Newtonian Dynamics in Astrophysics

Roberto Capuzzo Dolcetta,

Prague, December 2022

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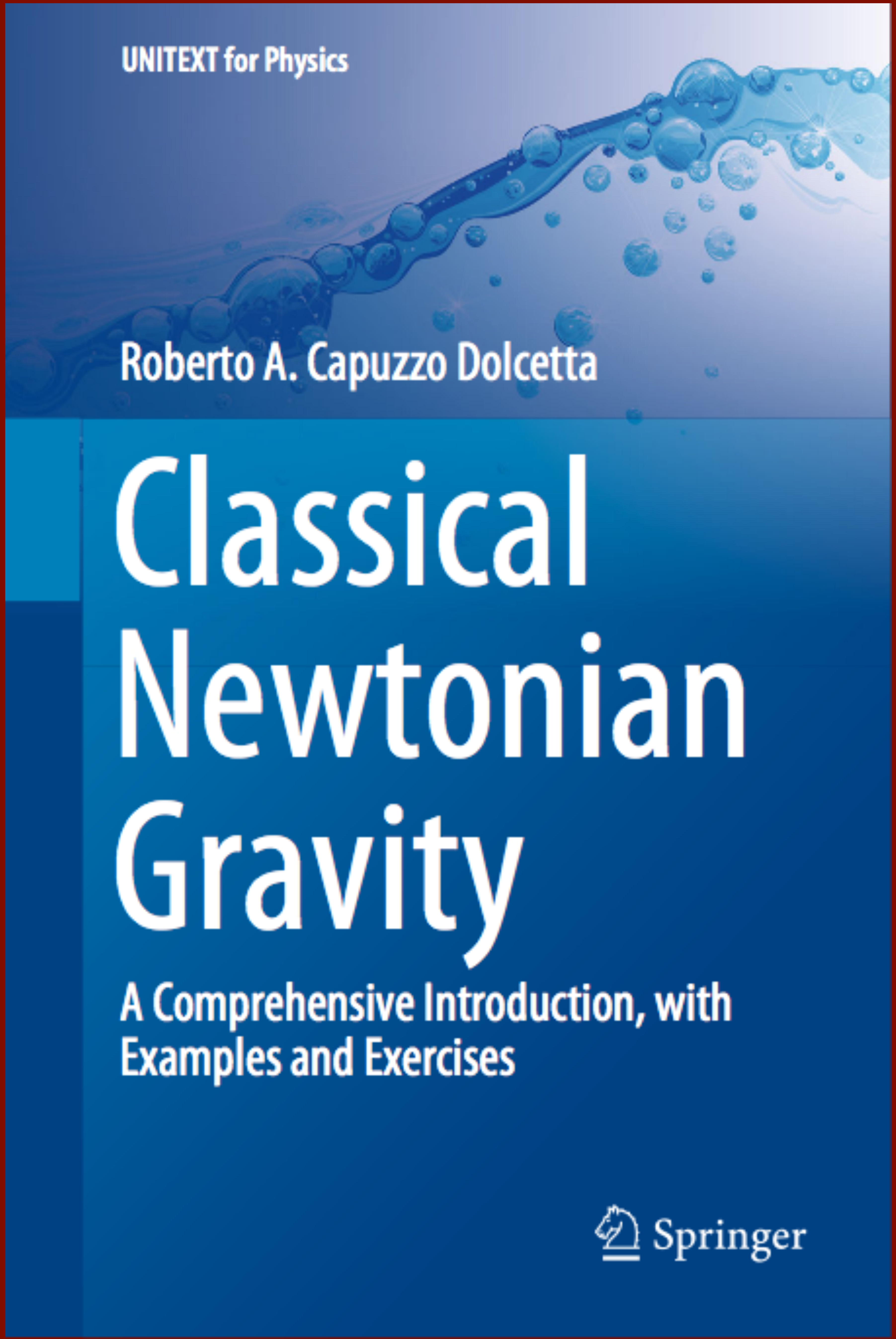


Force	range	simbol	mass	Charge	spin
Gravity	**long**	**G**	**no**	**0**	**2**
Electromag.	long	γ	no	0	1
Weak nucl..	short	$W^{\pm}$, Z^0	yes	±1, 0	1
Strong nucl.	short	g	yes	0	1

Long range forces:

Gravitational (Newton)

Electrostatic (Coulomb)

Gravity: Newtonian or Relativistic?

It depends upon what's under study

Newtonian regime:

$\beta = v/c \ll 1$; $\delta = (2GM/c^2)/r \ll 1$

Earth gravity

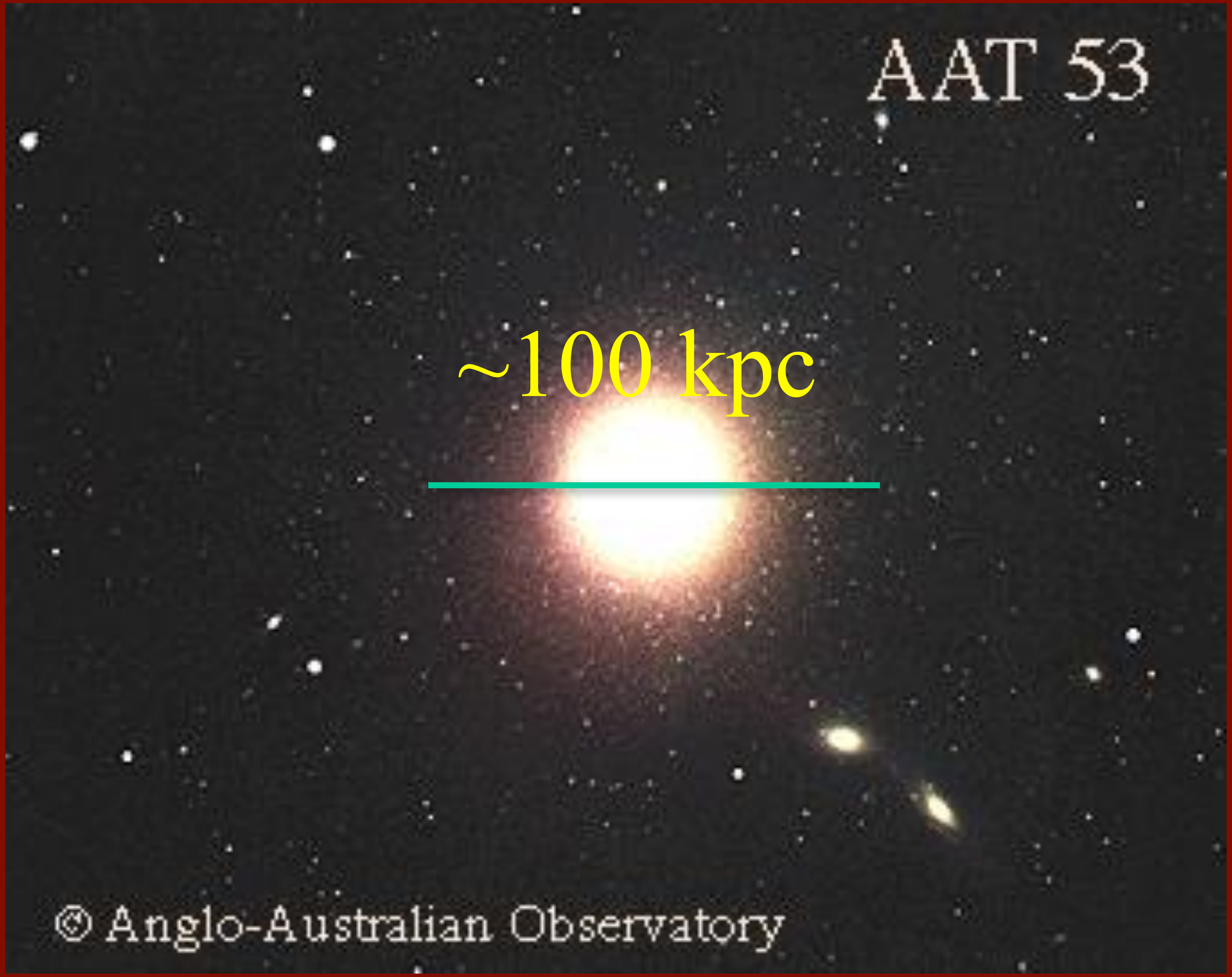
Perturbation of a *stable* equilibrium

Water? Test “particle”

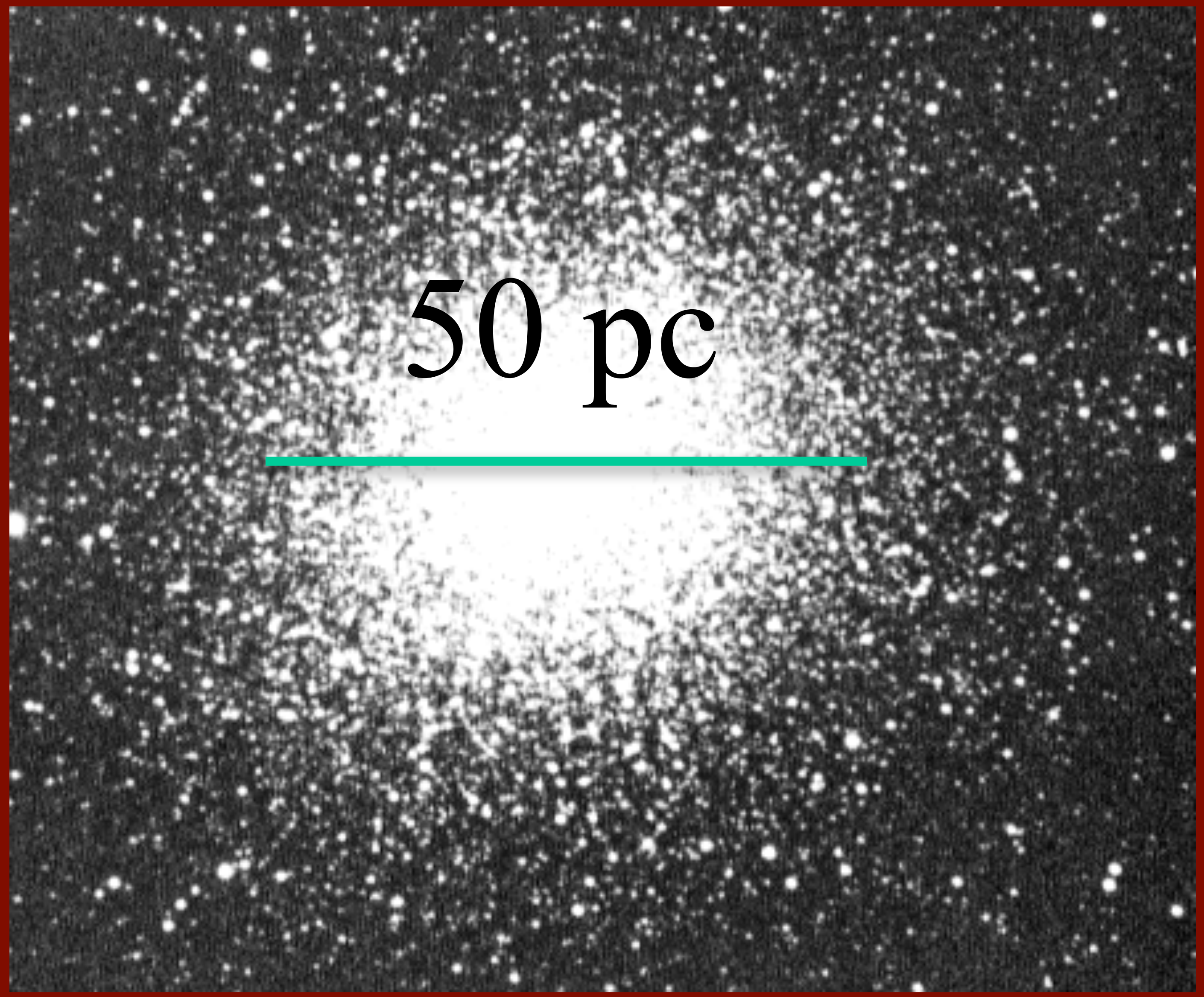
Perturbation of an *unstable* equilibrium

Snow? Test “particle”

Astronomical gravity



The giant elliptical M 87



Globular cluster

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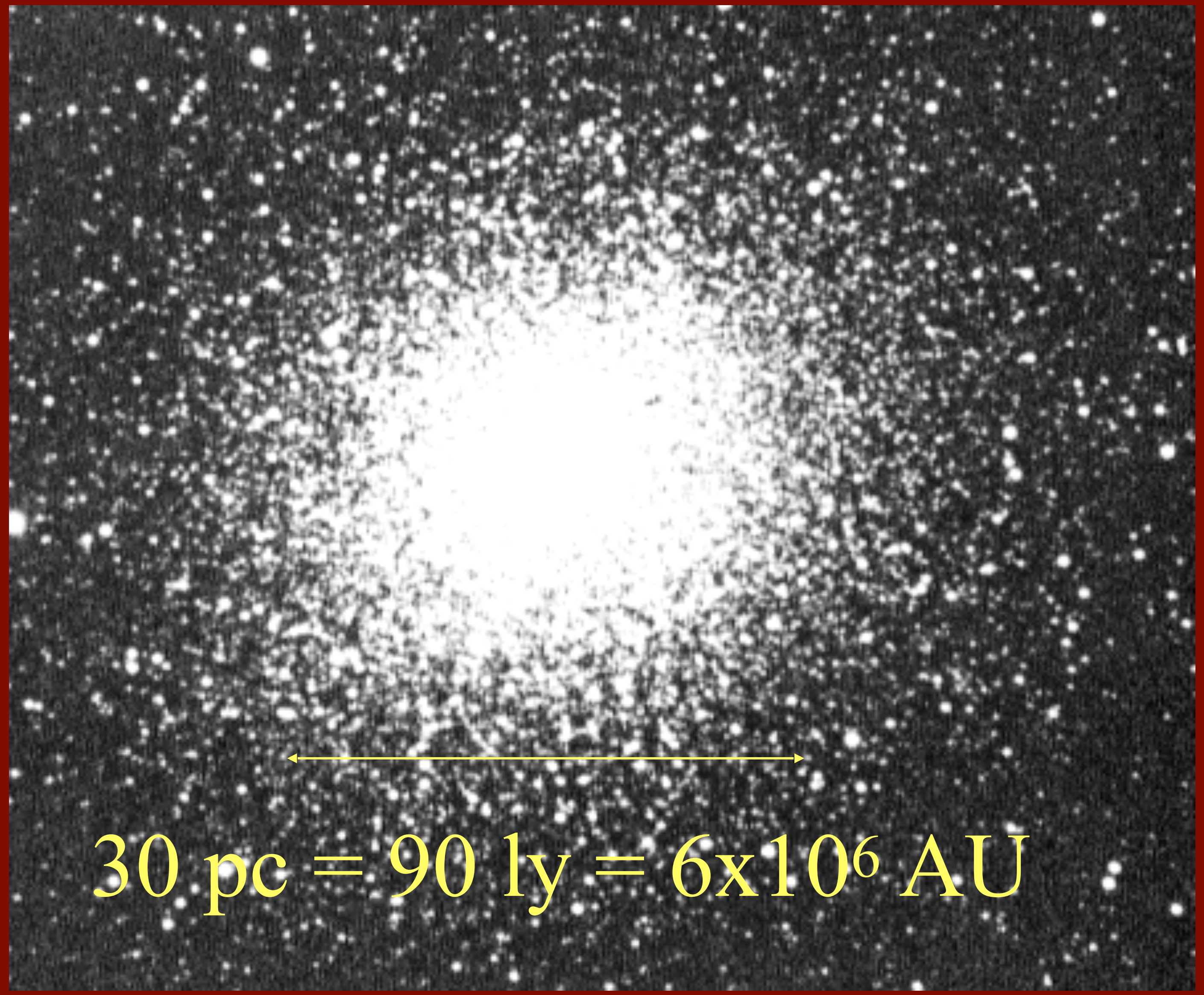
Peculiarity of astrophysics is the role of self-gravity

