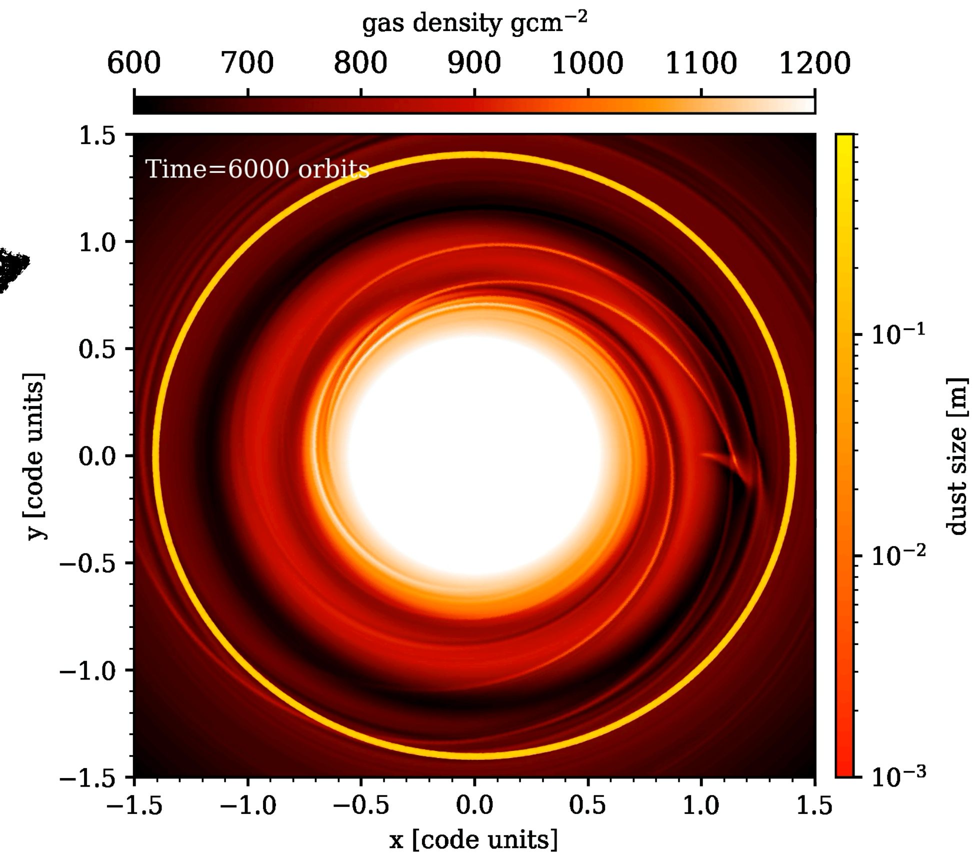
Selected chapters on astrophysics

Lecture 1

¨Overview of recent advances in planetary migration: from theoretical models to high-resolution 3D multi-fluid simulations¨

PhD. Raúl O. Chametla



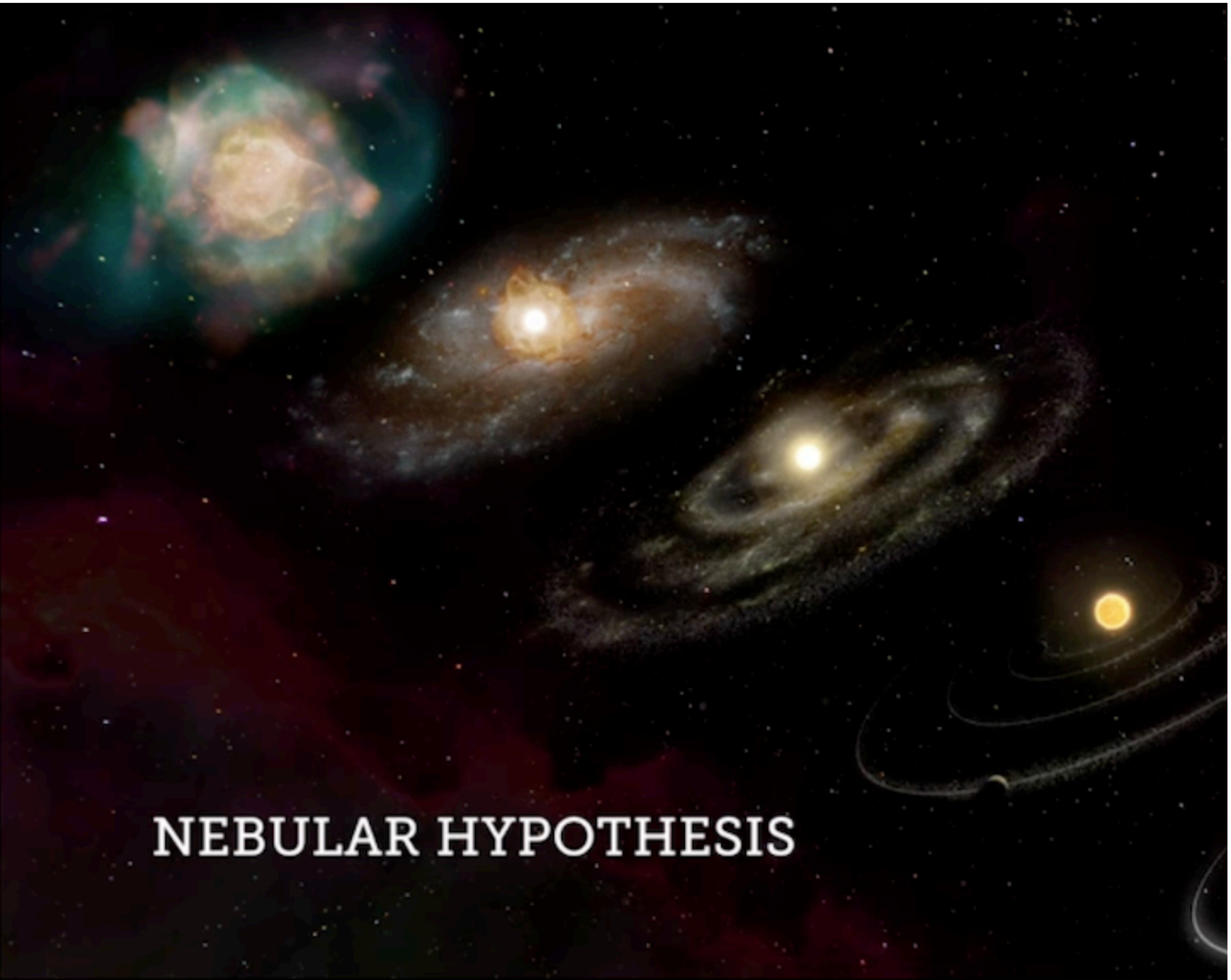
$$\Gamma_m = m\pi^2 \left(\frac{\Sigma_m}{r dD/dr} \left[r \frac{d\Phi_m}{dr} + \frac{2\Omega}{\Omega - \Omega_p} \Phi_m \right] \right)_{r=r_L}$$

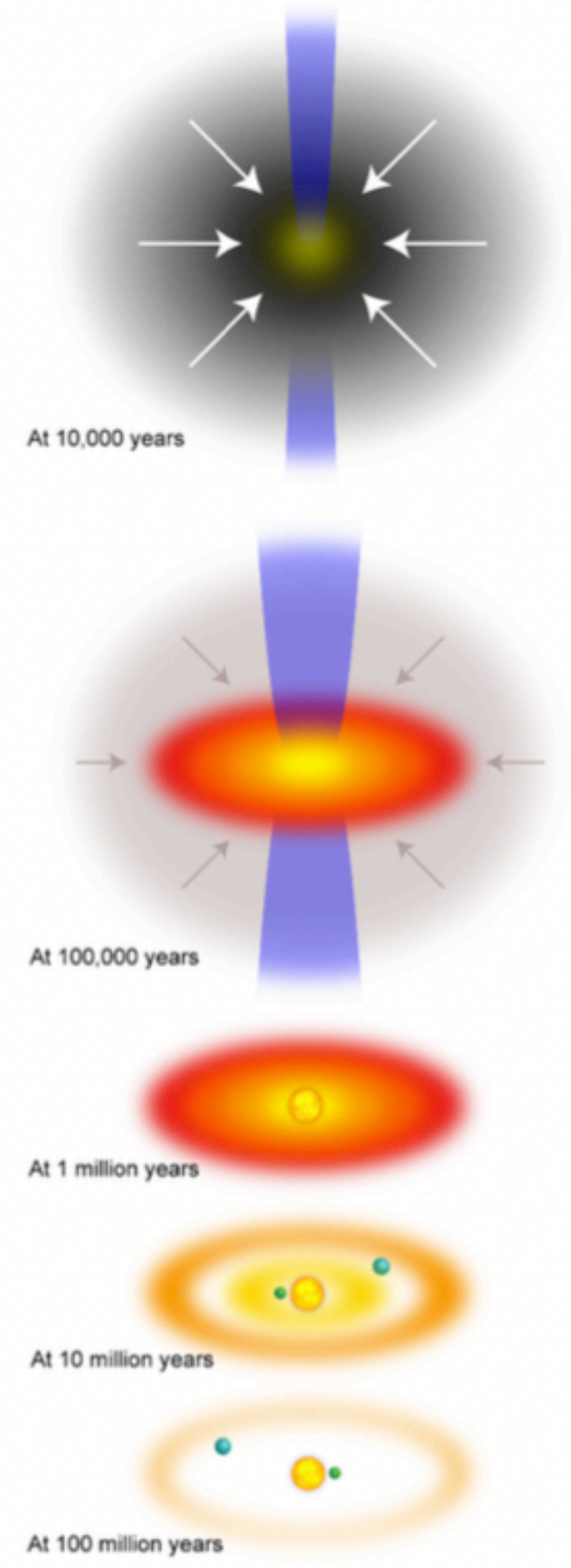
$$D(r) = \kappa(r)^2 - m(\Omega_p - \Omega(r))$$

Outline

- *Introduction*
- *Types of planetary migration*
- *Numerical models on planetary migration*
- *An introduction to FARGO3D and PLUTO codes*
- *Numerical tools*

