Newtonian Dynamics in Astrophysics

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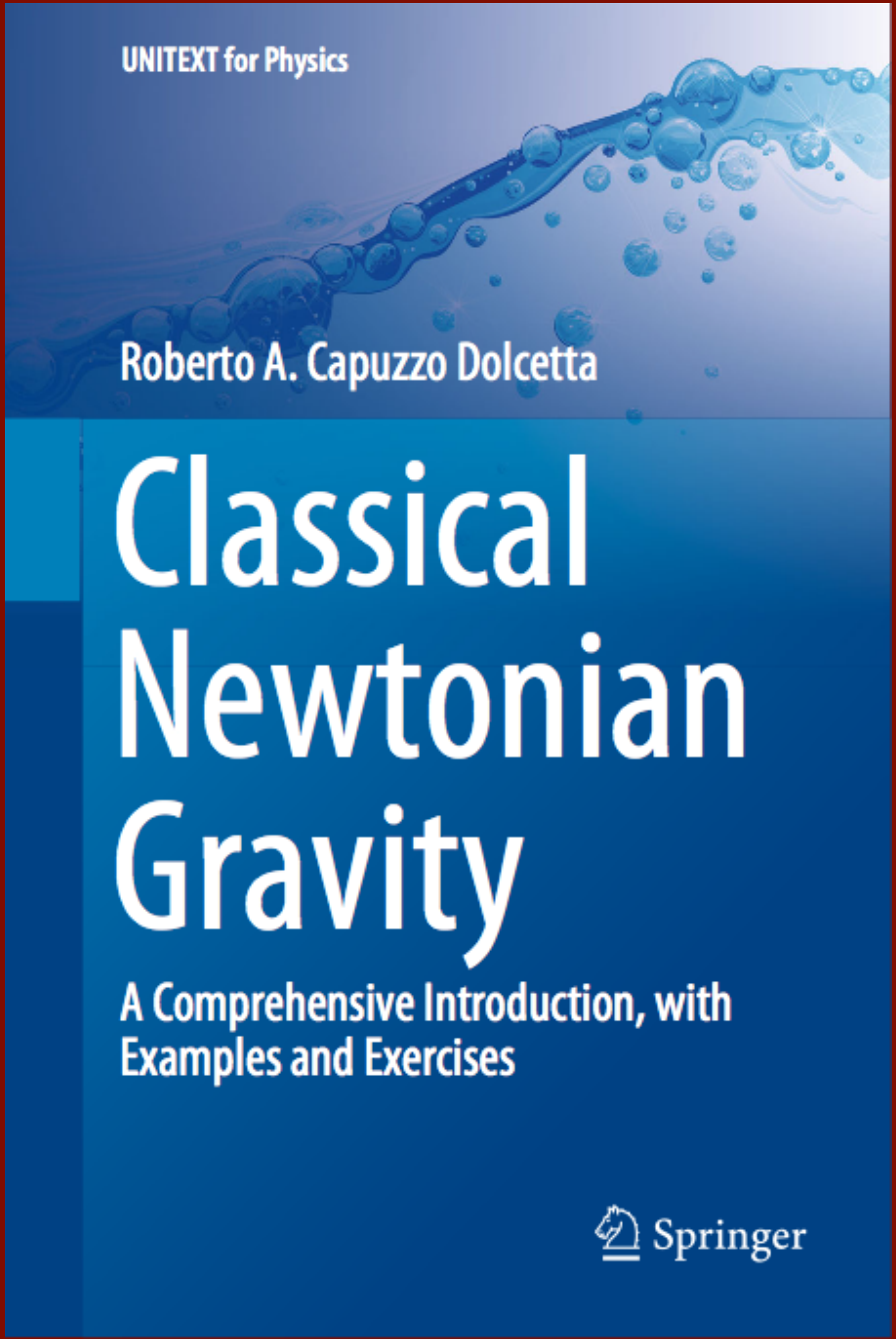

UV divergence approach via *potential smoothing*

$$U_{ij} \propto \frac{1}{r_{ij}} \rightarrow \frac{1}{\sqrt{b^2 + r_{ij}^2}},$$

which leads to $F_{ij} \rightarrow 0 \; for \; r_{ij} \rightarrow 0$

This corresponds to substitution of point masses with Plummer *clouds* of siz

$$\rho(r) = \frac{\rho_0}{\left[1 + (r/b)^2\right]^{5/2}}$$

proper choice of b is crucial ⟶ $b = \alpha \langle d_{cn} \rangle \sim \frac{\alpha}{\sqrt[3]{n}}$

Two-body regularization

Another way to solve the UV divergence was suggested in 1906 by mathematician Levi-Civita.