$\alpha \equiv$ auto grav/ext grav

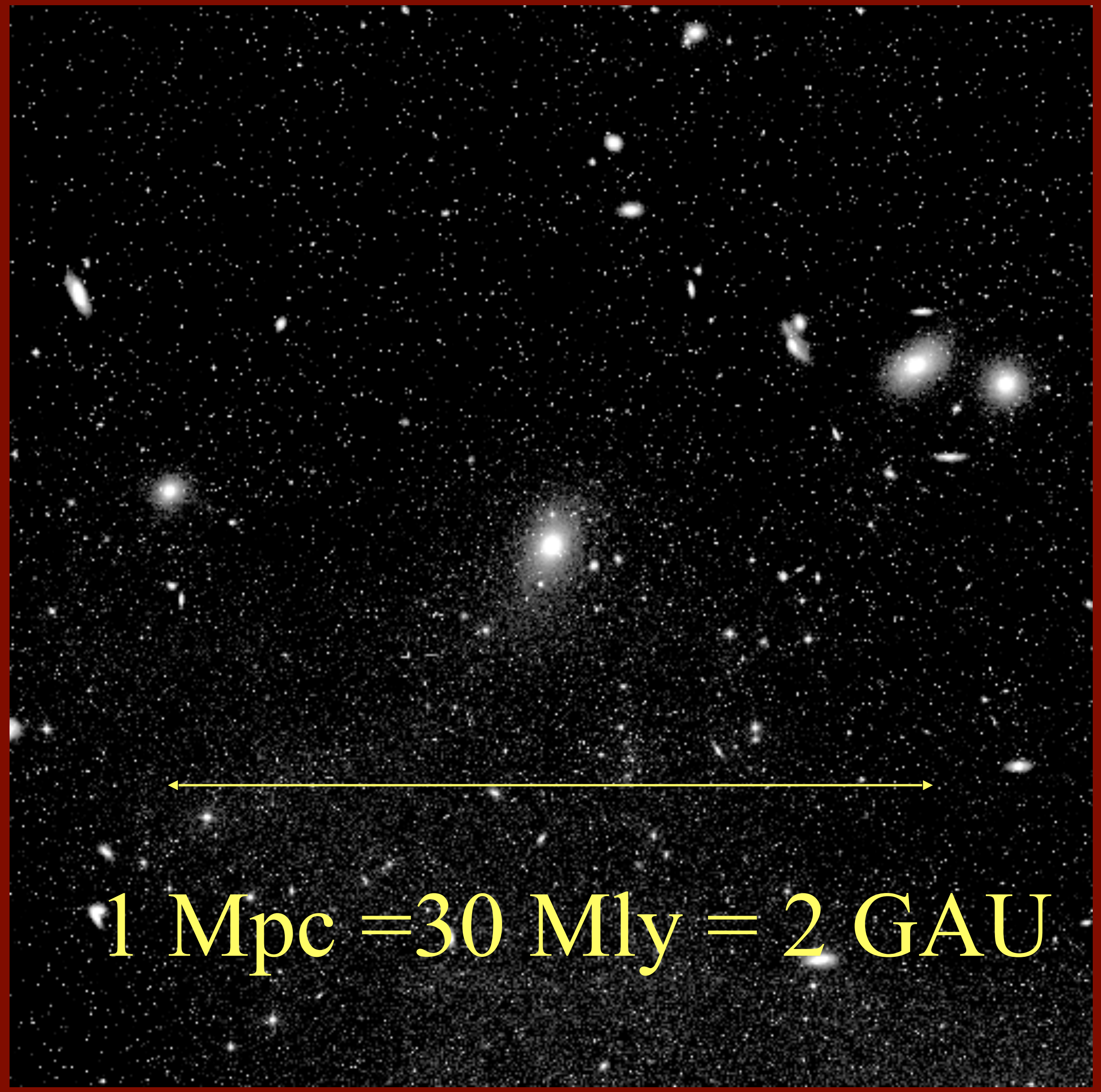
lake of Lipno

$\alpha \sim 10^{-8}$



AG: M 13

$\alpha \sim 10^{-2}$



cluster of galaxies

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1) *IR divergence* (U_{ij} never vanishes) $\Rightarrow O(N^2)$

2) *UV divergence* ($\lim_{r_{ij}\to 0} U_{ij} = \infty$) $\Rightarrow \Delta t \to 0$

1) IR divergence: the Olbers' paradox (1826, already introduced by Bentley)

$$I = \int_S \frac{I' dn}{r^2} = \lim_{r\to\infty} \int_0^r \frac{I' \rho_0 4\pi r^2}{r^2} dr = \infty!$$

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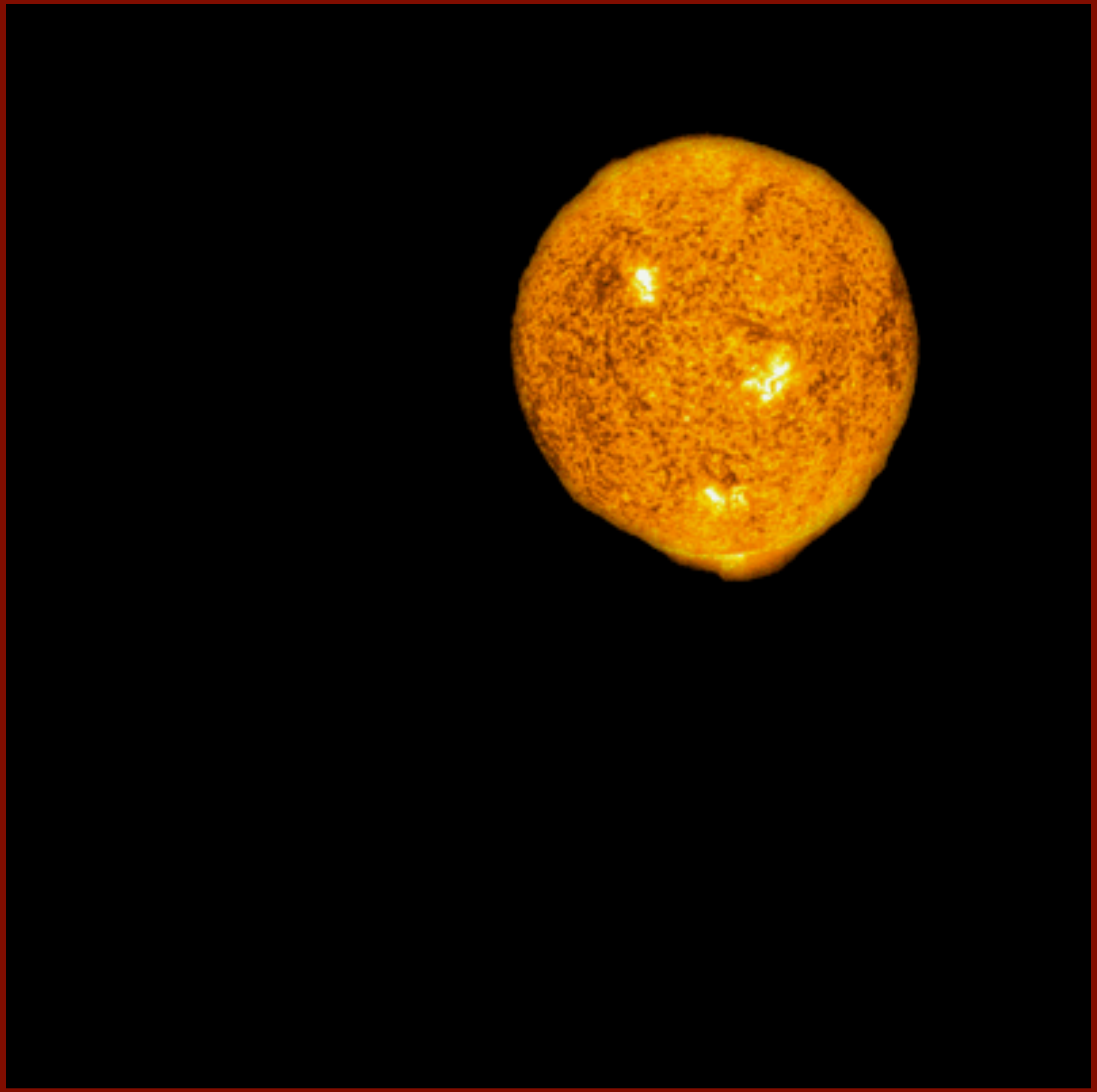
The night sky should appear uniform and luminous



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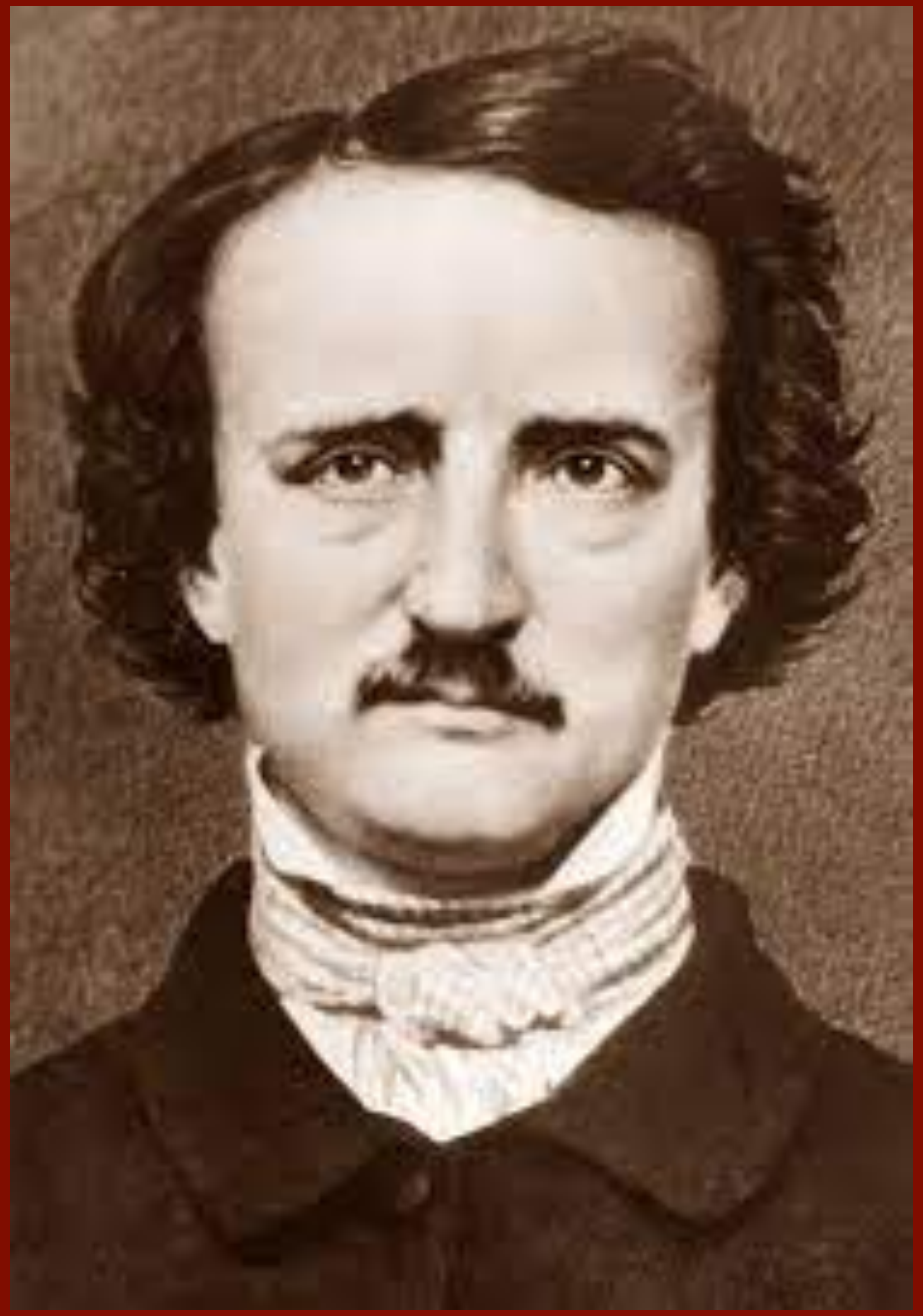
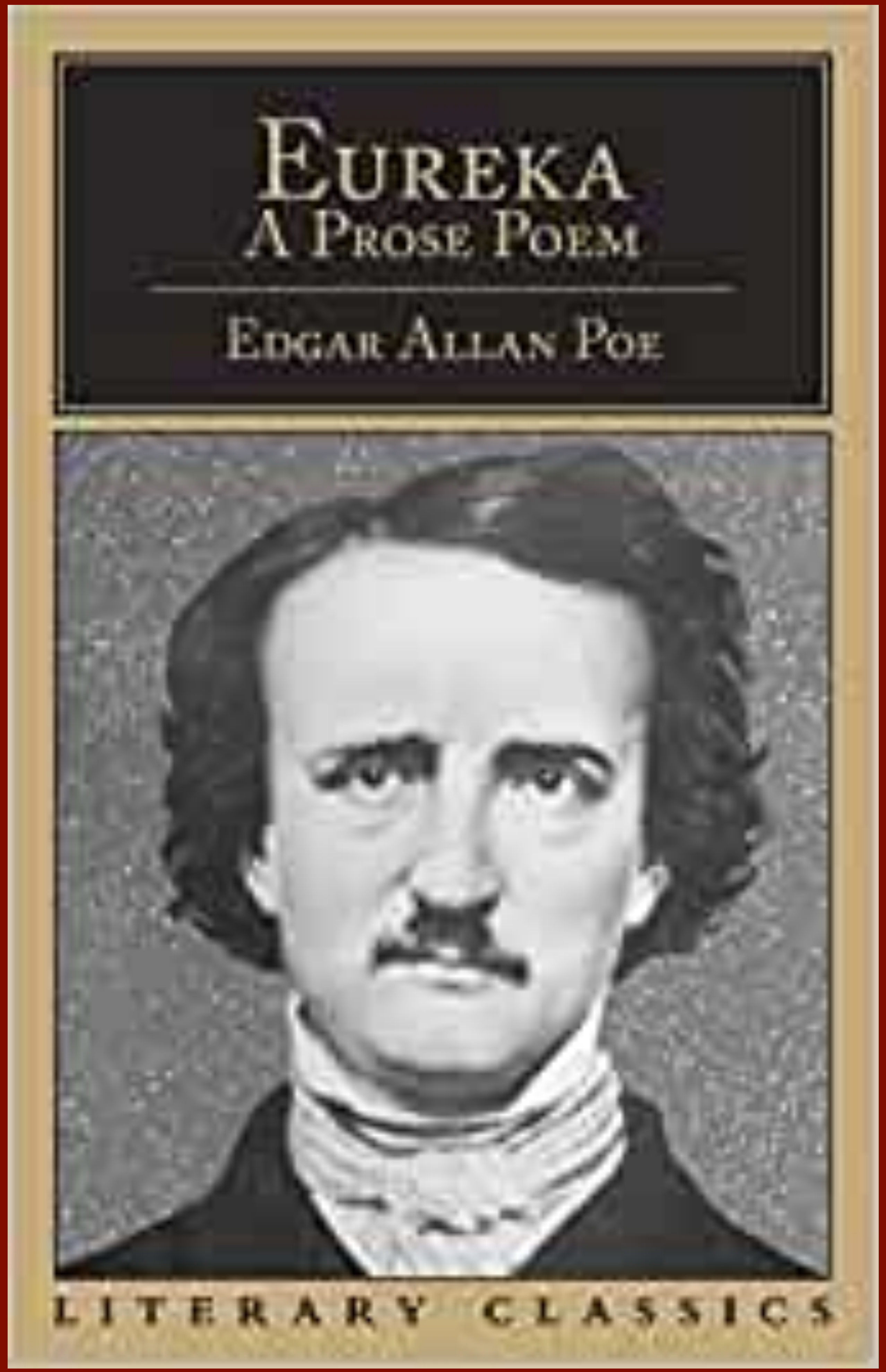
An intuition for the solution:

"Were the succession of stars endless, then the background of the sky would present us *a uniform luminosity*, like that displayed by the Galaxy –since there could be absolutely no point, in all that background, at which *would not exist a star*. The only mode, therefore, in which, under such a state of affairs, we could comprehend *the voids which our telescopes find in innumerable directions*, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all."