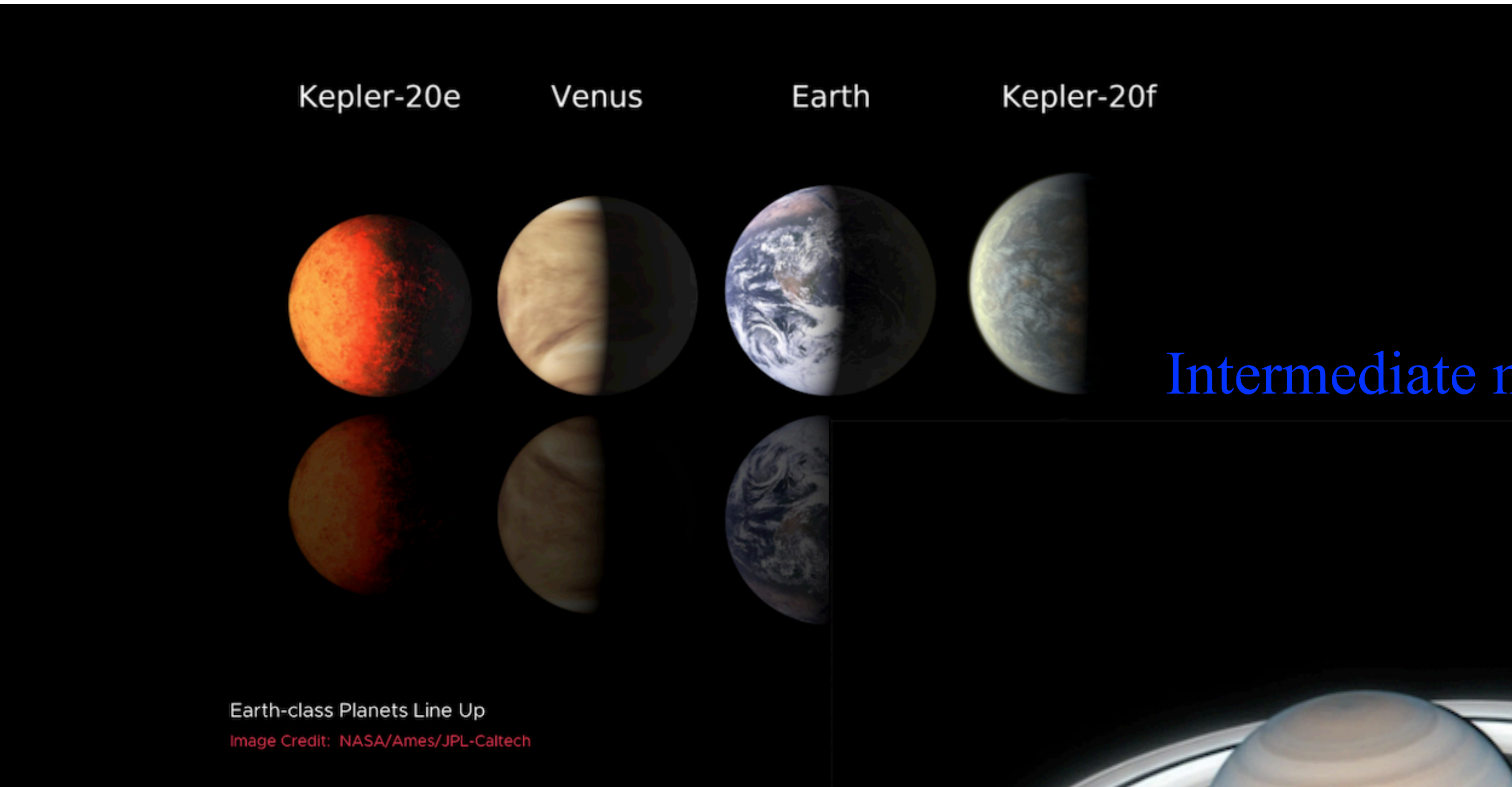
Introduction



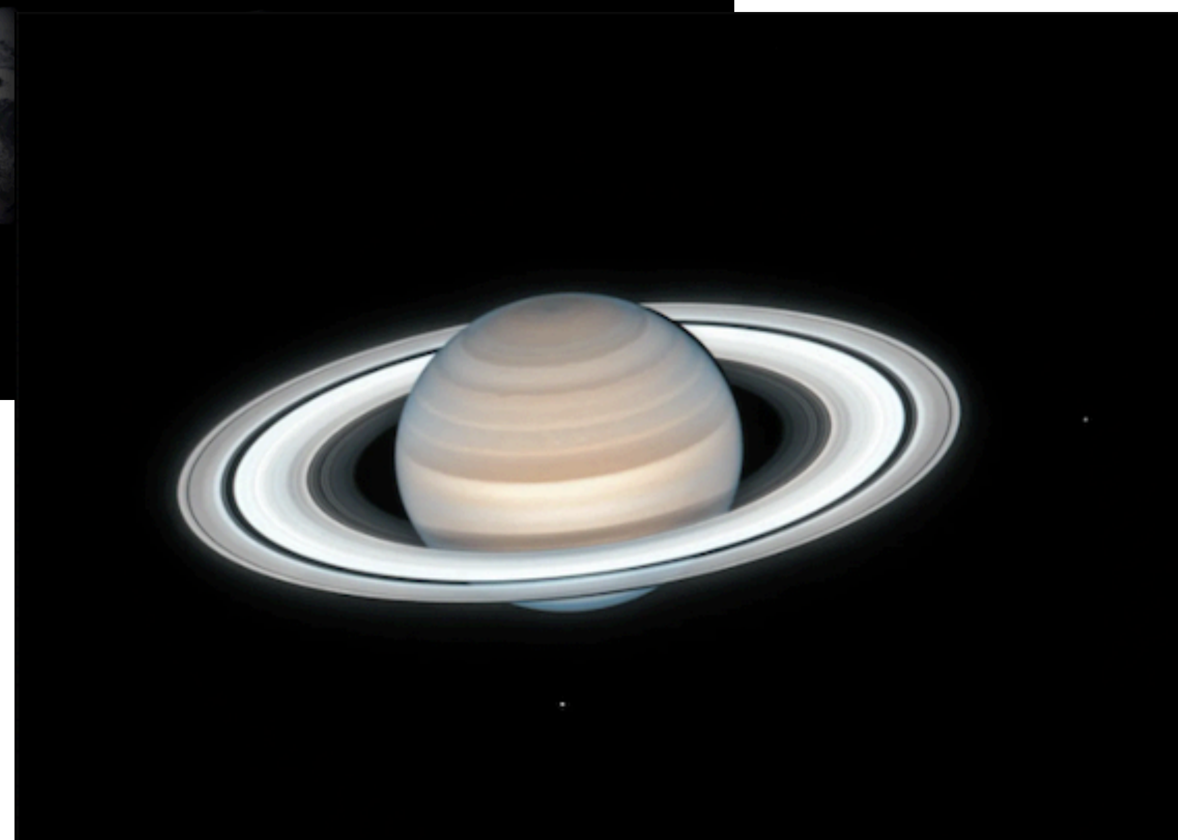


Introduction

Low mass planets $M_p \leq 15M_{\oplus}$

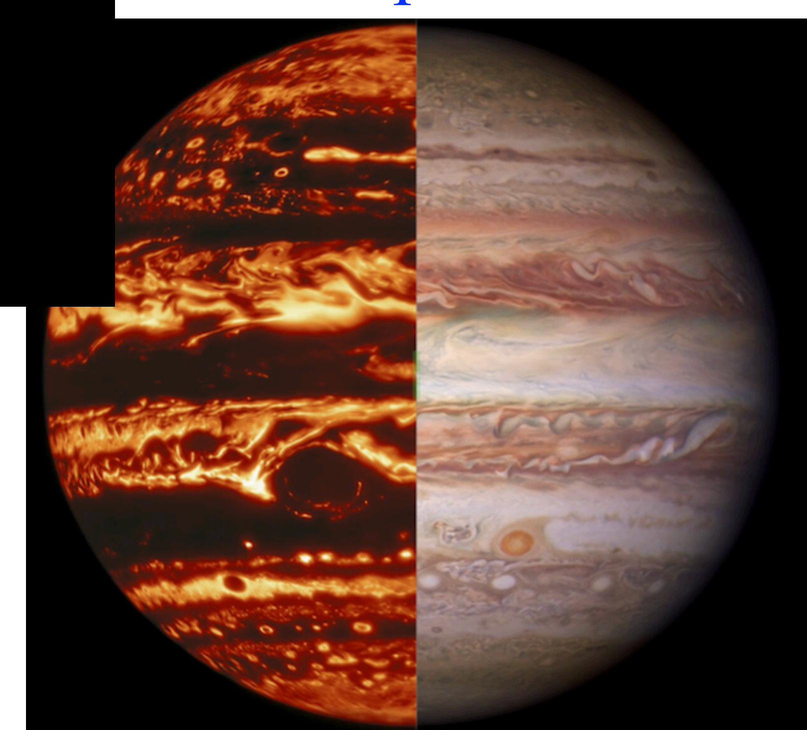


Intermediate mass planets $15M_{\oplus} > M_p < 1M_{Jup}$

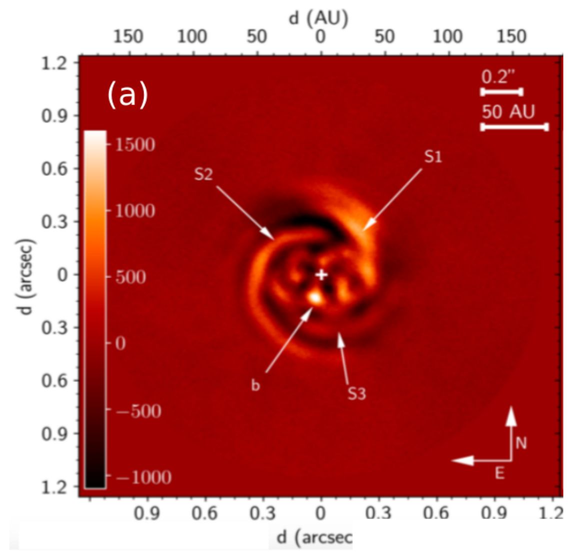


$M_p \geq 1M_{Jup}$

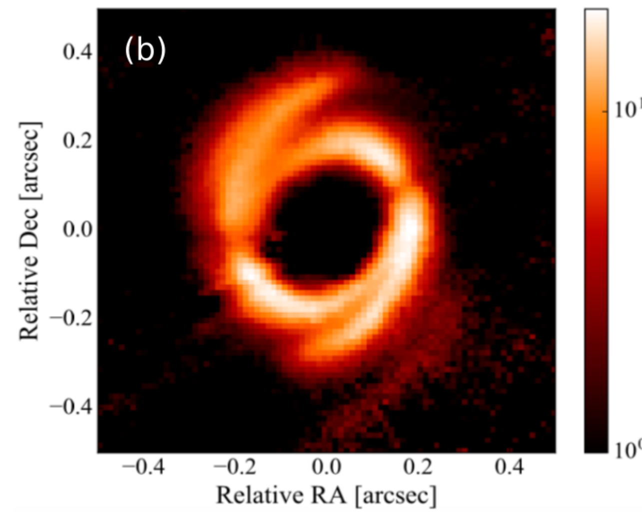
Massive planets



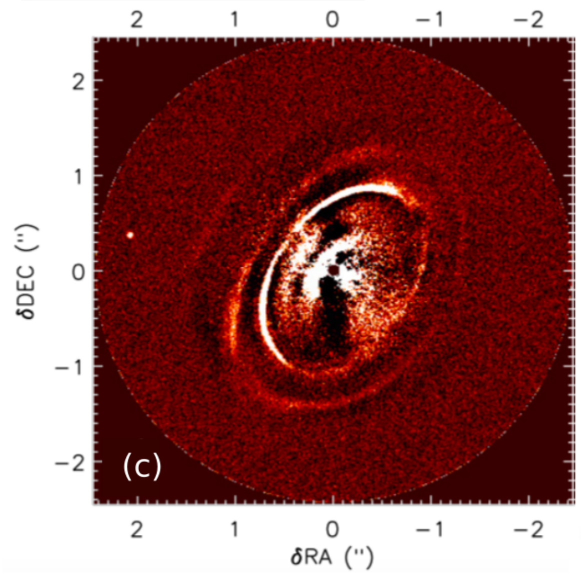
Observed Protoplanetary Disks



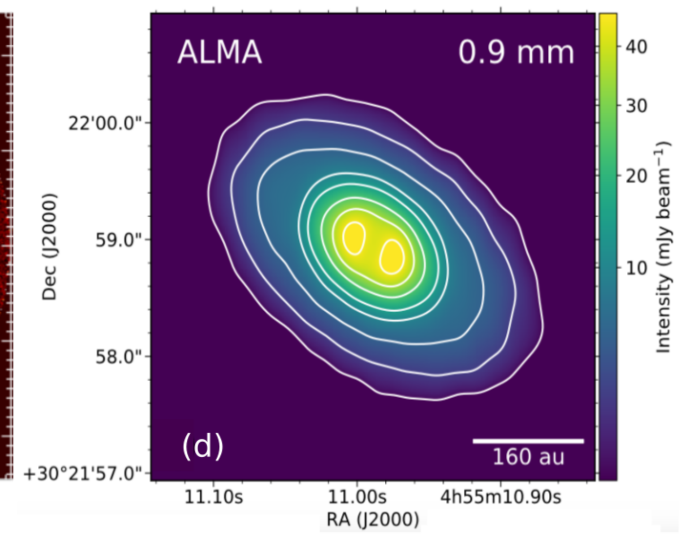
(a) MWC 758 (Reggiani et al. 2018).