It is the *2-body regularization*.

The idea behind is:

- to obtain regular differential equations of motions
  and *not* to obtain regular solutions of singular equations

singular eq. of motion: $$\ddot{\mathbf{r}} = -\frac{\mathbf{r}}{r^3}.$$

space-time transform.: $$d\tau = r^{-n}dt,\ n\ arbitrary,$$

$$\dot{\mathbf{r}} = \frac{d\mathbf{r}}{d\tau}\frac{d\tau}{dt} = \frac{d\mathbf{r}}{d\tau}r^{-n},$$

$$\frac{d^2\mathbf{r}}{d\tau^2} - nr^{-1}\frac{dr}{d\tau}\frac{d\mathbf{r}}{d\tau} = -r^{2n-3}\mathbf{r},$$

choosing n=1 $$\mathbf{r}'' - \frac{rr'}{r}\mathbf{r}' = -\frac{\mathbf{r}}{r},$$

in 1D $$x'' - \frac{x'^2}{x} = -1,$$

$$x'' - 2Ex = 1,\ \rightarrow\ u'' - 2Eu = 0, \qquad t(\tau) = \int_0^\tau x(\tau)d\tau$$

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A crucial numerical problem

Evaluating the *Euclidean* distance

$$|\mathbf{r}_i - \mathbf{r}_j| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$$

is expensive…

Need solving the non linear equation: $f(x) = x^2 - r_{ij}^2 = 0$

Various algorithms can be implemented: *Erone*, *Bombelli*, *Newton*, all computatio
demanding… ! ~ 30 flop per pair

Problem #1: Force evaluation $\mathbf{F}_{ij}=\nabla U_{ij} \rightarrow$ 35 flop

With a PE da v= 40 G*flop/sec*,
$t_{ij}=8.75\times10^{-9}$ *sec*

n_f = n. of op. *per* time step

$$n_f = \binom{N}{2} 35\,flop = \frac{N(N-1)}{2} 35\,flop$$

$$t = n_f / \mathrm{v}$$

↗ $N=1000 \rightarrow n_f=1.75\times10^{7}$ *flop*
$t = 4.4\mathrm{x}10^{-4}$ *sec*

→ $N=10^6 \rightarrow n_f=1.75\times10^{13}$ *flop*
$t = 7.3$ *min*

↘ $N=10^{11} \rightarrow n_f=1.75\times10^{23}$ *flop*
$t = 4.4\times10^{12}$*sec* = 0.14 *Myr!*

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Problem #2: Extension of simulations

Coarse-grain: violent relaxation t_{cr}

Fine-grain: "collisional" relaxation t_{rel}

Time scales: $t_{cr} \sim \frac{R^{3/2}}{\sqrt{GM}} < t_{rel} \sim \frac{1}{10}\frac{N}{\ln N} t_{cr} < t_{ev} \sim 150\ t_{rel}$,

- Choosing N=10^6, to reach *relaxation* takes ~7238 t_{cr}
- With 1000 time steps per t_{cr} this means $7.238 \times 10^6 \times 7.3\ \text{min} \simeq 100\ \text{yr!}$

The age of a globular cluster (~13 Gyr) is ≈ $2{\times}10^5\ t_{cross}$≈ 2760 years of computation!

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Profiling in a typical gas+stars simulation

	CPU time (%)
Gravitational forces evaluation	**60**
Fluid-dynamical quantities evaluation	**25**
Time integration	**15**

Solutions:

Resort to grid methods, like P3M methods (Poisson's eq. on a grid via FFT *and* a local direct summation)

or

Resort to multipole expansions ⟶ tree algorithms

Resort to supercomputers

or (partially…)

to *accelerators* like the Graphic Processing Units (GPUs)

nvidia Tesla V100S

- 21×10^9 transistors
- 5120 cores, 1.3 Ghz/core, 32 GB memory
- speed (teraFLOPS): 8.2 double p., 16.4 single p., 130 deep learning
- power 250 W.
- Cost ~ 11,000 €

End of Lectures