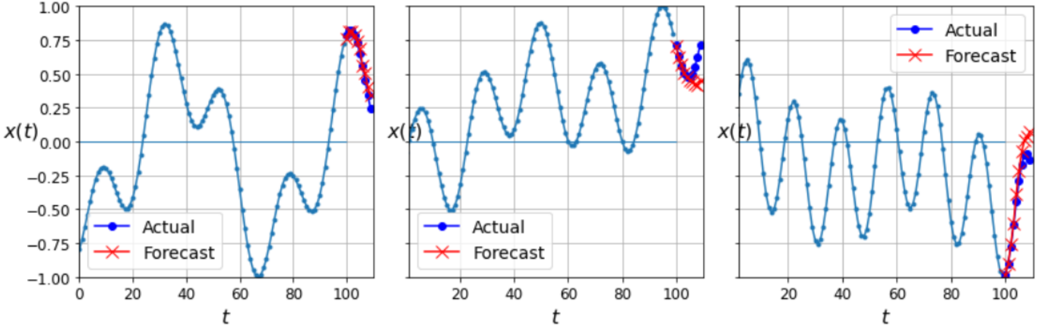

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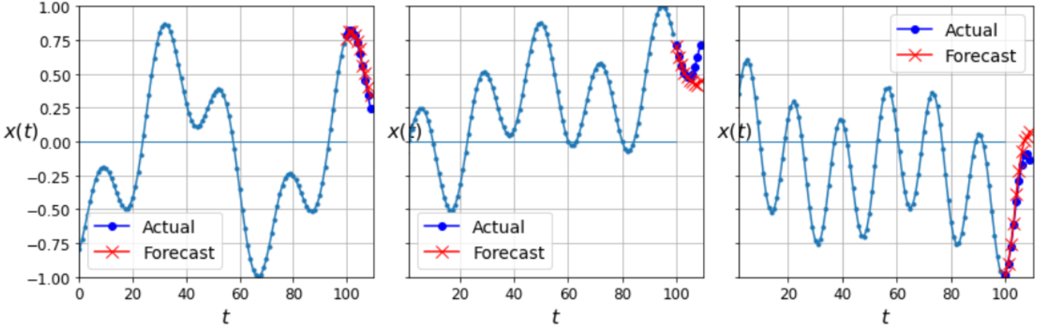

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WaveNet