E. A. Poe
1809 -1849



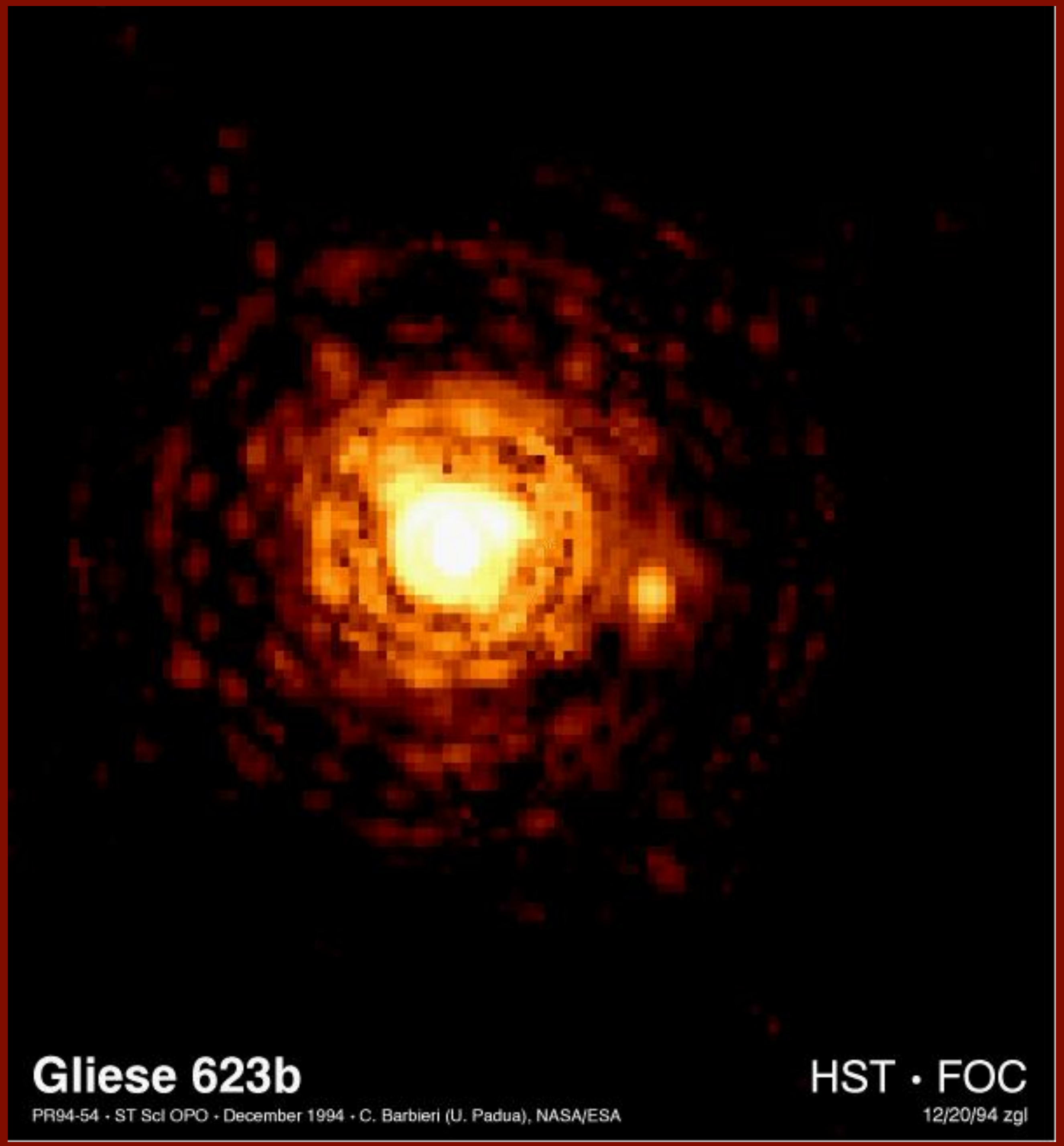
1848

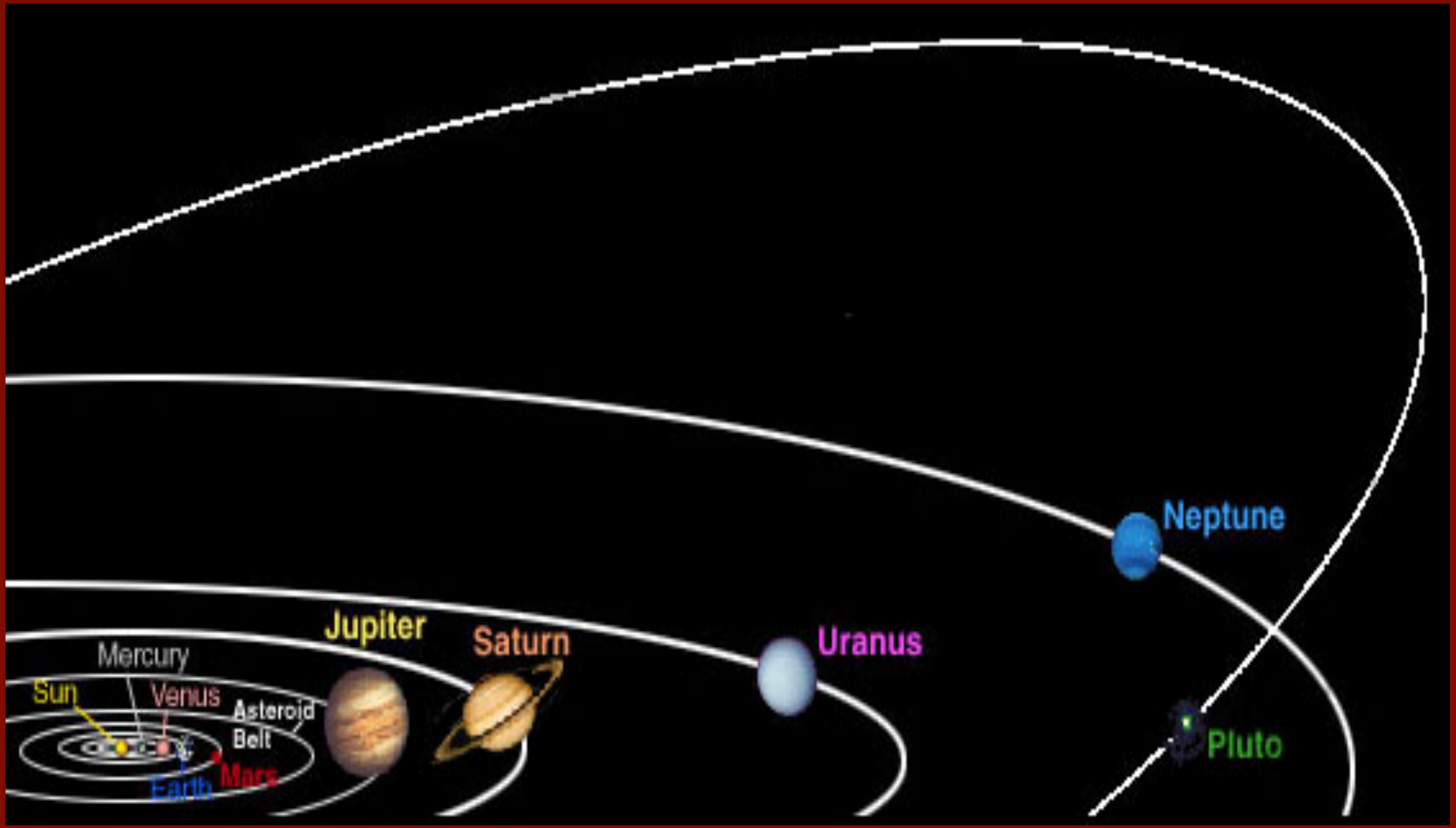
Astronomical systems

Solar and stellar systems are composed by N=2 up to $N=10^{12}$ stars, often embedded in a gaseous cloud...Multi-phase gravitational N-body problem...

Binaries...
N=2

Solar system ...
N=10

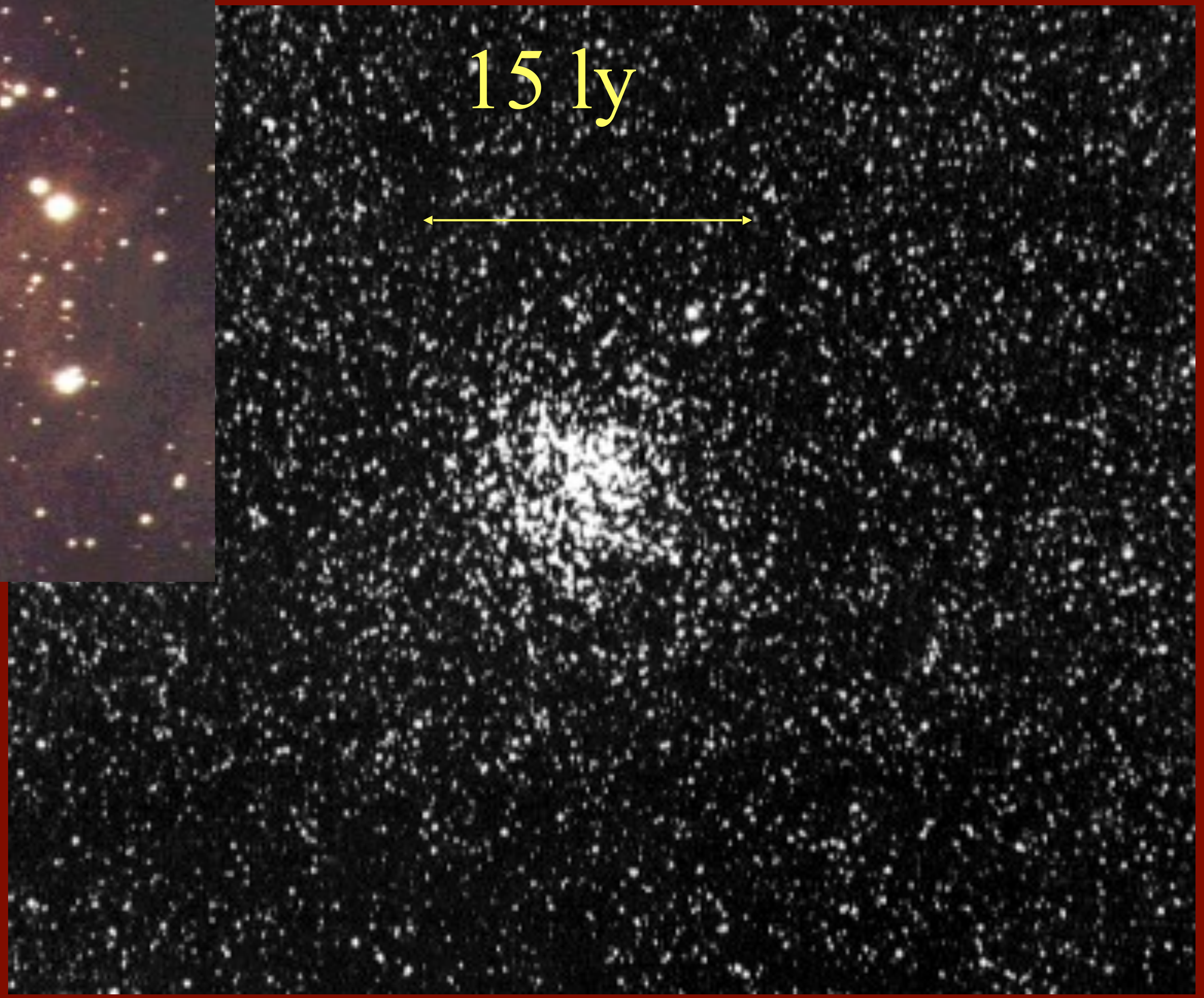




Small *open clusters*
N=50

embedded in their *mother cloud*...like
M16

Large *open clusters*
N=1000



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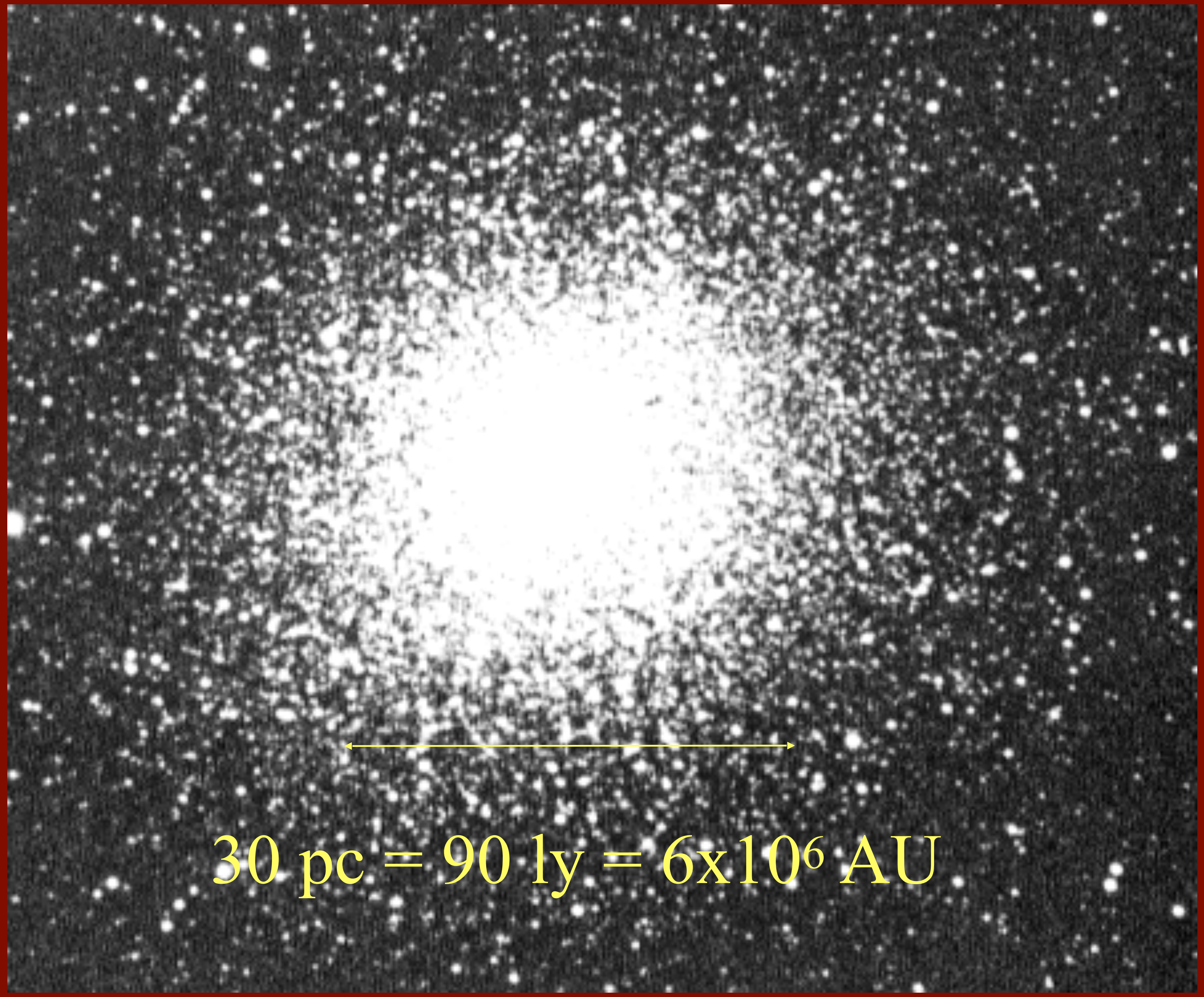
M 5