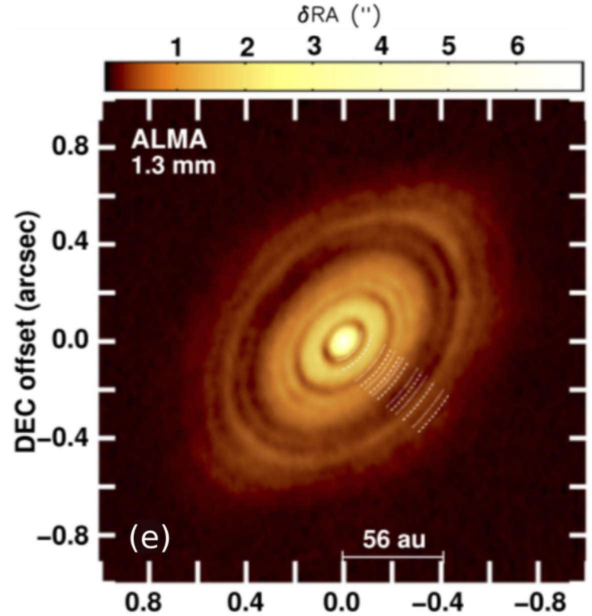
(b) HD100453 (Benisty et al. 2017).



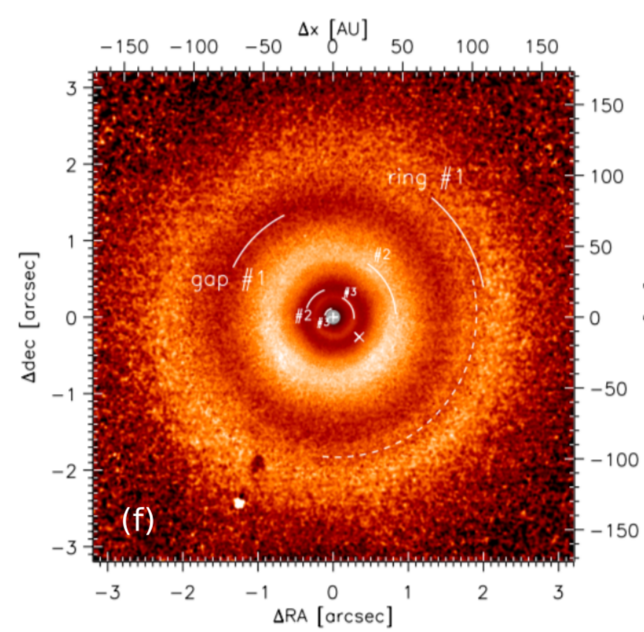
(c) RX J1615.3-3255 (Boer et al. 2016).



(d) GM Aurigae (Macías et al. 2018).



(e) HL-TAU (Carrasco-González et al. 2016).

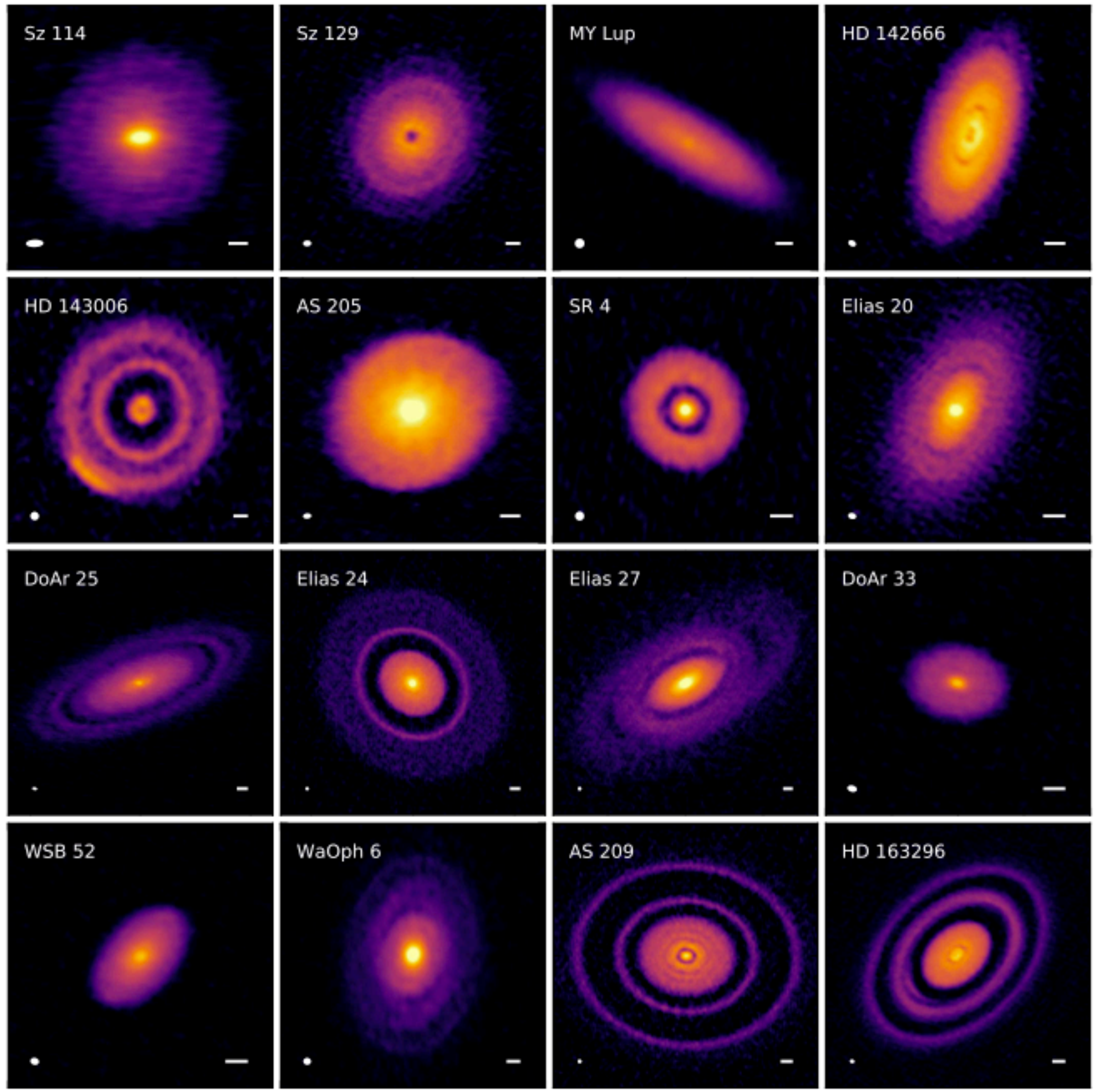


(f) TW Hya (Boekel et al. 2017).

Protoplanetary gaseous dusty discs

Andrews et al. (2018)

Protoplanetary discs are composed of gas and a small fraction of dust (~ 1% of the total gas).



Recent observational studies show different structures in dust density. These can be caused by the interaction of the protoplanetary disk with planets of different masses forming and/or migrating.

Figure 1. Gallery of continuum emission images (1.25 mm) for the discs in the DSHARP sample.

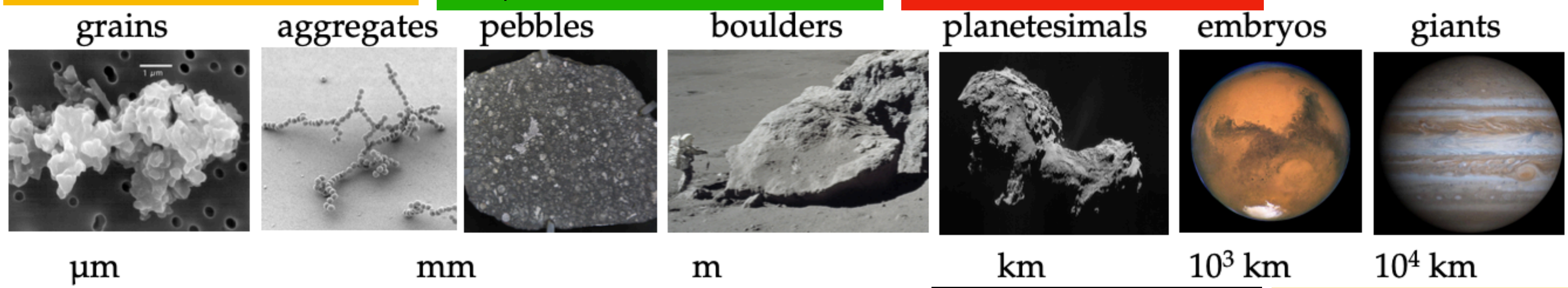
Dust in protoplanetary disks

Grains - up to 1 µm.
These are either inherited from the collapse of the parent molecular cloud or have been condensed from the cooling protoplanetary nebula.

Pebbles - sub-mm to meter.
We simply mean compact objects larger than grains but smaller than boulders. Chondrules (~100 µm) and Calcium-Aluminium Inclusions (CAIs; ≈cm), both found in meteorites, are examples.

Planetesimals - sub-km to several $\sim 10^2$ km.
These are bodies that start to bind material by their gravity. They are traditionally considered to be the planetary 'building blocks'

Rocky planets (not shown in this Figure) - up to several Earth masses.



Aggregates/fractals -
~µm up to a unspecified upper size.
The first product of dust coagulation.
They can be very porous.

Boulders - meter size.
The meter-size scale is approximately the dividing line between particles that are strongly influenced by the gas and particles that travel in Kepler orbits.

(planetary)Embryos
-10^3km to~$10M_\oplus$.
Bodies that emerge out of a planetesimal population due to a runaway growth coagulation process.

Gas giants - ? $100M_\oplus$.
Consist mostly of gas, but (in the core accretion model) thought to have formed only after the creation of a ≈10 Earth mass solid core.

Types of planetary migration

Basic mechanism of planetary migration

The presence of a planet orbiting the star creates a non-axisymmetric time varying gravitational potential.

The gas reacts to this perturbation in the potential by the formation of density waves.

These density waves create additional perturbations in the potential which are seen by the planet as well.

The torques originating from these perturbations change the planet's angular momentum and give rise to migration.

We here consider migration due to the gravitational interaction with the gaseous disk only.

Migration can also occur due to the interaction with the planetesimals disk.

Orbital decay due to direct gas drag is negligible at planetary masses.

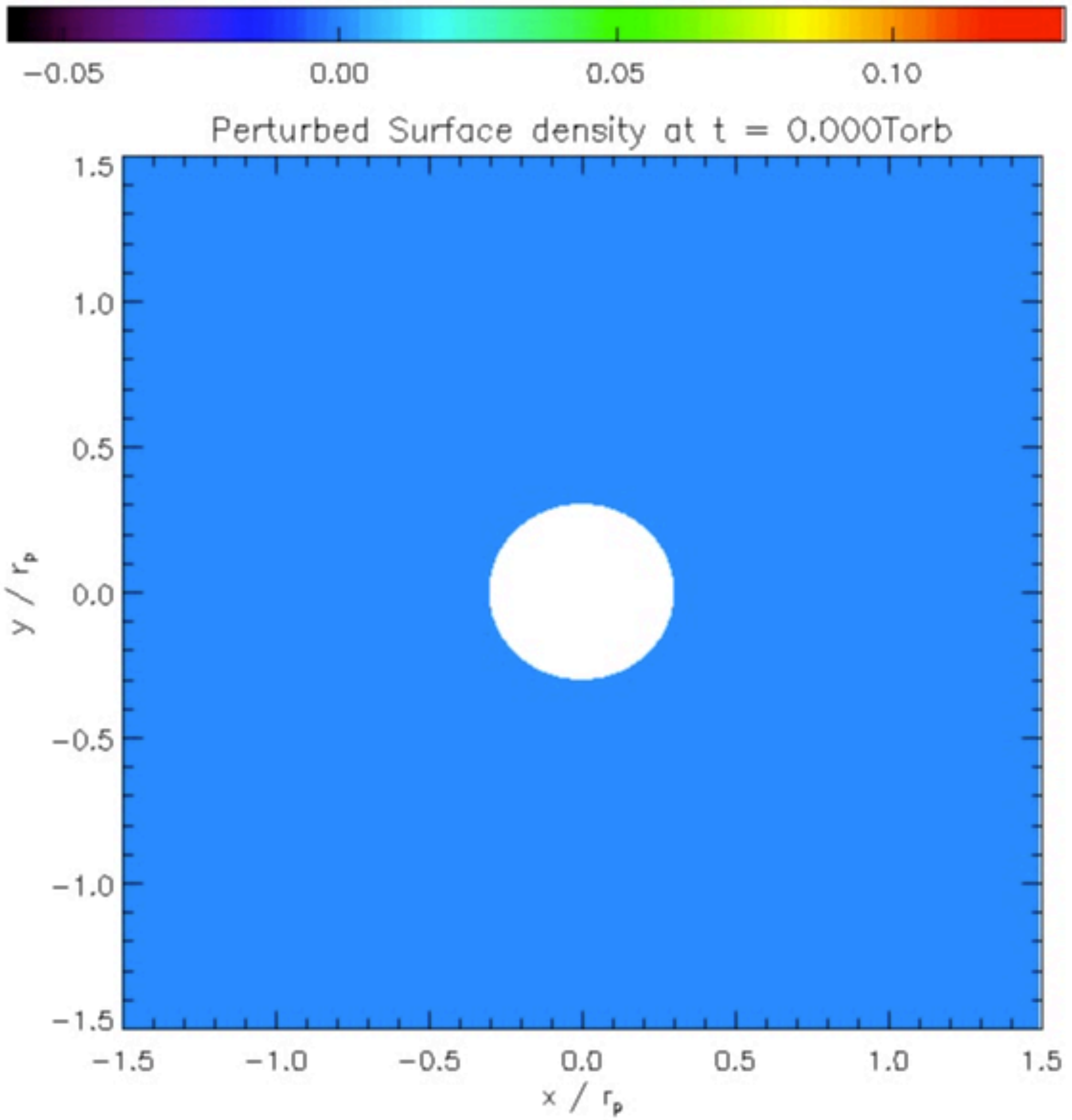
Some more used notation:

Ω_K	Keplerian orbital frequency
$q \equiv \frac{M_p}{M_\star}$	Mass ratio between the planet and the star
H	The disk thickness
$h \equiv H(r)/r$	Aspect ratio of the disk
$\Sigma(r)$	Surface density of the disk
ρ	Volumetric density of the gas
$\mathbf{v}$	Gas velocity
r_p	Orbital radius of the planet

Types of planetary migration

For small mass planets the density waves propagate through the disk

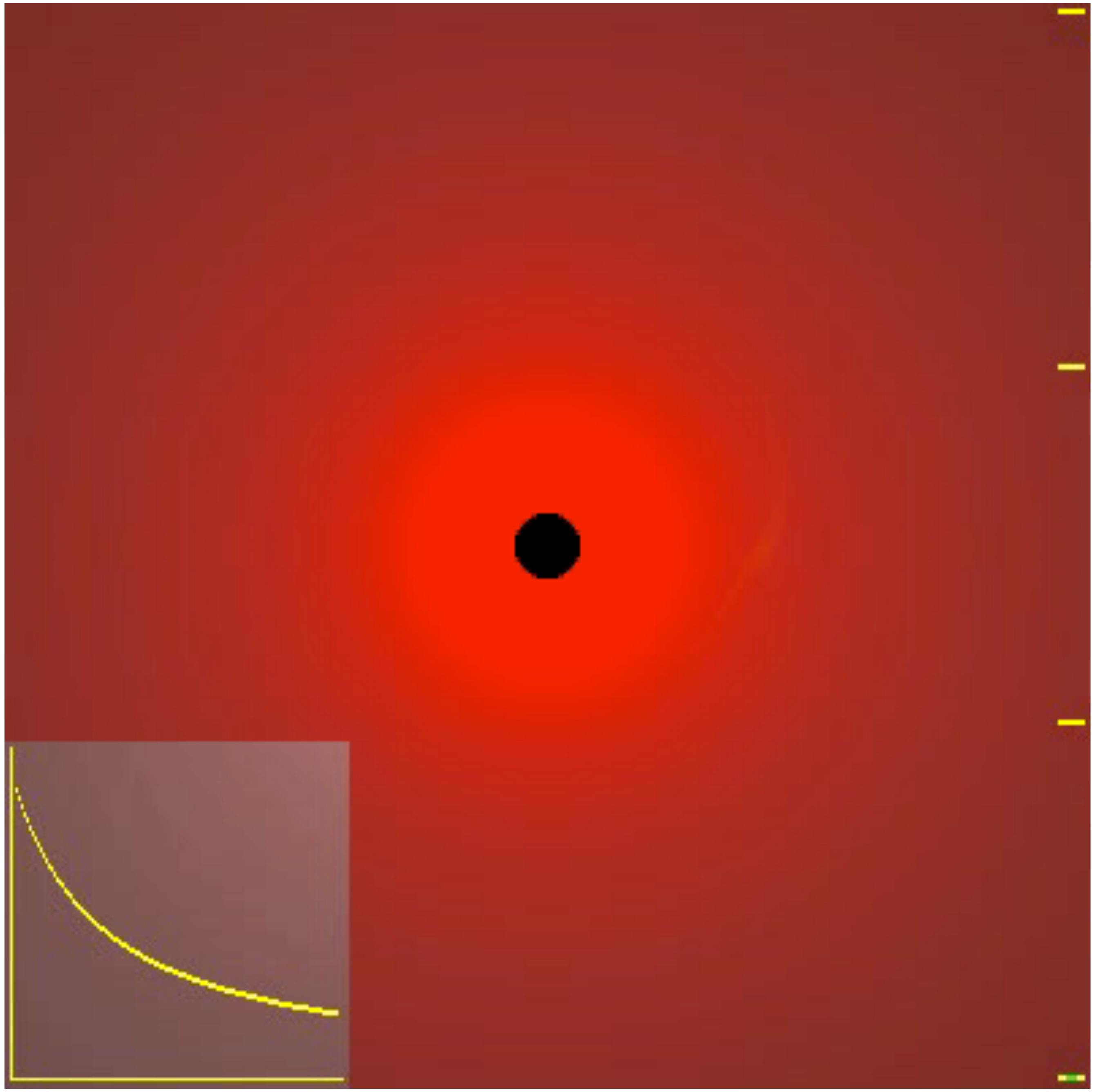
Type I migration
Migration mode of small mass planets, no gap



- For larger mass planets, a gap opens in the disk

Type II migration

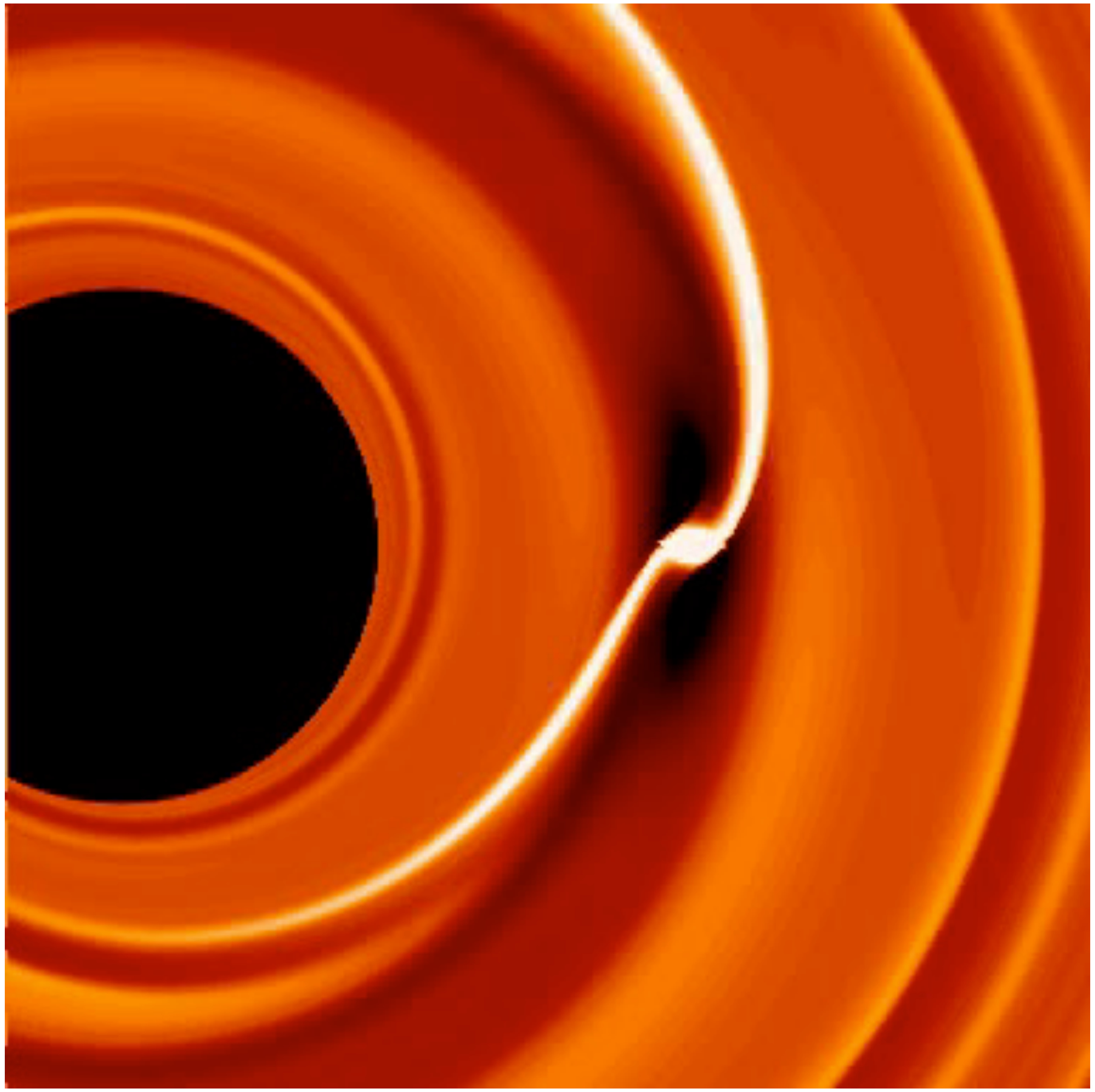
Migration mode of large mass planets, with gap

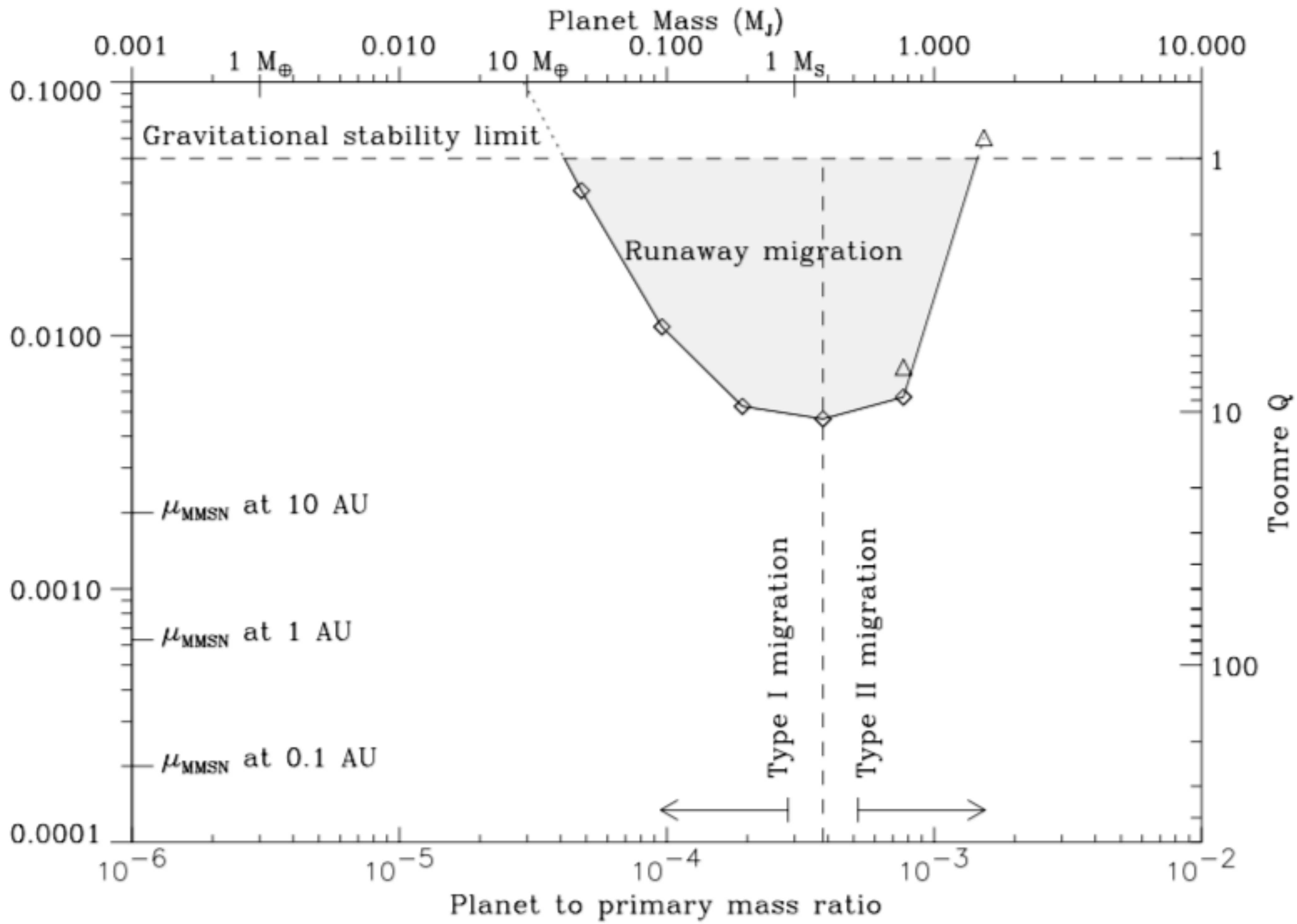


- In this type of migration, sub-giant planets with masses of the order of the mass of the planet Saturn are considered embedded in massive discs.