Globular clusters

$10^4 \leq N \leq 10^6$



M 13, in Hercules

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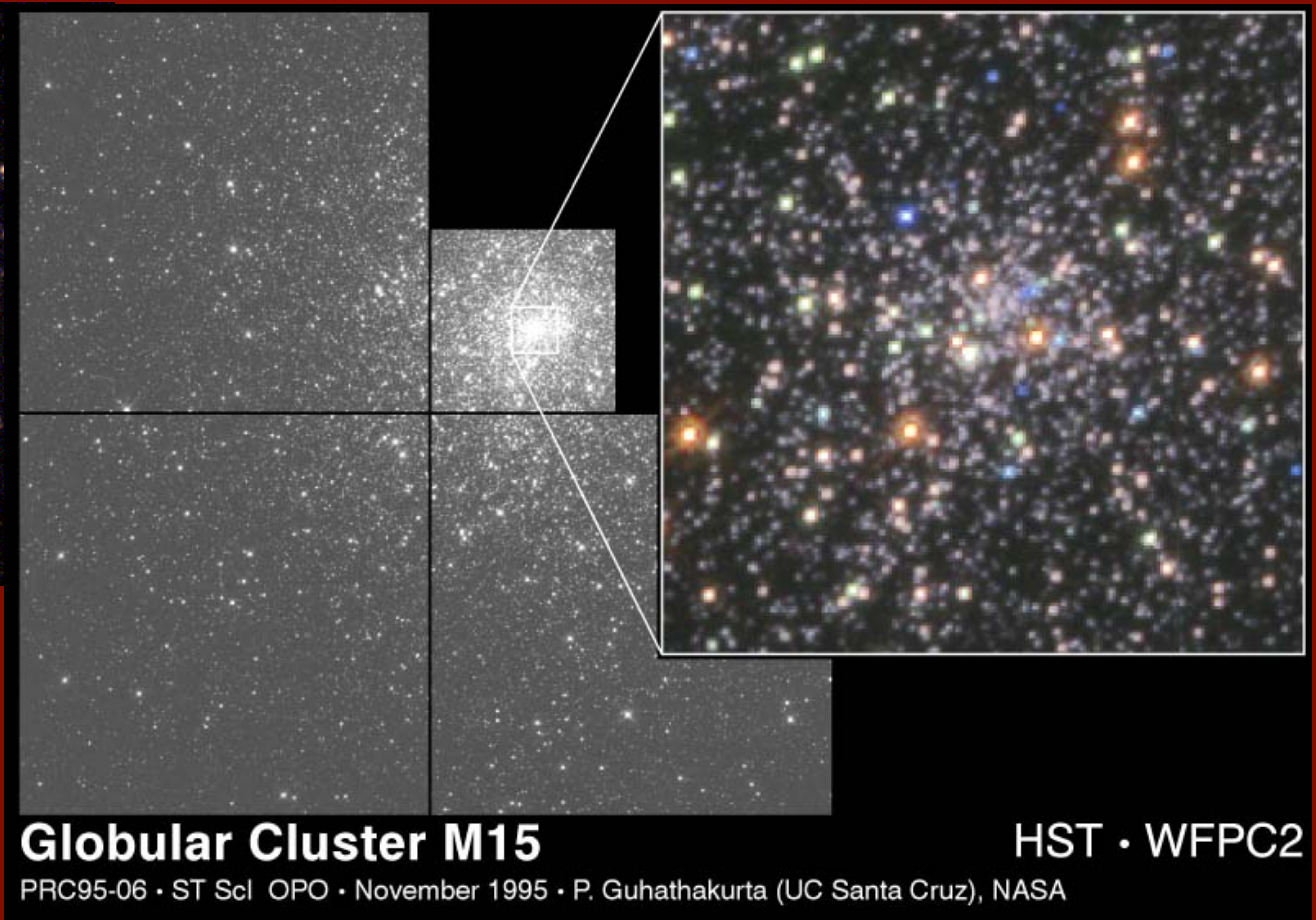
1 pc = 3 ly

M 15



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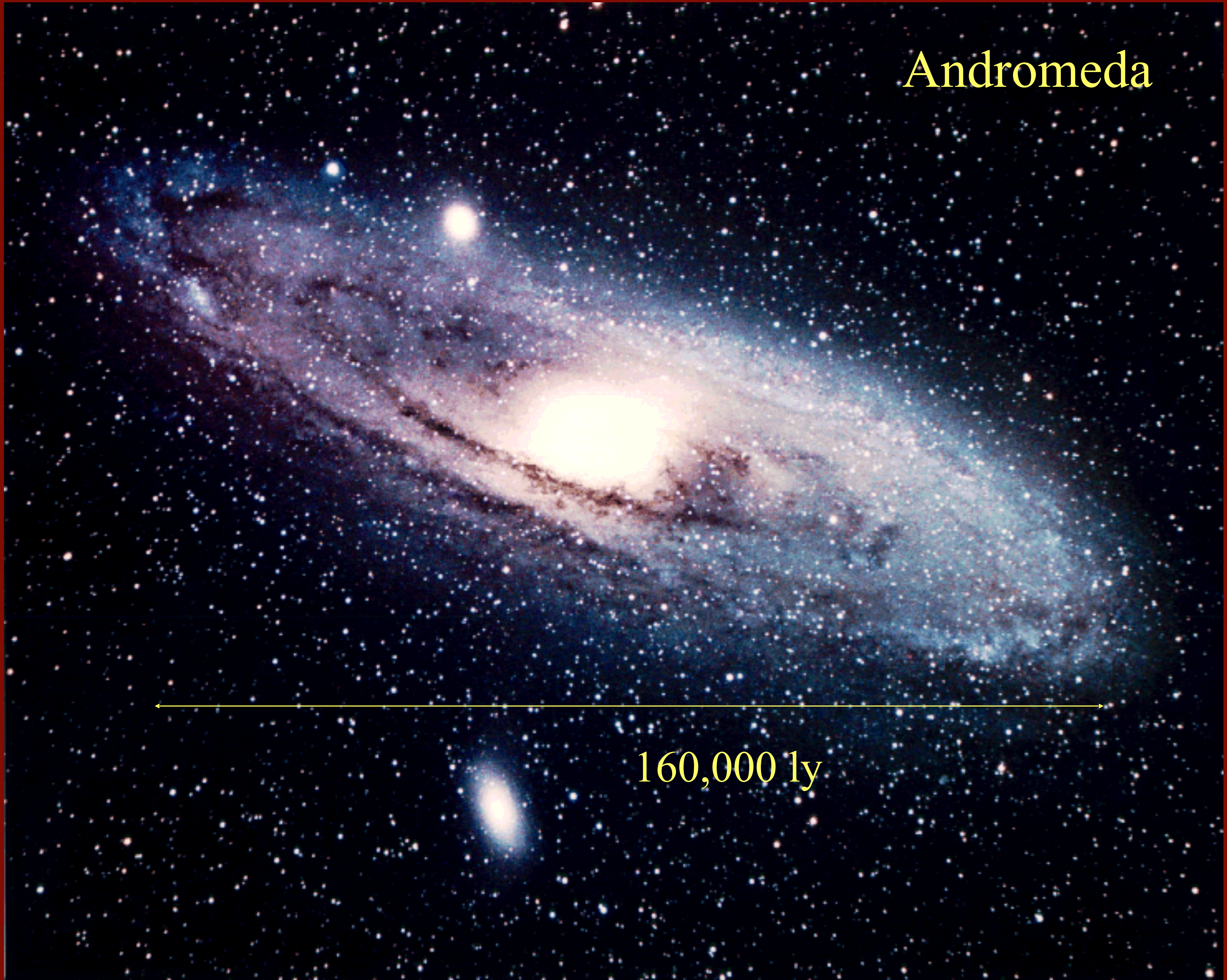
$N = 2\times10^{11}$



$N = 10^{12}$

M 87
a *giant* elliptical

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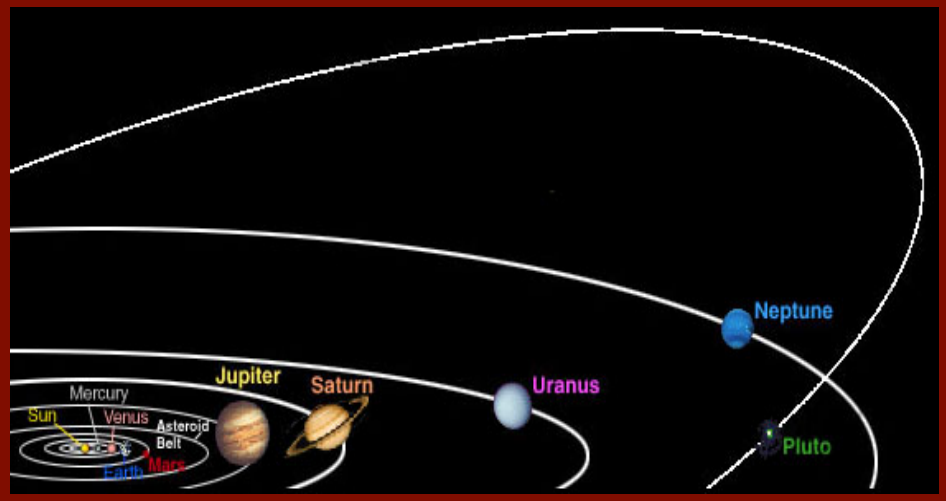
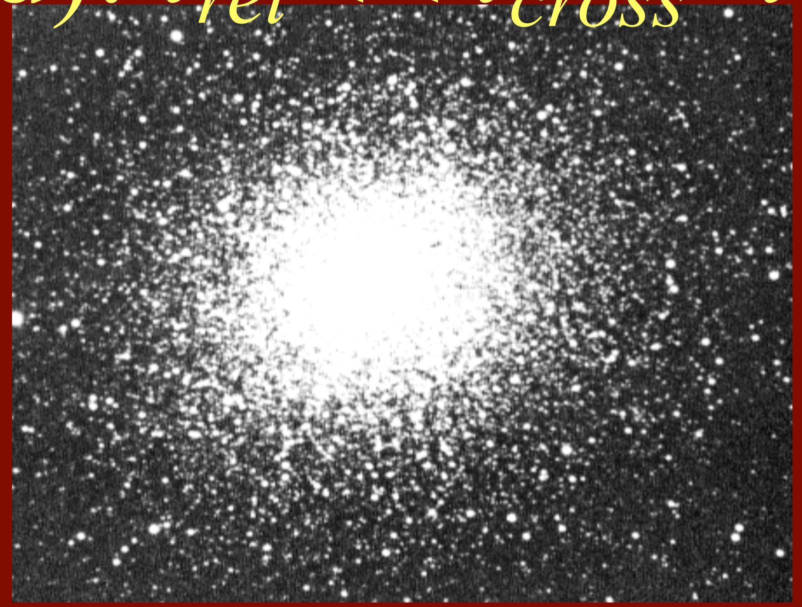
t = age of the system, t_{rel}= relax. time, t_{cross}= orb.time

Fluid (collision-dominated): $t_{rel} << t_{cross} < t$



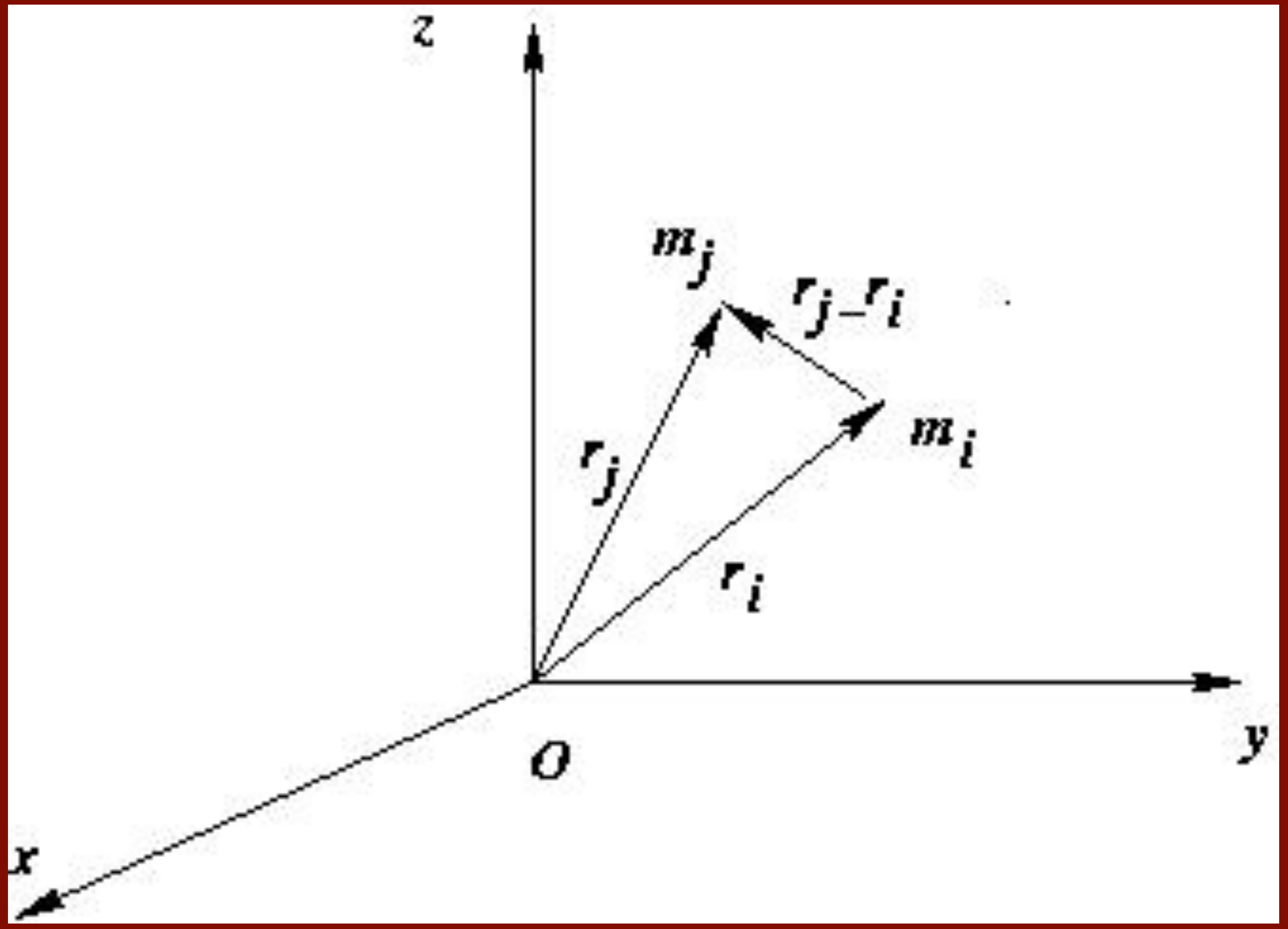


Few body ⟶ *Intermediate N* ⟶ *Many body*

Stellar system	Binaries, triple, Plan. systems	Open clusters	Globular clusters, galactic nuclei	Galaxies, Galaxy clusters
N	2,3, ≤10	≤10,000	$10^5 \div 10^9$	$\geq 10^9$
Regime	Deterministic	Collisional	Secularly collisional	Collisionless
Time-scales	$t_{cross} << t$	$t_{rel} \leq t_{cross} < t$	$t_{cross} < t_{rel} < t$	$t_{cross} < t << t_{rel}$
Gravity	Newtonian	Newtonian	Newtonian,general relativity	Newtonian, gen.relativity
Technique	Analytic, Perturbative, Direct N-body	Gas+Direct N-body	Fokker-Planck, Direct N-body	Tree-codes, PM, P3M

The *N*-body classic gravitational problem

Statement of the problem



$$\ddot{\mathbf{r}}_i = -G\sum_{\substack{j=1\\ j\neq i}}^{N} \frac{m_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}\left(\mathbf{r}_i - \mathbf{r}_j\right)$$

$$\mathbf{r}_i(0) = \mathbf{r}_{i0}$$

$$\dot{\mathbf{r}}_i(0) = \dot{\mathbf{r}}_{i0}$$

Explicit solutions only for N=2. Under simplifying conditions for N=3.
As (unusable) series for $N \geq 3$ and $L > 0$.