Type III migration (migration runaway)

Gap is partially opened.





Planet Mass (M_J)
0.001
1 $M_\oplus$
0.010
10 $M_\oplus$
0.100
1 M_S
1.000
10.000
0.1000
0.0100
0.0010
0.0001
Gravitational stability limit
Runaway migration
μ_{MMSN} at 10 AU
μ_{MMSN} at 1 AU
μ_{MMSN} at 0.1 AU
Type I migration
Type II migration
1
10
100
Toomre Q
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
Planet to primary mass ratio

Migration type I: Linear theory

The hydrodynamical equations that govern the flow are the continuity, momentum and energy equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \cdot \mathbf{v} \right) = -\nabla P + \mathbf{F}_{ext},$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{v}) = -P \nabla \cdot \mathbf{v}$$

$$\rho = \rho_0 + \rho_1$$

$$e = e_0 + e_1$$

$$P = P_0 + P_1$$

$$\mathbf{v} = (u, v, w)$$

$$u = u_1$$

$$v = v_0 + v_1$$

$$w = w_1$$

Small perturbations

Resonant torques

Imagine a planet in orbit about a star. The rotation frequency of the planet is given by its Keplerian frequency,

$$\Omega_p = \sqrt{\frac{GM_\star}{r_p^3}}$$

Then, there are special resonant places in the disk. Two types must be distinguished:

1) Corotation resonance: located where $\Omega = \Omega_p$

   If the disc is Keplerian (i.e., if we neglect gas pressure and self-gravity), then the corotation resonance is found at the planet's orbital radius.

2) Lindblad resonances: located where $m(\Omega - \Omega_p) = \pm\,\kappa$

- with m an integer.

- And $$\kappa^2 = R\frac{d\Omega^2}{dR} + 4\Omega^2$$

a) for the + sign (rotation faster than planet): The *inner* Lindblad resonances

b) the - sign (rotation slower than planet): The *outer* Lindblad resonances

The actual position of these resonances can be readily obtained:

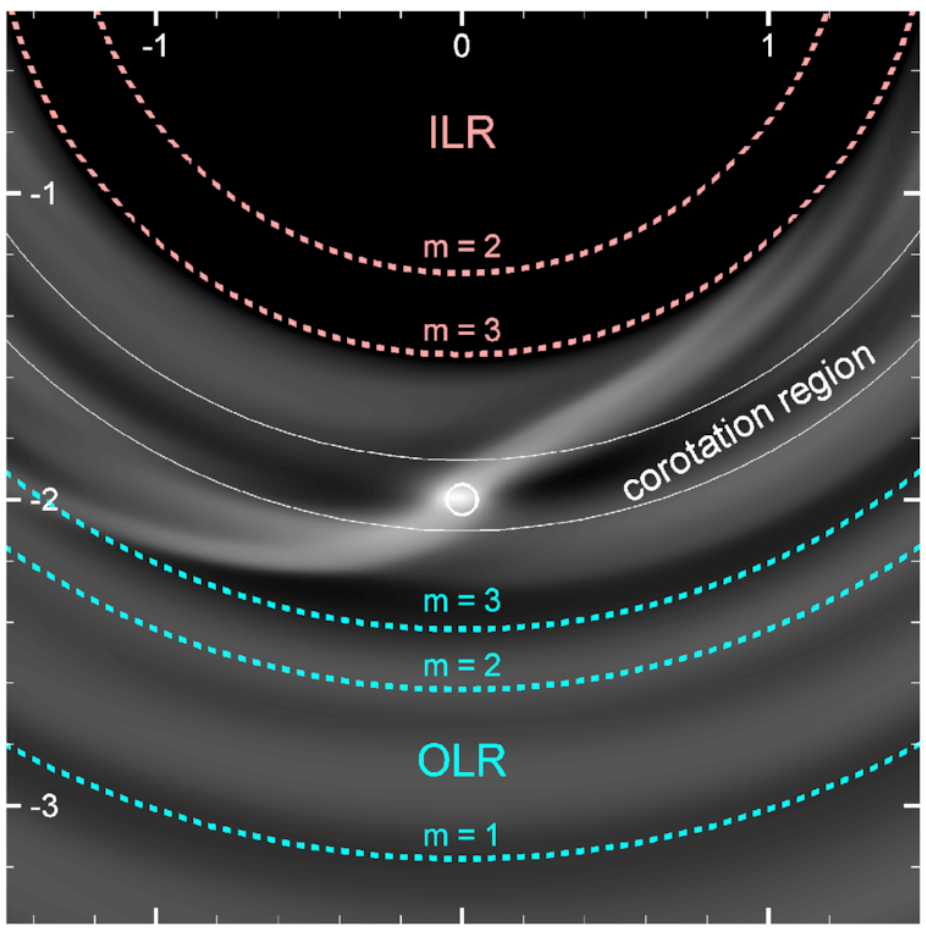


Figure. A schematic diagram of various resonance locations for a planet of mass $M_p = 15\, M_\oplus$.

$$r_{ILR} = r_p \left(\frac{m}{m-1} \right)^{-2/3}$$

$$r_{OLR} = r_p \left(\frac{m}{m+1} \right)^{-2/3}$$

THE ASTROPHYSICAL JOURNAL, **233**:857–871, 1979 November 1

THE EXCITATION OF DENSITY WAVES AT THE LINDBLAD AND COROTATION RESONANCES BY AN EXTERNAL POTENTIAL[1]

PETER GOLDREICH[2] AND SCOTT TREMAINE[3]
Received 1978 September 22; accepted 1979 May 2

ABSTRACT

We calculate the linear response of a differentially rotating two-dimensional gas disk to a rigidly rotating external potential. The main assumptions are that the sound speed is much smaller than the orbital velocity and that the external potential varies on the scale of the disk radius. We investigate disks both with and without self-gravity.

The external potential exerts torques on the disk only at the Lindblad and corotation resonances. The torque is positive at the outer Lindblad resonance and negative at the inner Lindblad resonance; at corotation the torque has the sign of the radial gradient of vorticity per unit surface density. The torques are of the same order of magnitude at both types of resonance and are independent of the sound speed in the disk.

The external potential also excites density waves in the vicinity of the Lindblad and corotation resonances. The long trailing wave is excited at a Lindblad resonance. It transports away from the resonance all of the angular momentum which is deposited there by the external torque. Short trailing waves are excited at the corotation resonance. The amplitudes of the excited waves are the same on both sides of the resonance and are small unless the disk is almost gravitationally unstable. No net angular momentum is transported away from the corotation region by the waves. Thus the angular momentum deposited there by the external torque accumulates in the gas.

We briefly discuss the behavior of particle disks and prove that the external torques on particle disks are identical to those on gas disks.

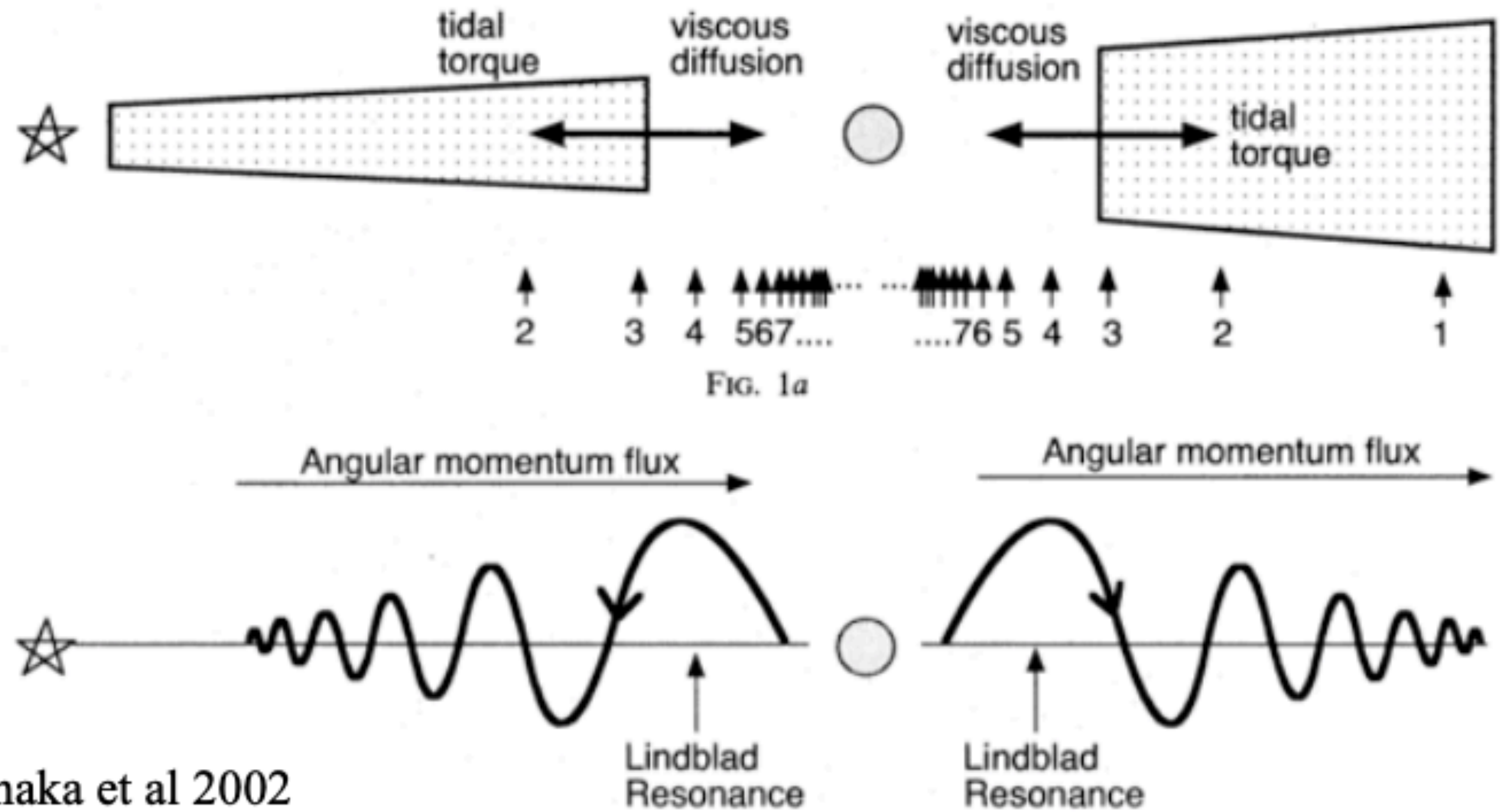
Subject headings: galaxies: structure — hydrodynamics — stars: stellar dynamics

A full analysis considers the evolution of linear perturbations in a fluid disk, and was first applied to planet-disc interactions by Goldreich & Tremaine (1979).

The first step is to linearize the hydrodynamic equations, and to decompose the perturbation to the Keplerian potential due to the planet into Fourier modes. For example, the potential of the planet is expressed in terms of azimuthally periodic components characterized by *m*, the inverse wave number.

$$\Phi_m(r, \phi, t) = \Phi_m(r)\cos[m(\phi - \Omega_p t)]$$

The next step is to compute the response of the disc to the perturbations, and from this it is possible to calculate the total torque on the planet by summing up the torques originating at each resonance.



Tanaka et al 2002

The important result of such complex calculations is that the total angular momentum exchange between disc and planet can be expressed as the sum of the torques exerted at discrete resonances in the disc.

These resonances correspond to the points in the disc where the planet excites waves, which take the form of spiral density waves.

The importance of the resonances can be understood by the following argument: the torques at non-resonant locations in the disc do not interfere constructively, and consequently cancel out when averaged over the orbit.

Using such an approach, Goldreich & Tremaine (1979) derived an expression for the torque at the Lindblad resonance:

$$\Gamma_m = m\pi^2 \left(\frac{\Sigma_m}{r dD/dr} \left[r \frac{d\Phi_m}{dr} + \frac{2\Omega}{\Omega - \Omega_p} \Phi \right] \right)_{r=r_L}$$

where $D(r) = \kappa(r)^2 - m(\Omega_p - \Omega(r))$.

Since then, several groups performed similar calculations. For example, Tanaka et al (2002) studied the three-dimensional interaction between a planet and a three dimensional isothermal disk. Assuming that the perturbation are small enough, they used linear theory to derive the relevant torques. After some significant calculations, they obtain the following expression for the torque exerted on the planet:

$$\Gamma = -(1.364 + 0.541\sigma)\left(\frac{M_p}{M_\star}\frac{r_p\Omega_p}{c_s}\right)^2 \Sigma_p r_p^4 \Omega_p^4$$

Here σ is defined by the exponent of the assumed power law surface density of the disk

$$\Sigma = \Sigma_0 \left(\frac{r}{r_0}\right)^\sigma$$

The corresponding inward migration timescale is given by:

$$\tau_{mig} = (2.7 + 1.1\sigma)^{-1}\frac{M_\star}{M_p}\frac{M_\star}{\Sigma_p r_p^2}\Omega_p^{-1} \propto \frac{1}{M_p}$$

Type I migration

The migration is determined by the net torque exerted onto the planet (Goldreich & Tremaine 1979; Tanaka et al. 2002). This torque can be written:

$$\Gamma_{Total} = \sum_{ILR} \Gamma_{LR} + \sum_{OLR} \Gamma_{LR} + \Gamma_{CR}$$

This torque can be expressed in a general way as:

$$\Gamma_{Total} = \frac{1}{\gamma}(c_0 + C_1\sigma + C_2\beta)\Gamma_0$$

where Γ_0 is given as:

$$\Gamma_0 = \left(\frac{q}{h}\right)^2 \Sigma_p^2 r_p^2 \Omega_p^2$$

and C_0, C_1 and C_3 are constants and σ, β are the exponents of the assumed power-law distribution of density and temperature.

- We can now calculate the migration velocity.

The conservation of angular momentum implies:

$$\frac{dJ_p}{dt} = \Gamma_{Total}$$

The angular momentum of the planet is given by:

$$J_p = M_p(GM_\star r_p)^{\frac{1}{2}}$$

From which we obtain the migration velocity:

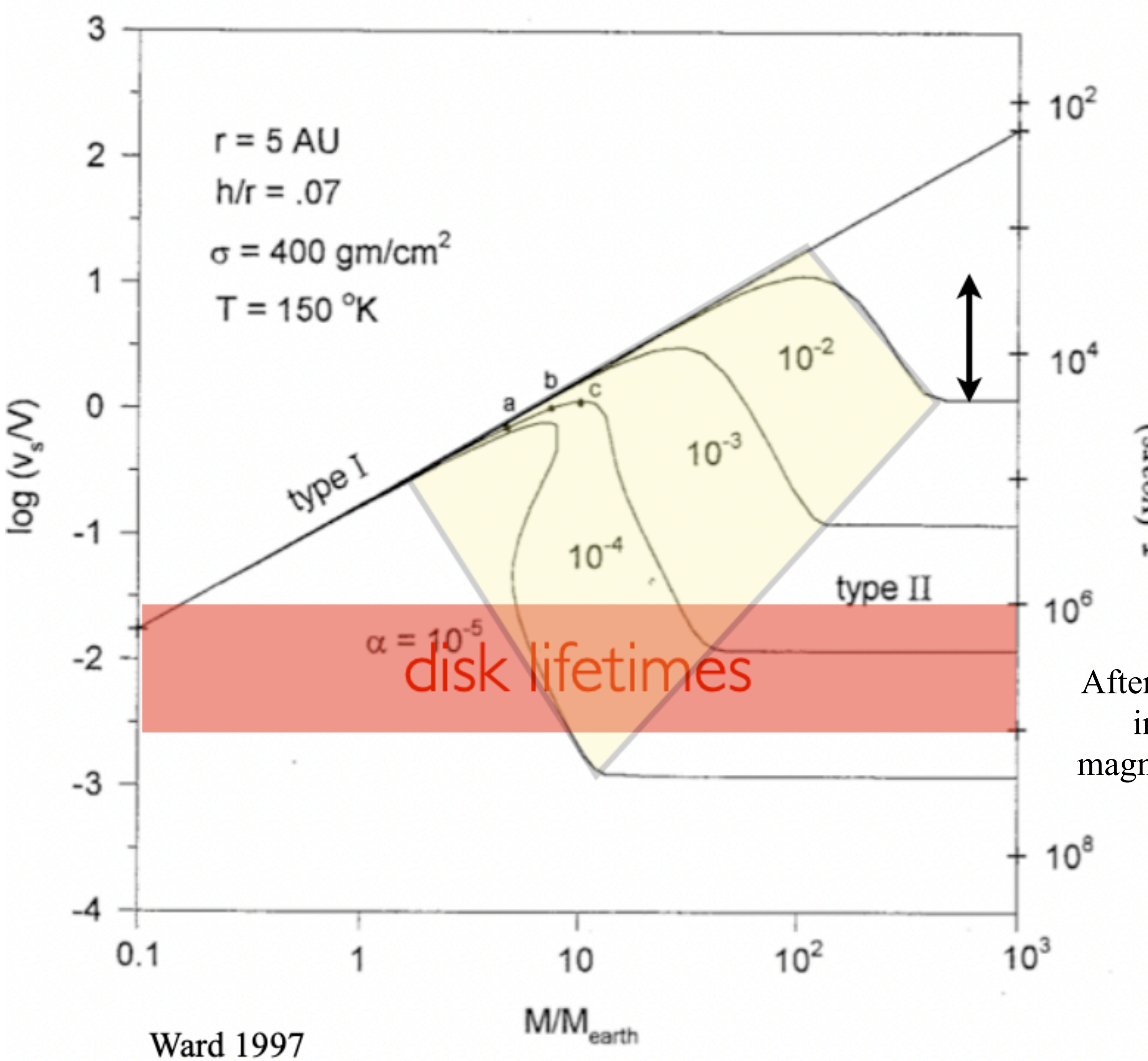
$$\frac{dr_p}{dt} = -2r_p\frac{\Gamma_{Total}}{J_p}$$

Depending upon the sign of the torque (i.e. upon the signs of the C_i and the slopes of the density and temperature distribution), the migration can proceed inwards or outward.

For an isothermal disk with a MMSN profile ($\sigma = 1.5$) migration is inwards and rapid.

Migration timescales

Note that before the transition to type II sets in, the migration timescale in type I are very short, $\sim 10^4$yrs.

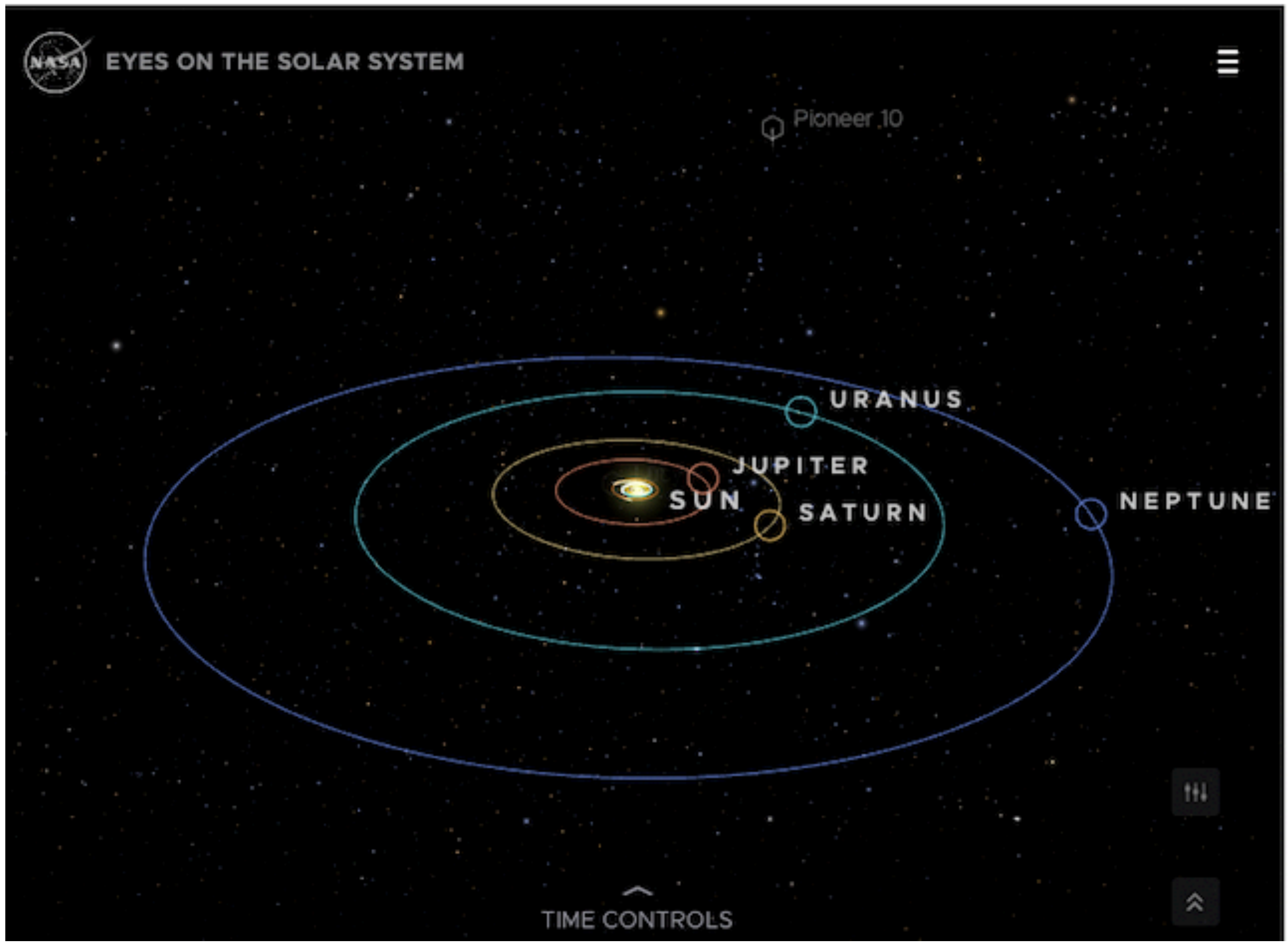


Ward 1997

Afterwards, migration timescale increase by 1-2 orders of magnitude from type I to type II.

For type I this means: Planets seem to migrate so fast that they should all fall into the star within the lifetime of the disk (unless they grow extremely rapid)!