The system can be transformed into a system of 6*N 1st* order ODEs letting $\mathbf{v}_i = \dot{\mathbf{r}}_i$, *i=1,2,…,N :*

$$\begin{cases} \dot{\mathbf{r}}_i = \mathbf{v}_i , \\ \dot{\mathbf{v}}_i = G \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{r_{ij}^3} , \\ \mathbf{r}_i(0) = \mathbf{r}_{i0} , \\ \mathbf{v}_i(0) = \dot{\mathbf{r}}_{i0} . \end{cases}$$

In any case the system:

- is of *complexity* O(N^2);
- is far from *linearity*;
- is poorly *constrained* in phase-space —> chaotic (ergodic behavior) .

10 constants of motion, independent of *N*

$$E = \frac{1}{2}\sum_{i=1}^{N} m_i v_i^2 - \frac{1}{2}\sum_{(i,j)=1, i\neq j}^{N} G\frac{m_i m_j}{r_{ij}} = E_0,$$ *Energy first integral*

$$\mathbf{L} = \sum_{i=1}^{N} \mathbf{r}_i \wedge m_i \dot{\mathbf{r}}_i = \mathbf{L}_0,$$ *Angular momentum first integral*

$$\mathbf{Q} = \sum_{i=1}^{N} m_i \dot{\mathbf{r}}_i = \mathbf{Q}_0,$$ *Momentum first integral*

$$\mathbf{r}_{com} = \frac{\sum_{i=1}^{N} m_i \mathbf{r}_i}{M} = \frac{\sum_{i=1}^{N} m_i \mathbf{r}_{i0}}{M} + \frac{\mathbf{Q}_0 (t - t_0)}{M}.$$ *Center of mass constant*

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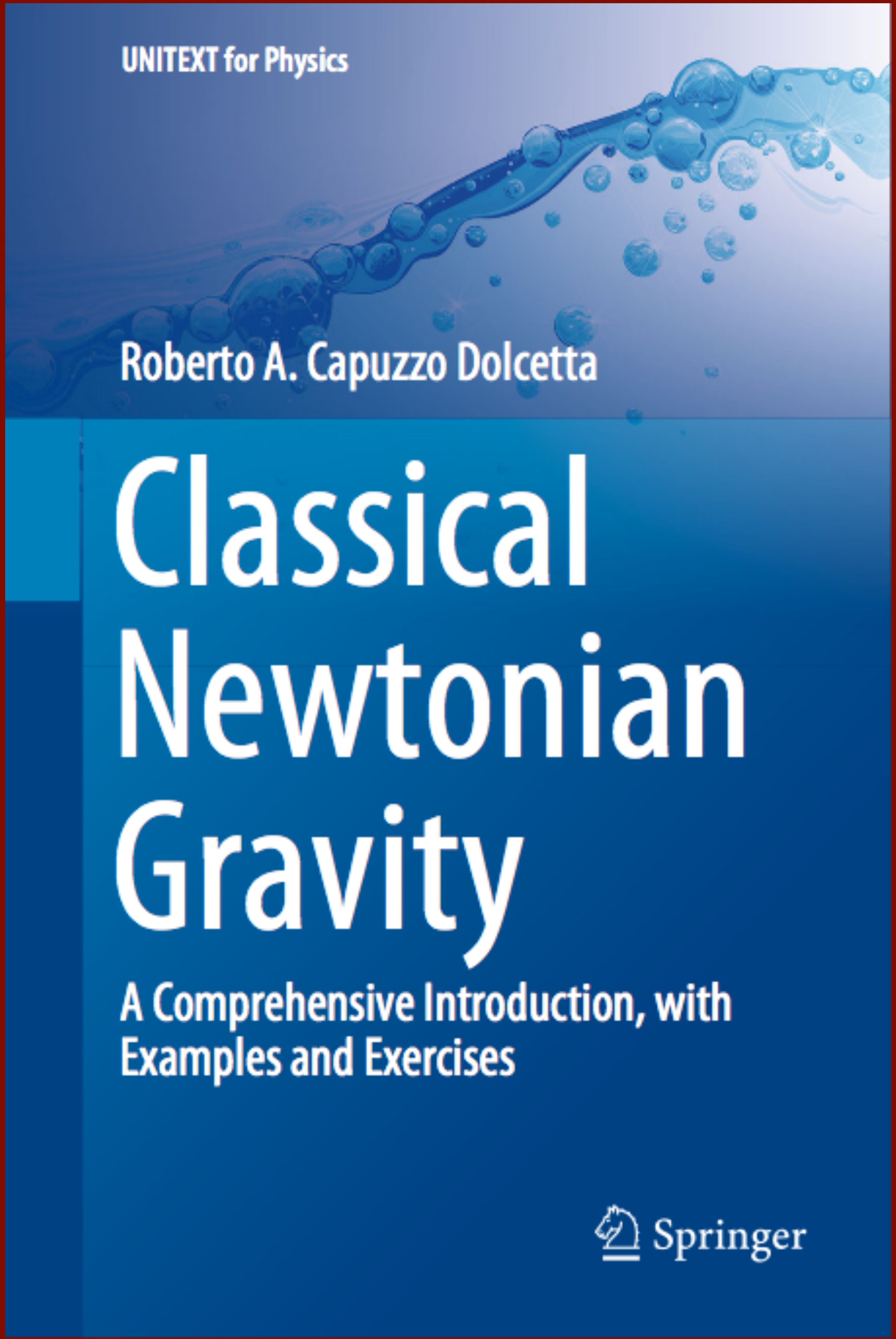

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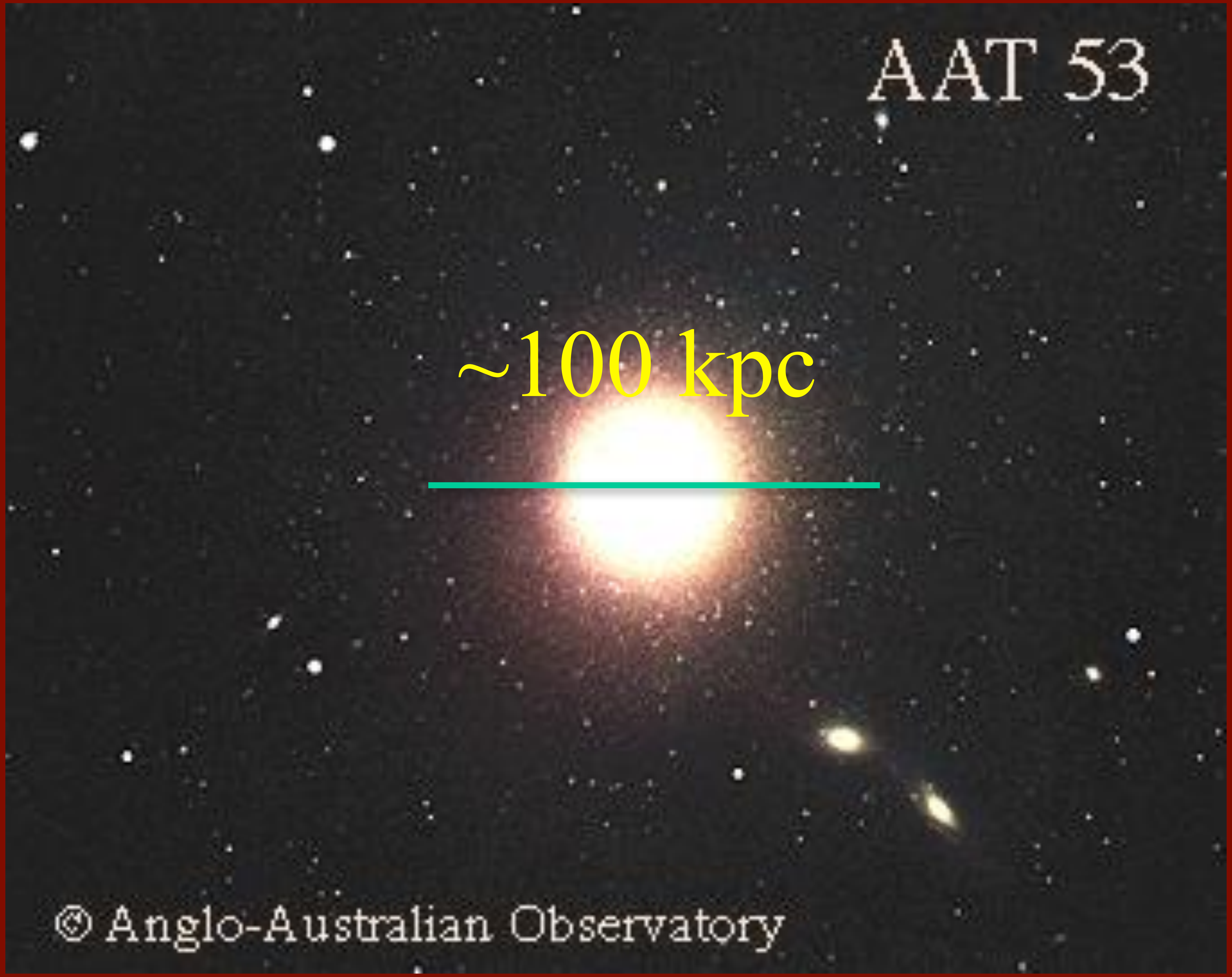

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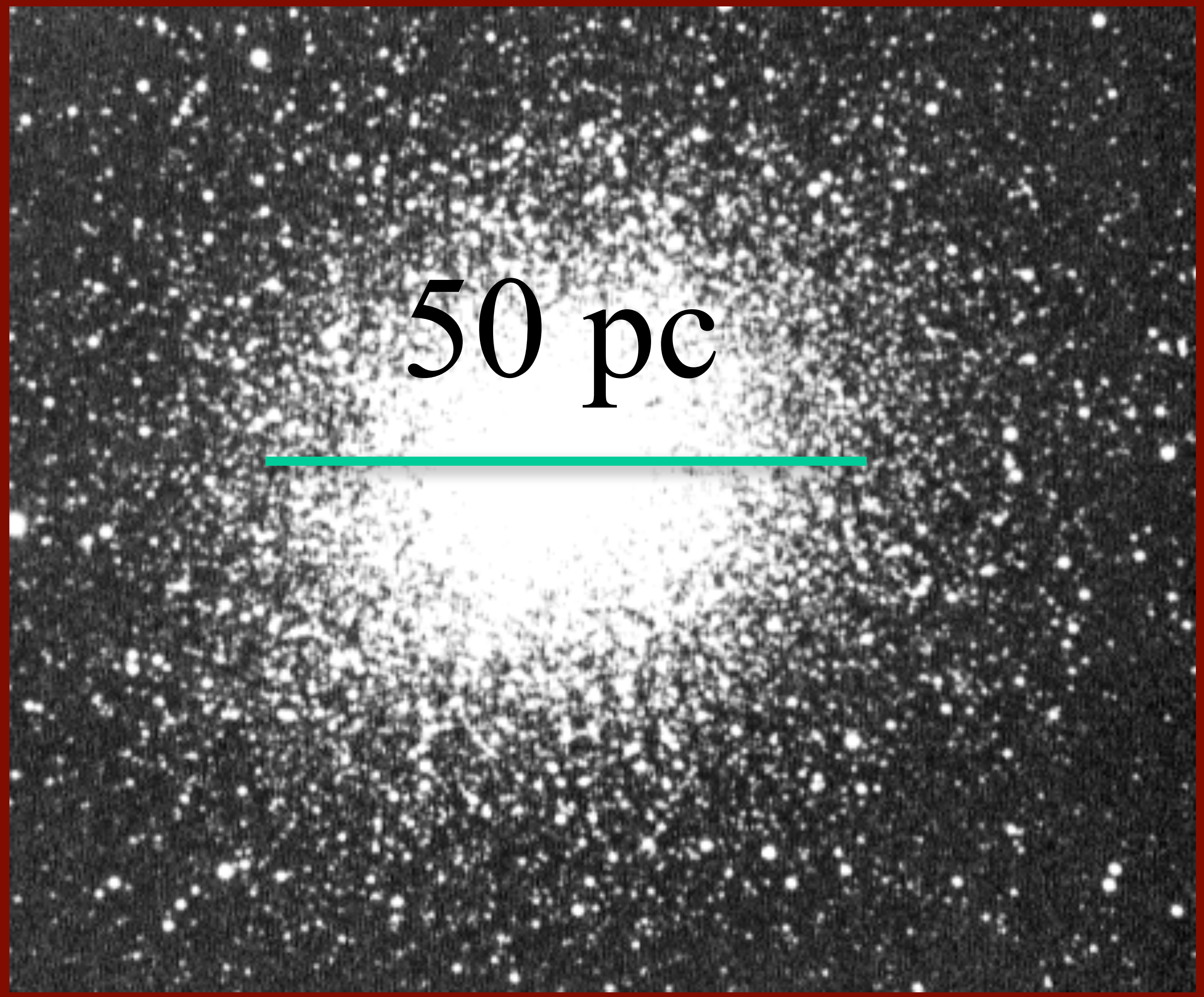

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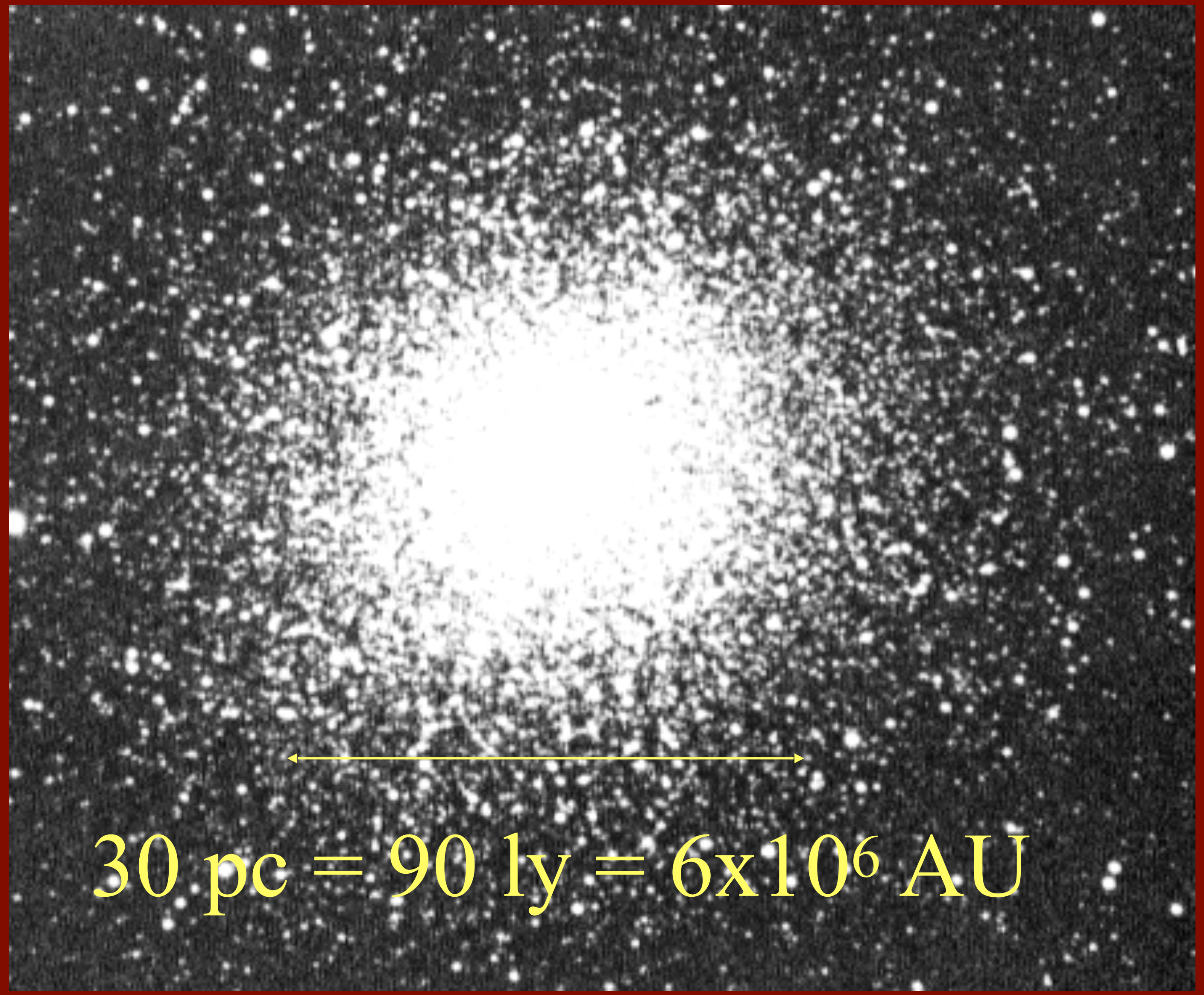

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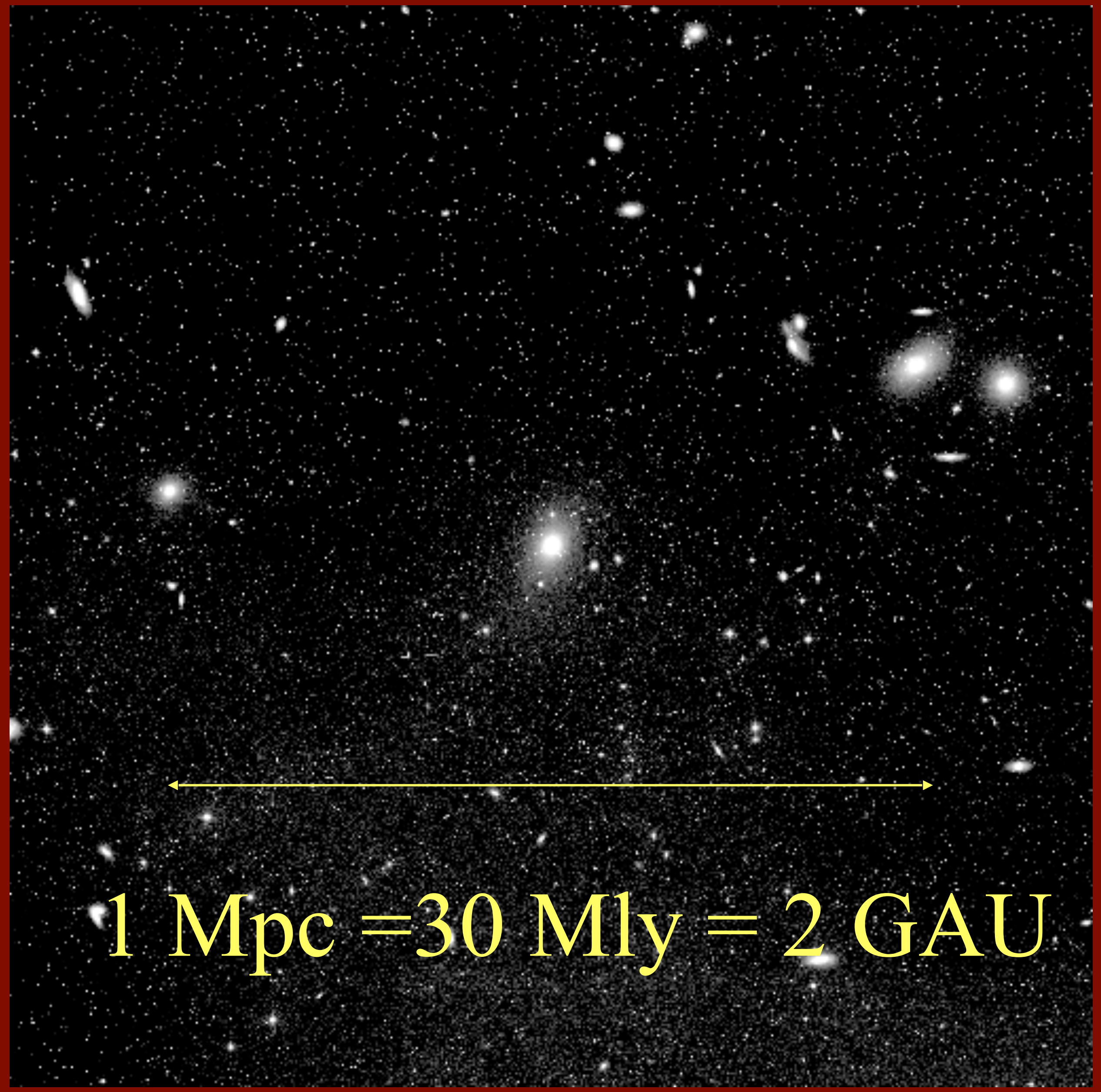

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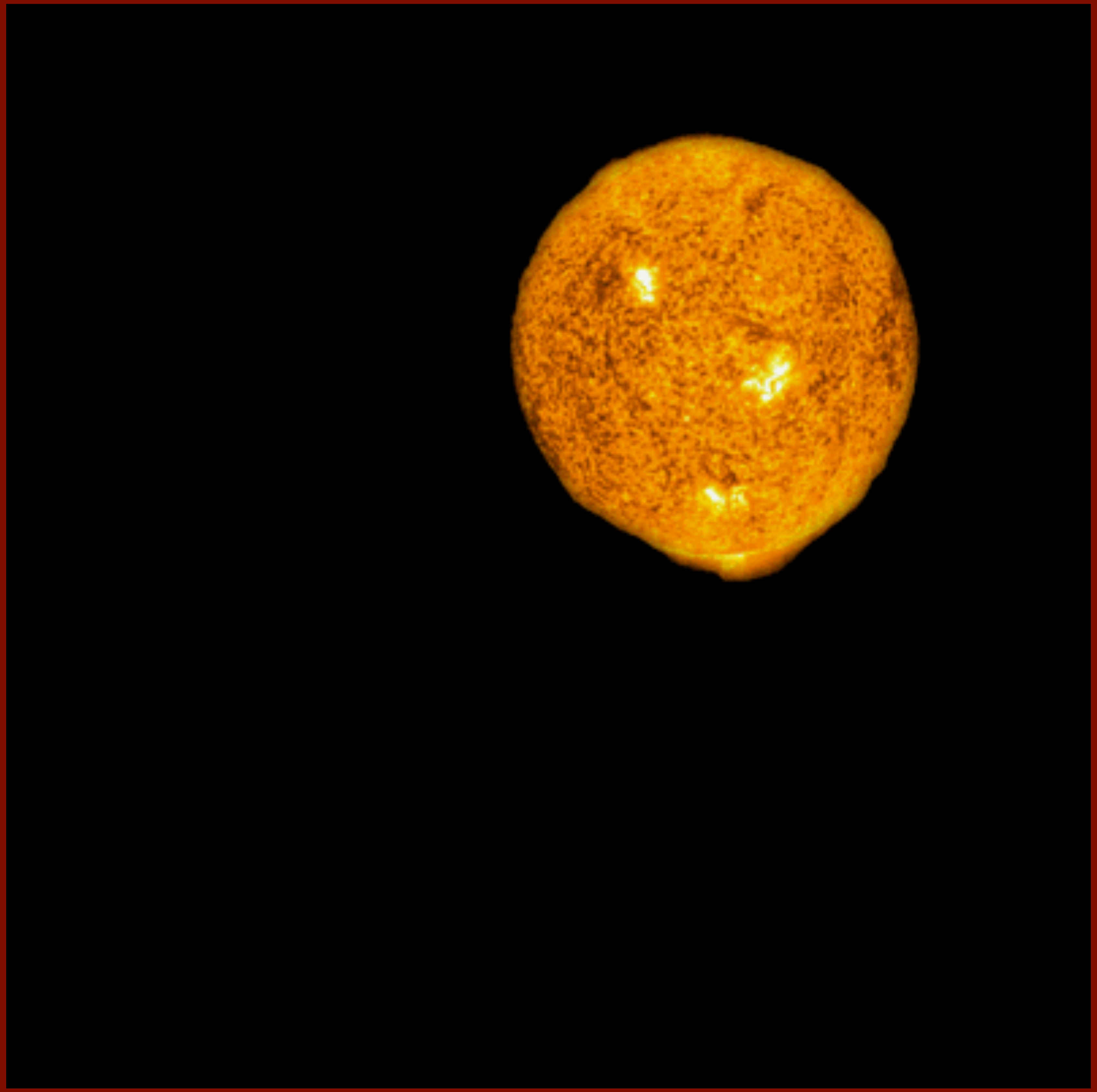

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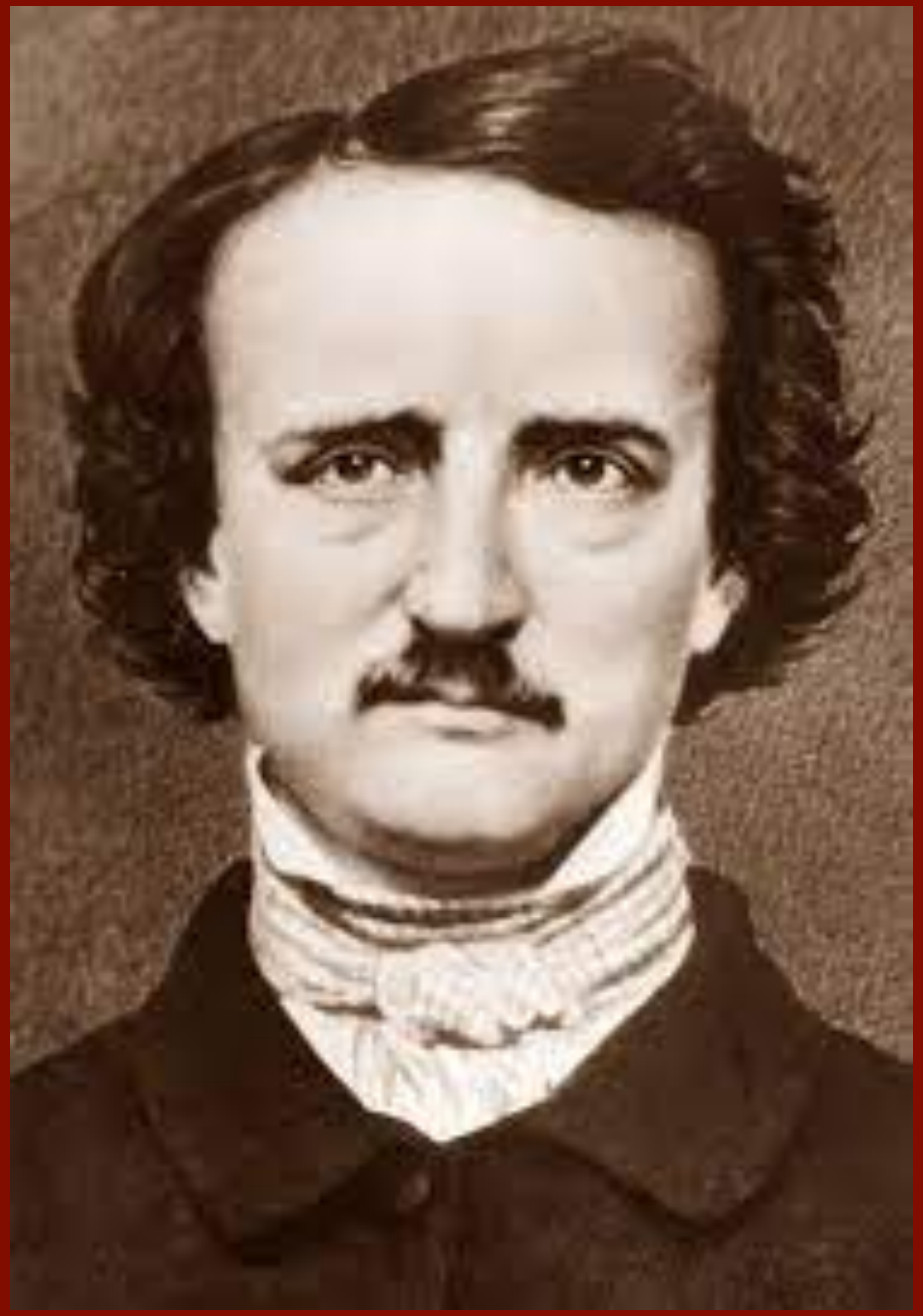

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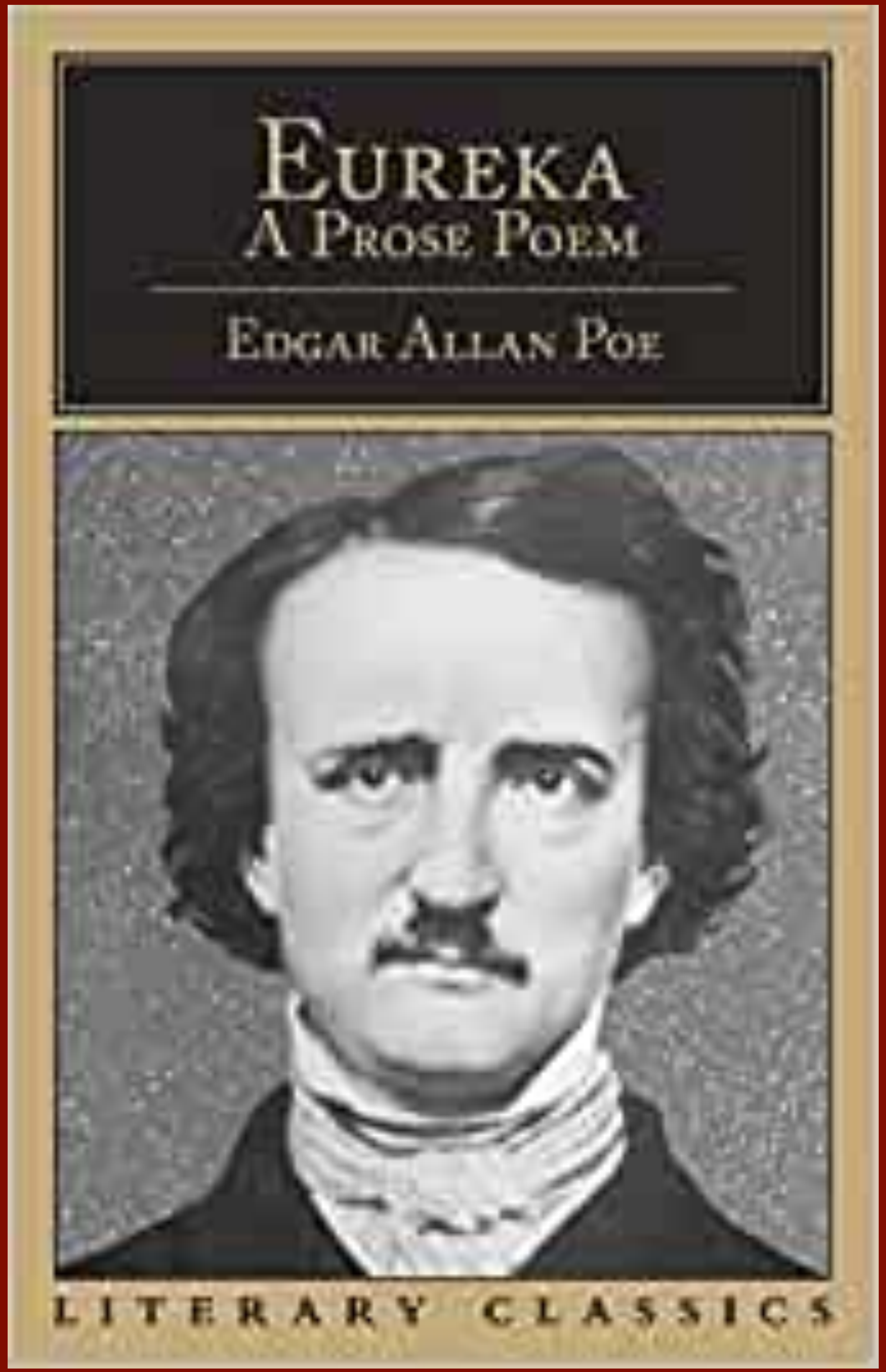