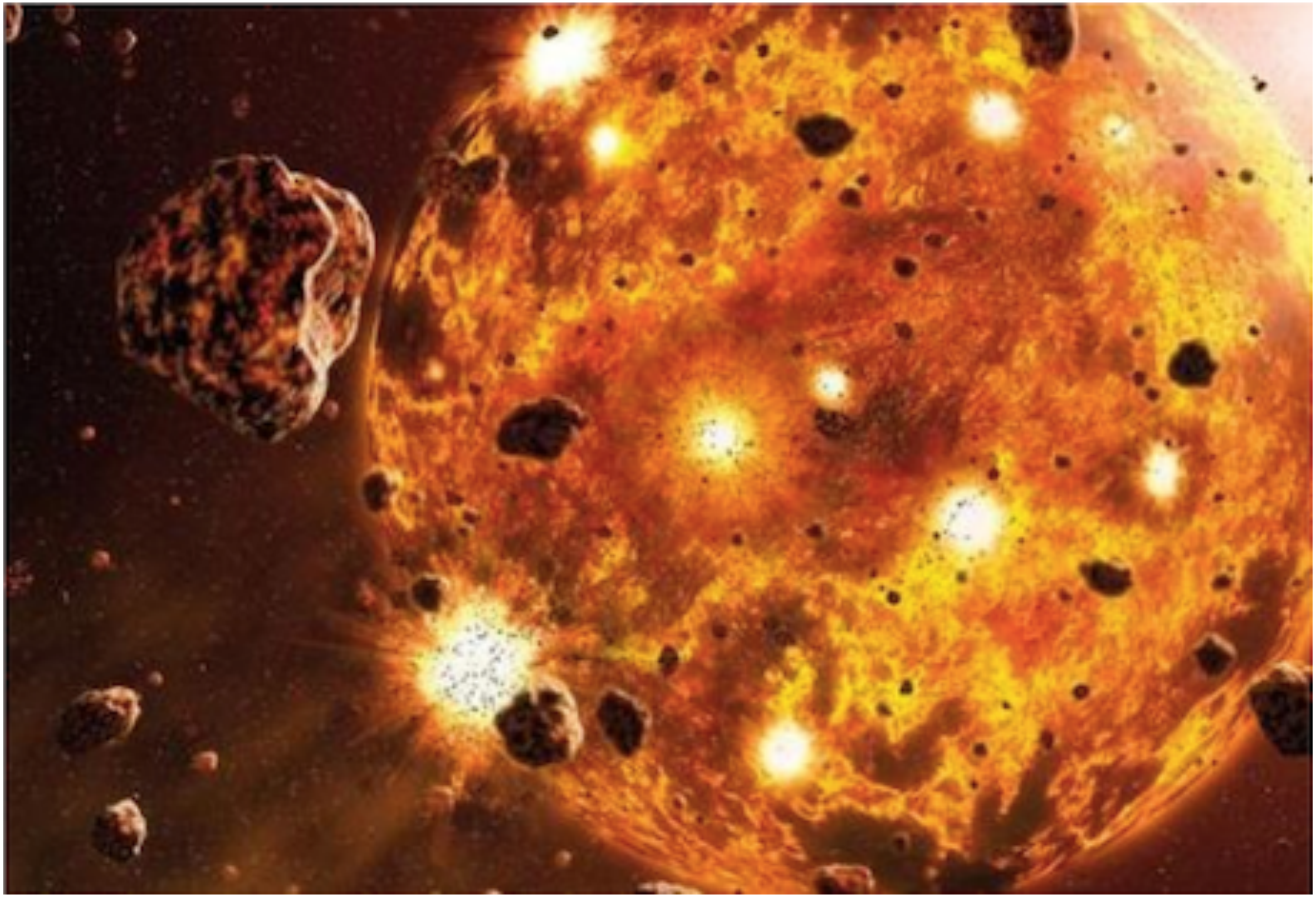
- If this were true, planetary cores should not survive in the disc, in evident contrast with the observation that many planetary systems (like ours:)



These very short migration timescales represent another major issue in modern planet formation theory.

Therefore, simple linear theory for isothermal disks cannot be the final word!

Coorbital thermal torques on low-mass protoplanets
(Linear Theory)



Coorbital Thermal Torques on low-mass protoplanets

Masset (2017)

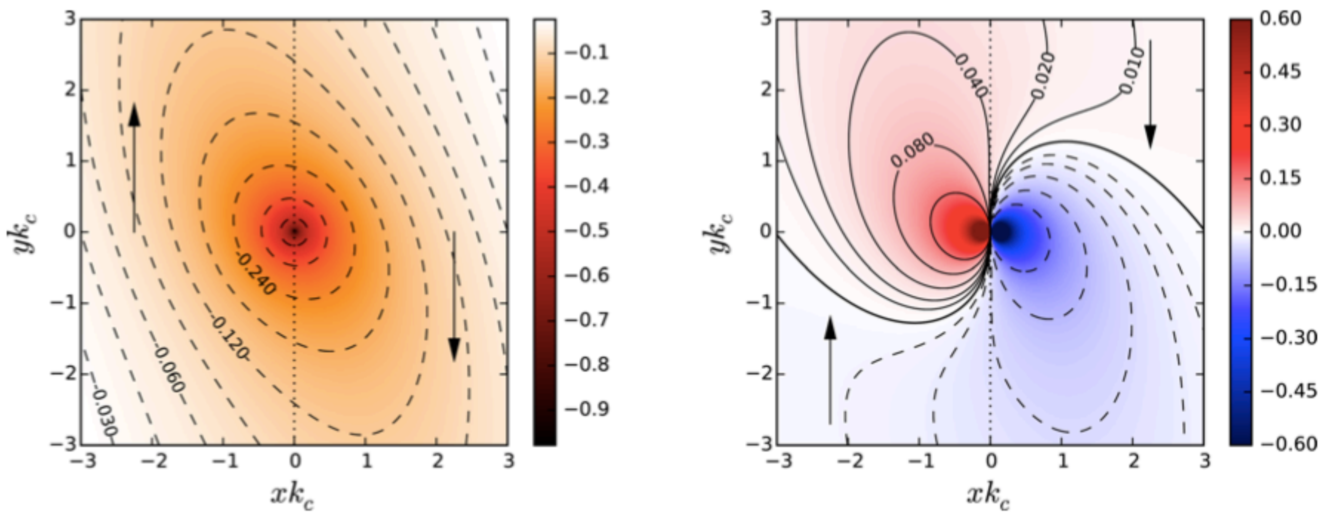


Figure 1. Perturbation of surface density $\sigma'^{(0)}$ in units of $\gamma(\gamma-1)L/\chi c_s^2$ due to the singular heat release $L\delta(\mathbf{r})$ (left) and perturbation $\sigma'^{(1)}$ arising from the heat 'dipole' $-L\delta'(x)\delta(y)\delta(z)$, in units of $\gamma(\gamma-1)Lk_c/\chi c_s^2$ (right). The map of the right can also be regarded as the derivative of the perturbation of surface density with respect to the planet position x_p. These maps have been obtained by summing 40 000 Fourier components in geometric sequence from $k_y = 10^{-4}k_c$ to $k_y = 10^4 k_c$. The levels on the left map are in geometric sequence with a ratio of $\sqrt{2}$ from -3×10^{-2} to -0.48, while the levels on the right map are in geometric sequence with a ratio of 2 from $\pm1\times10^{-2}$ to ±0.16. The thicker contour corresponds to the null level. The vertical arrows depict schematically the Keplerian flow. When the distance to the planet is largely smaller than λ_c (i.e. for $|xk_c| \ll 1$ and $|yk_c| \ll 1$), diffusion dominates and the perturbation has spherical symmetry. For distances comparable to or larger than λ_c, advection takes over and the perturbation is distorted under the action of the Keplerian flow.

The distance to the planet's corotation is considered to be much smaller than the size of the perturbation

$(x_p << \lambda_c)$

$$x_p = \left(\frac{\sigma}{3} + \frac{\beta+3}{6}\right) h^2 r_p$$

$$\lambda_c = \sqrt{\frac{\chi}{q\Omega_p\gamma}}.$$

5.5 A simple expression for the total thermal torque

The total thermal torque is the sum of the heating torque and of the cold thermal torque:

$$\Gamma^{\rm total}_{\rm thermal} = \Gamma^{\rm heating}_{\rm thermal} + \Gamma^{\rm cold}_{\rm thermal}, \tag{142}$$

while the total torque acting on the planet is

$$\Gamma^{\rm total} = \Gamma^{\rm total}_{\rm thermal} + \Gamma_{\rm adiabatic}. \tag{143}$$

The heating torque given by equation (109) can be cast in a simple form using the critical luminosity L_c of equation (132). We obtain

$$\Gamma^{\rm heating}_{\rm thermal} = 1.61\frac{\gamma-1}{\gamma}\frac{x_p}{\lambda_c}\frac{L}{L_c}\Gamma_0, \tag{144}$$

while the total thermal torque is

$$\Gamma^{\rm total}_{\rm thermal} = 1.61\frac{\gamma-1}{\gamma}\frac{x_p}{\lambda_c}\left(\frac{L}{L_c}-1\right)\Gamma_0. \tag{145}$$

This expression can also be recast under the convenient form

$$\Gamma^{\rm total}_{\rm thermal} = 1.61\frac{\gamma-1}{\gamma}\eta\left(\frac{H}{\lambda_c}\right)\left(\frac{L}{L_c}-1\right)h\Gamma_0, \tag{146}$$

where η is given by equation (139), λ_c by equation (120), L_c by equation (132) and Γ_0 by equation (134).

Gap formation

The angular momentum is transported from the inner part of the disk to the planet and from the planet to the outer part of the disk. Hence, gas inside the planet looses angular momentum and moves inwards while gas outside gains angular momentum and moves outwards. For this mechanism to result in the opening of a gap, two conditions have to be met.

Condition I (thermal condition):

Hill sphere of a planet must be comparable to the disk scale height. If this is not true the disc will be able to accrete past the planet away from the disc mid-plane.

This can be expressed by the condition:

$$r_H = r_p \left(\frac{M_p}{3M_\star} \right)^{1/3} \geq H$$

Which implies a mass ratio planet/star of:

$$q = \frac{M_p}{M_\star} \geq 3 \left(\frac{H}{r} \right)_p^3 = 3h_p^3$$

Typically the disk aspect ratio is $h \approx 0.05$ and $q \geq 1.25 \times 10^{-4}$ corresponding to $M > 0.13 M_{Jupiter}$.

Condition II (viscous condition):

Viscous effect must not be able to close the gap.

This can be expressed by the condition:

$$\tau_{close} \geq \tau_{open}$$

In terms of torque, this condition is written

$$\left(\frac{dJ}{dt}\right)_{LR} \geq \left(\frac{dJ}{dt}\right)_{visc}$$

Considering the impulse approximation:

$$\frac{8}{27}\frac{G^2 M_p^2 r_p \Sigma}{9\Omega_p^2 b_{min}^3} \geq 3\pi\nu\Sigma r_p^2 \Omega_p$$

With $\nu = \alpha c_s H$ and $b_{min} = R_H$ we can get:

$$q \geq \frac{243\pi}{8}\alpha h^2$$

Typically $h \approx 0.05$, $\alpha = 10^2$ so that $q \geq 2.39 \times 10^{-3}$,

corresponding to $M > 2.5 M_{Jupiter}$.

Once the planet is massive enough to open gap, the gas is being pushed away from the planet and hence the torques diminish. The planet is kept in the middle of the gap, as if it were to be closer to the inner edge, it would gain angular momentum, and it would migrate back outwards, while the opposite effect would happen close to the outer edge. In a static disk, the planet would also be static. As the disk is itself evolving on the viscous timescale, also the planet's orbit is evolving on this timescale:

$$\tau_{mig} = \frac{r_p^2}{\nu} = \frac{r_p^2}{\alpha c_s H} = \frac{1}{\alpha}\left(\frac{r_p}{H}\right)^2 \Omega_p^{-1}$$

where we have used the fact that the viscosity is given by $\nu = \alpha c_s H$ and the sound speed is given as $c_s = H\Omega_p$.