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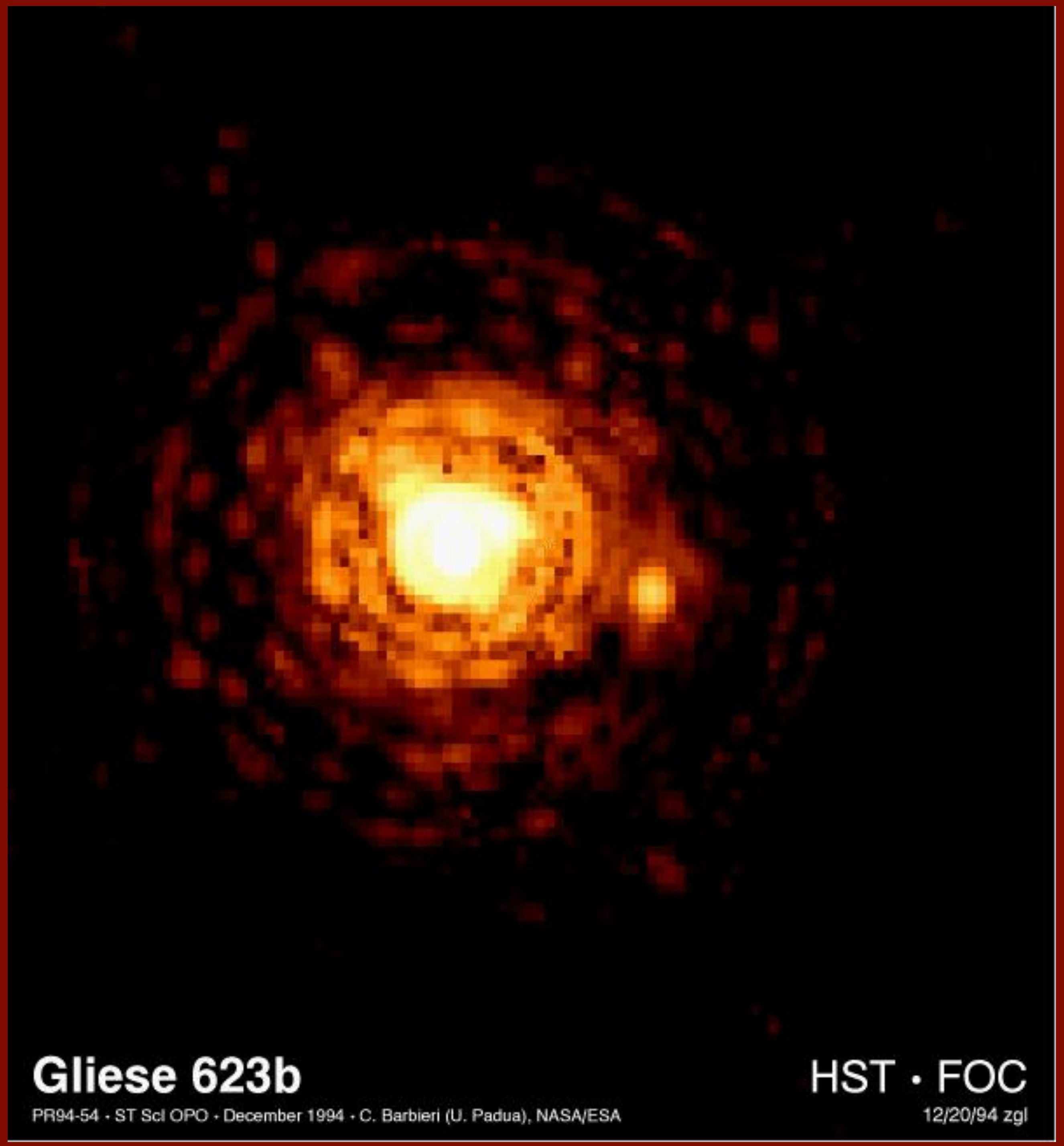
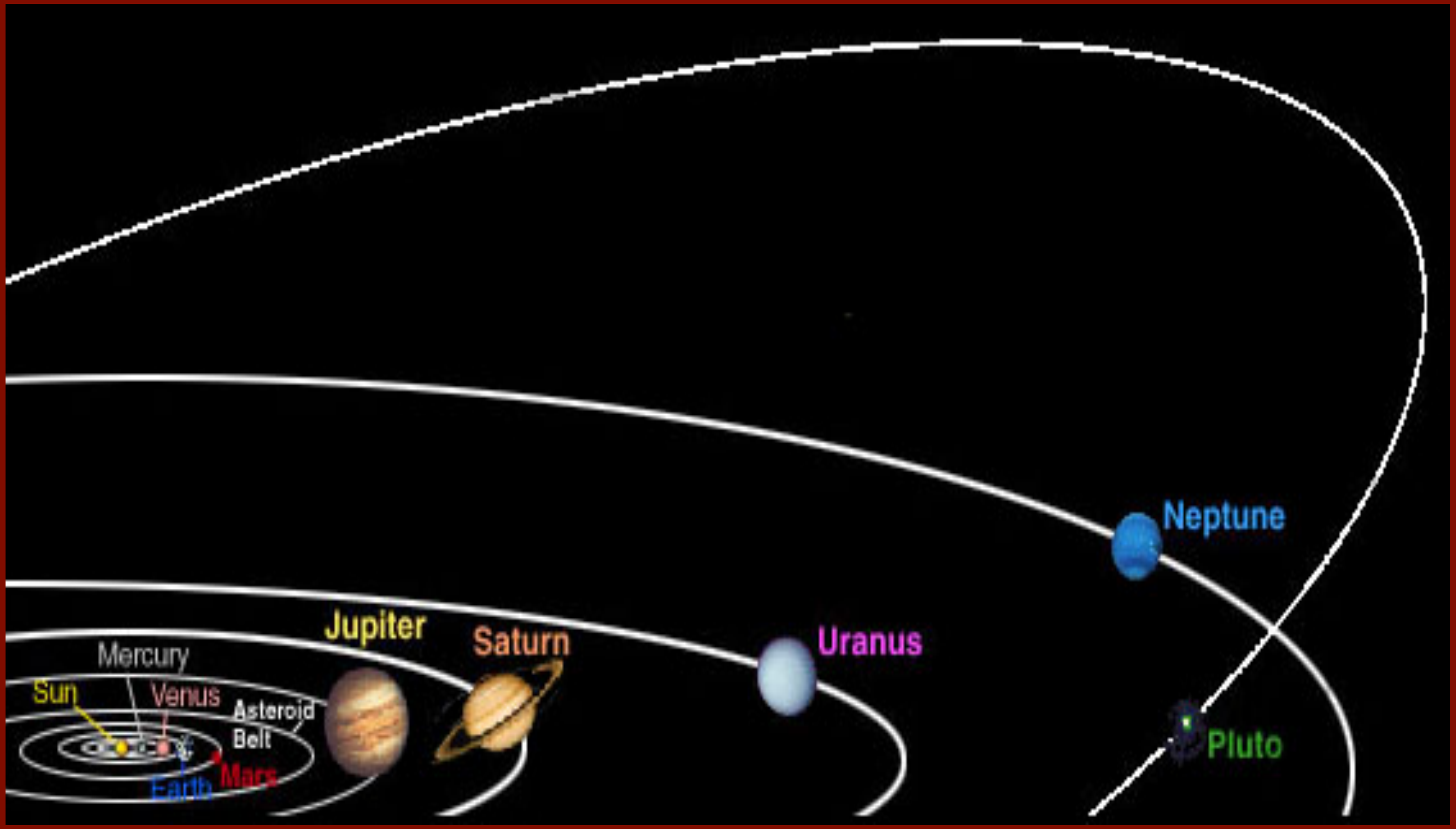


Solar system ...
N=10



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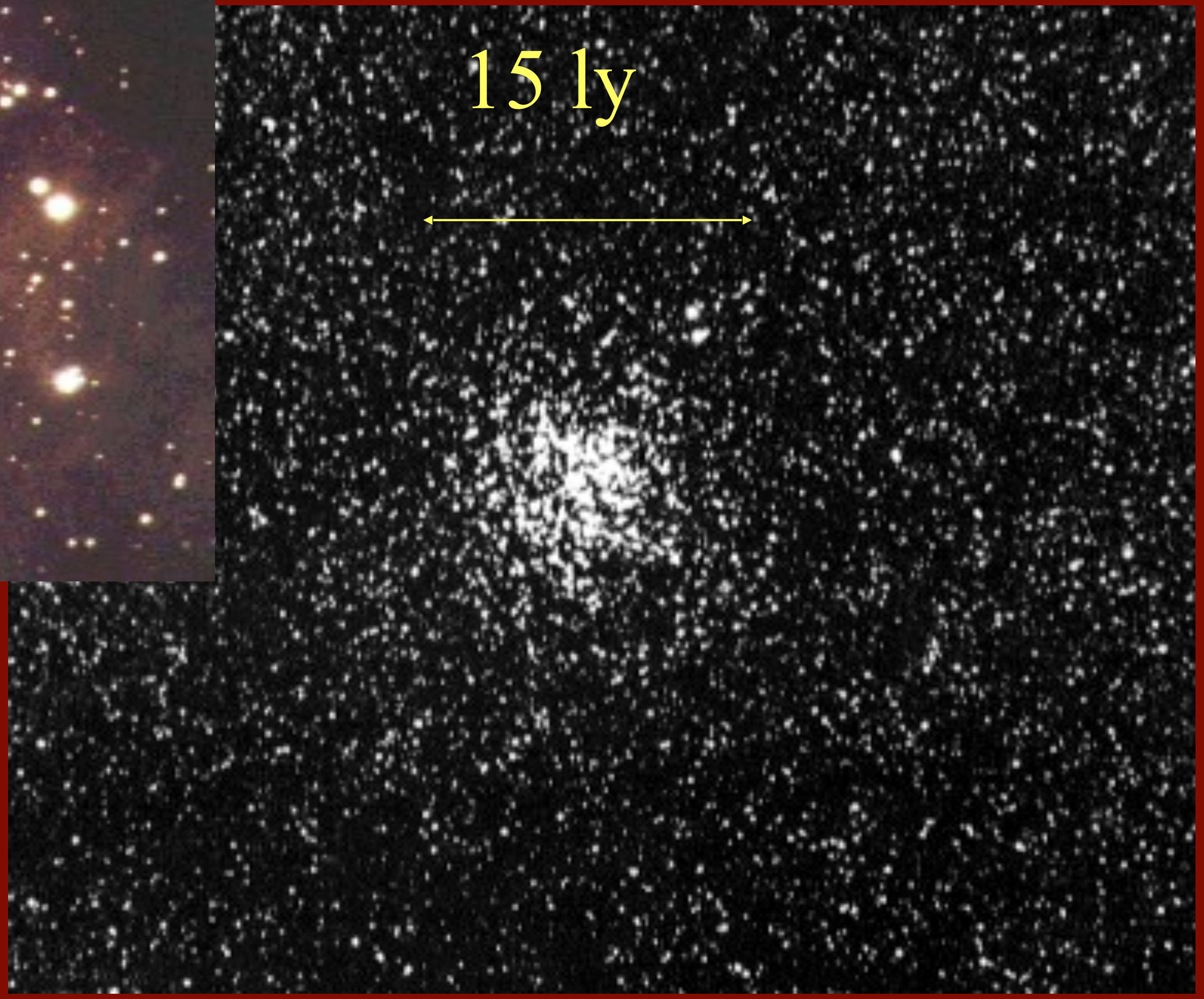

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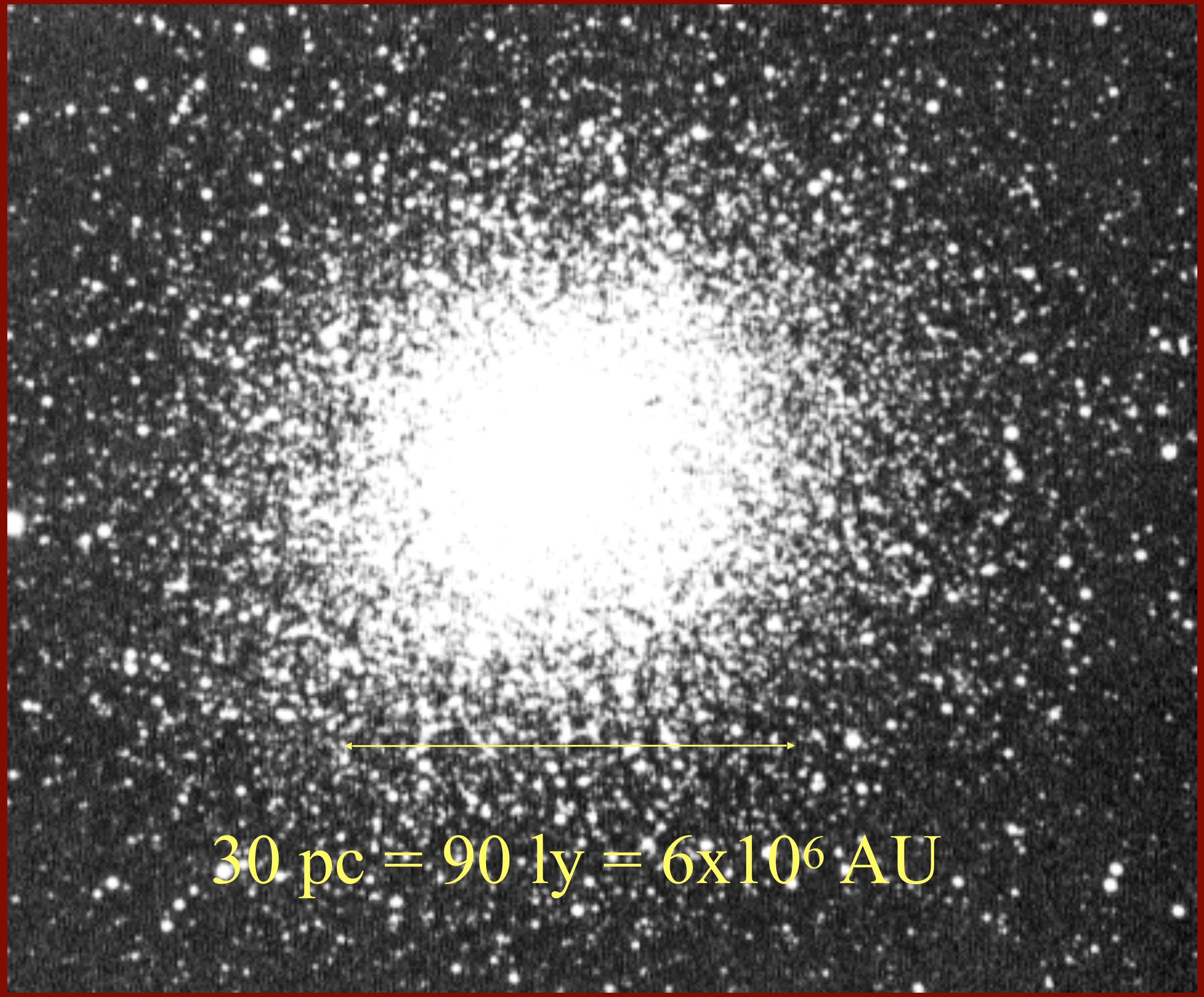

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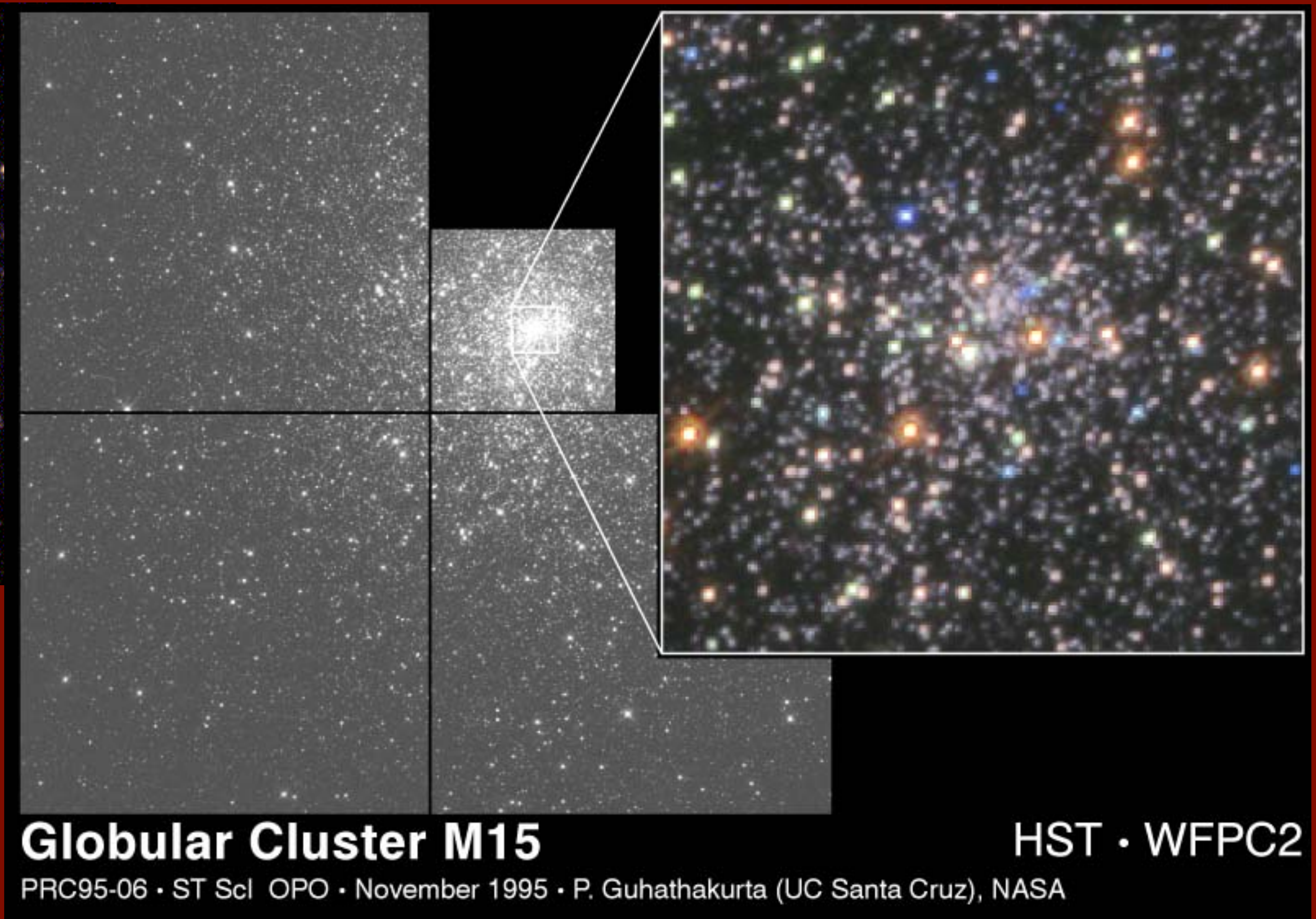

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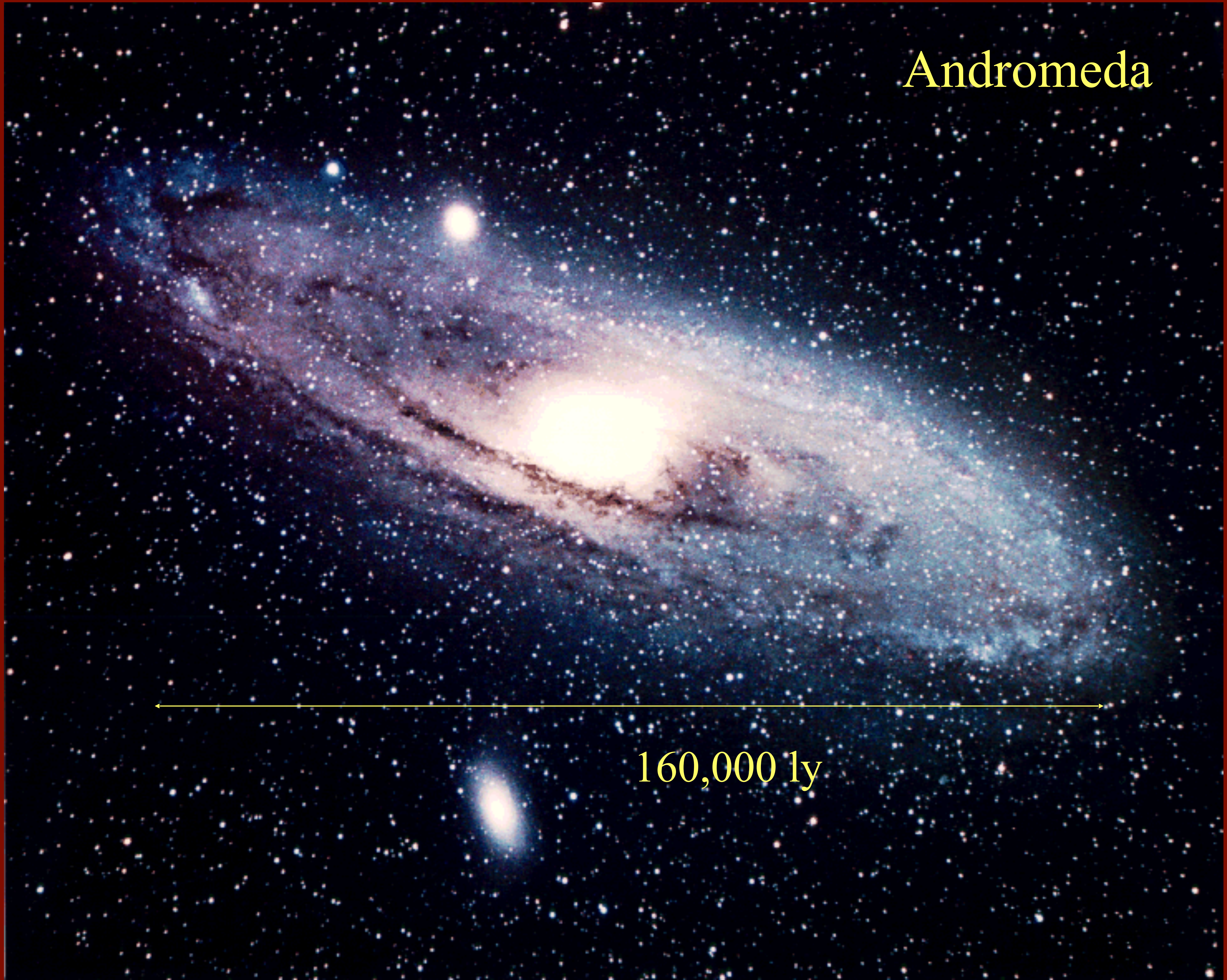

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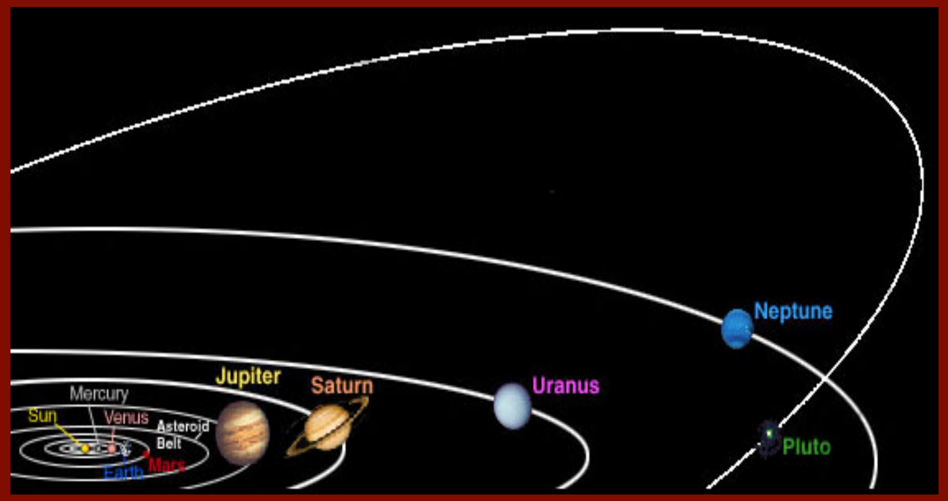

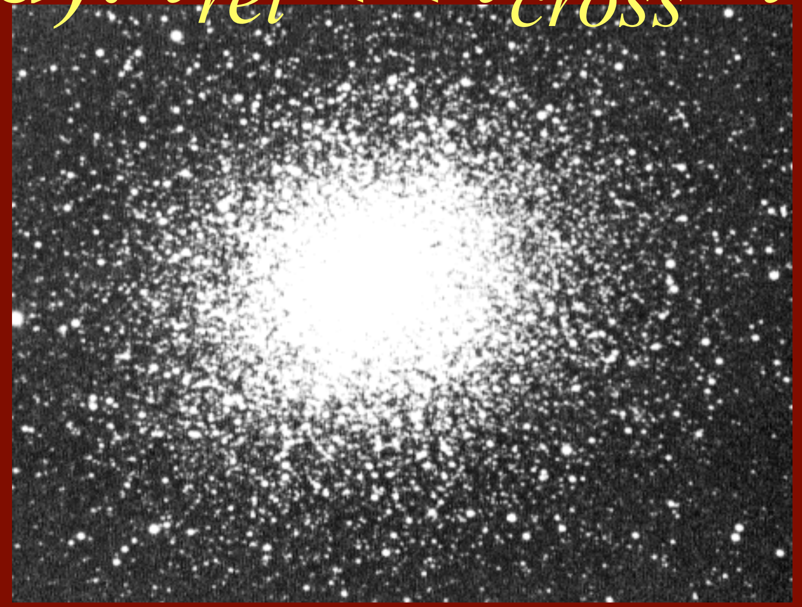

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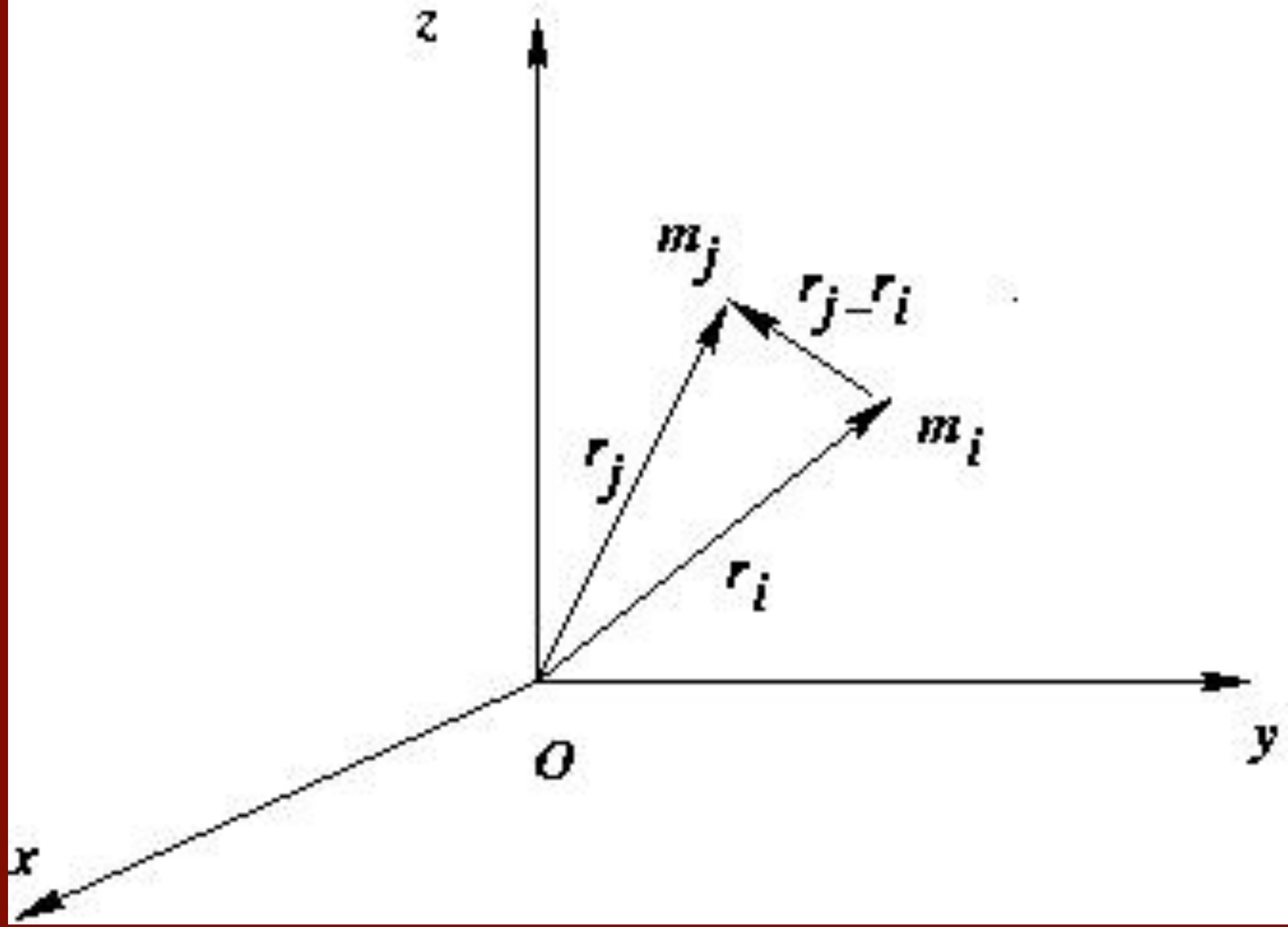

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