Note that in the case of type II migration, the migration timescale is independent of the mass of the planet and only depends upon the mass of the star and the characteristics of the disk. We have however seen that this simple picture is valid only if the planet is not too massive $\left(B = \frac{3\pi\Sigma_0 R_0^2}{M_p} >> 1 \right)$.

One therefore distinguishes two regimes:

Disk dominated type II (B>>1):

$$\tau_{mig} = \tau_{visc}$$

Planet dominated type II (B<<1):

$$\tau_{mig} \sim \tau_{visc} B$$

Therefore, in the planet dominated regime, migration is slower.

Observed gaps

The Astrophysical Journal, 693:L86–L90, 2009 March 10 doi:10.1088/0004-637X/693/2/L86

HIGH ANGULAR RESOLUTION RADIO OBSERVATIONS OF THE HL/XZ TAU REGION: MAPPING THE 50 AU PROTOPLANETARY DISK AROUND HL TAU AND RESOLVING XZ TAU S INTO A 13 AU BINARY

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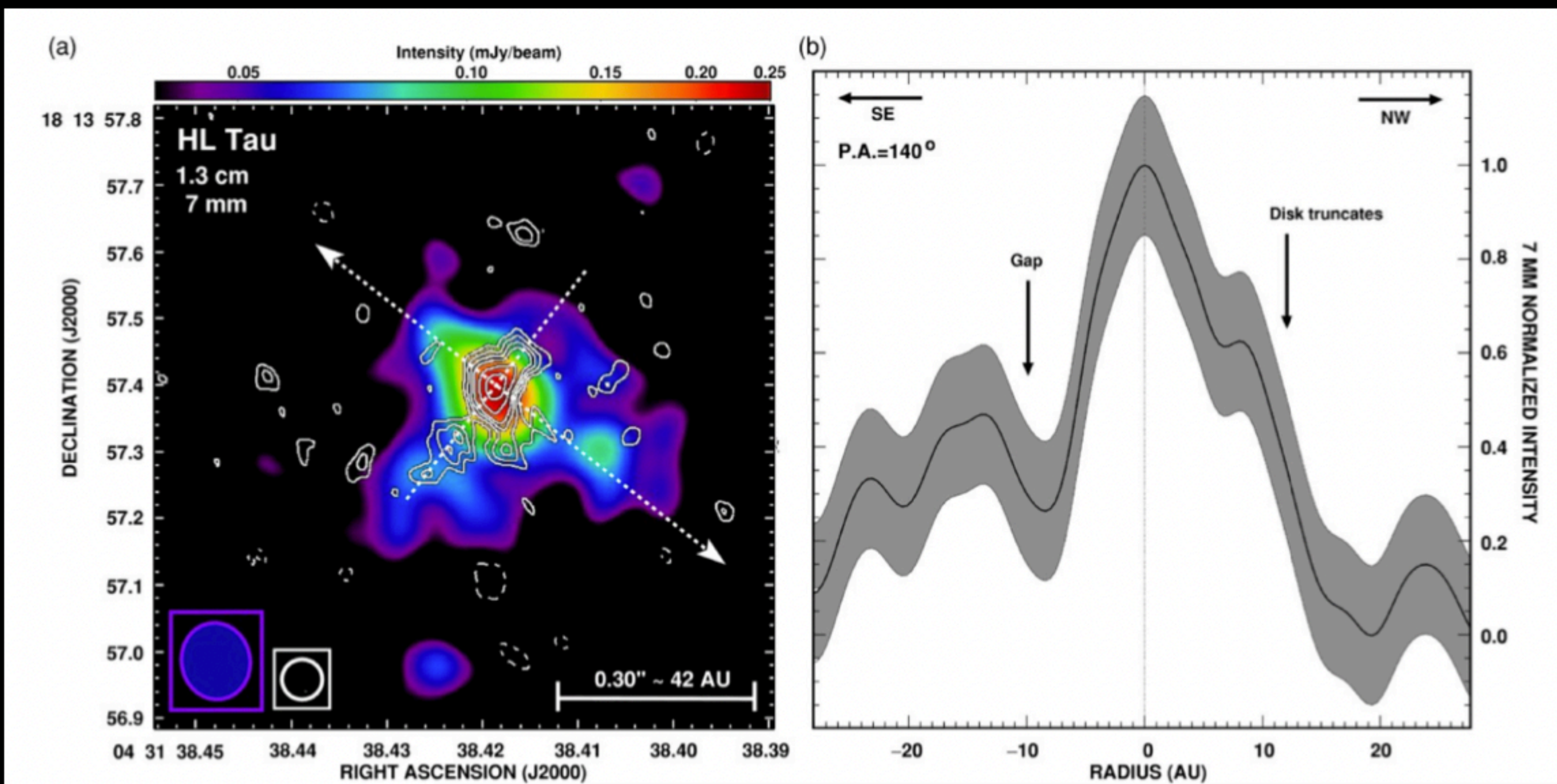
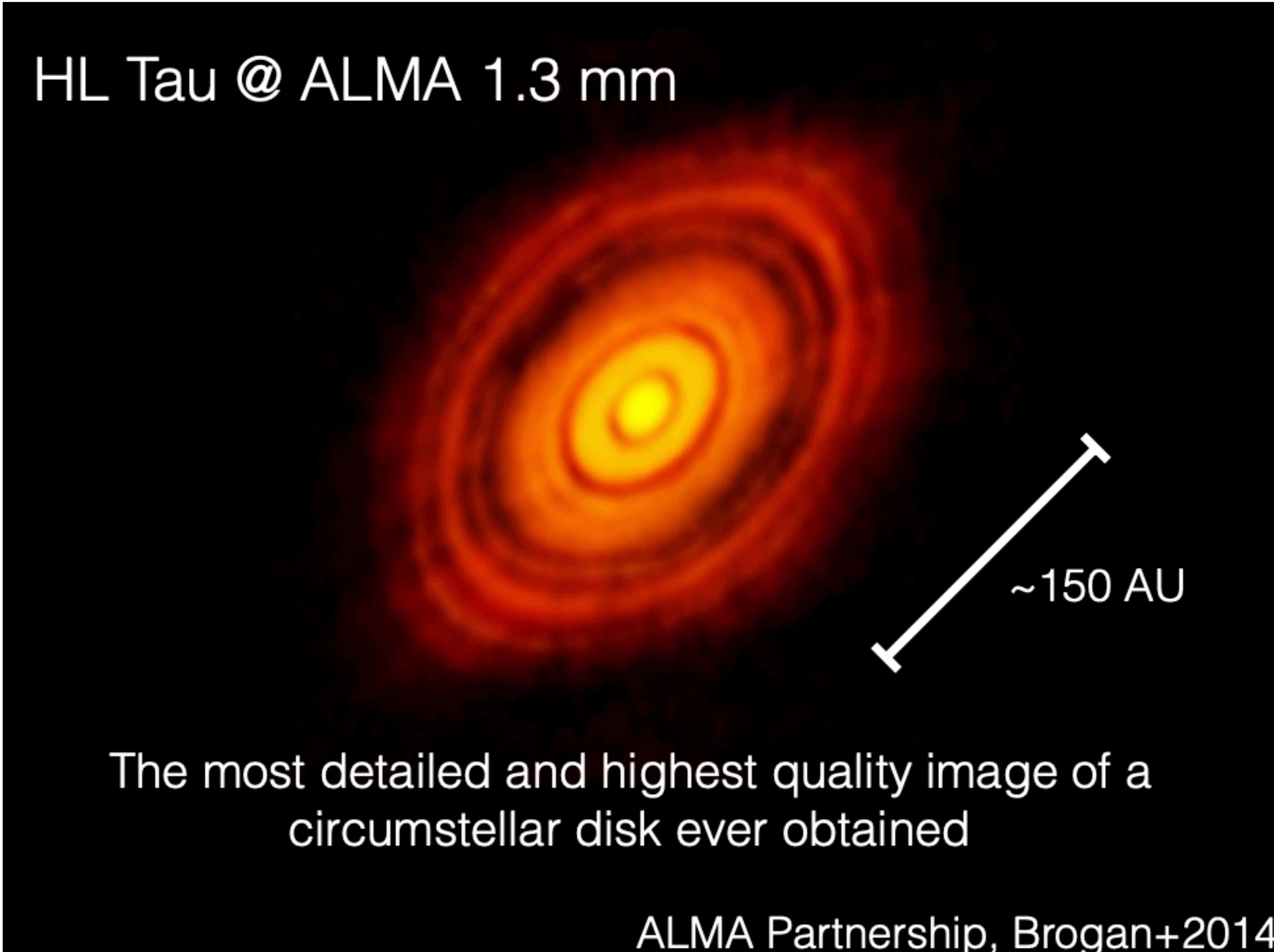
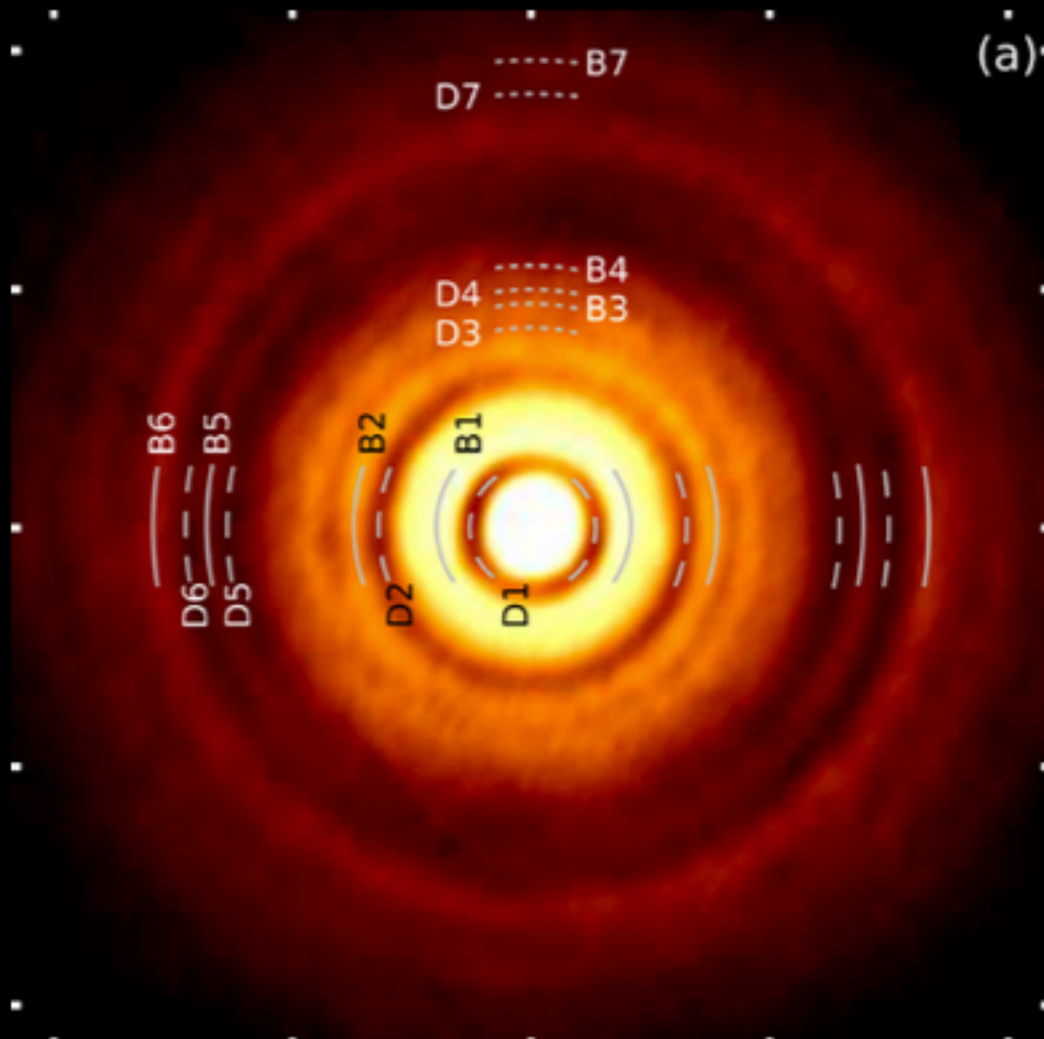


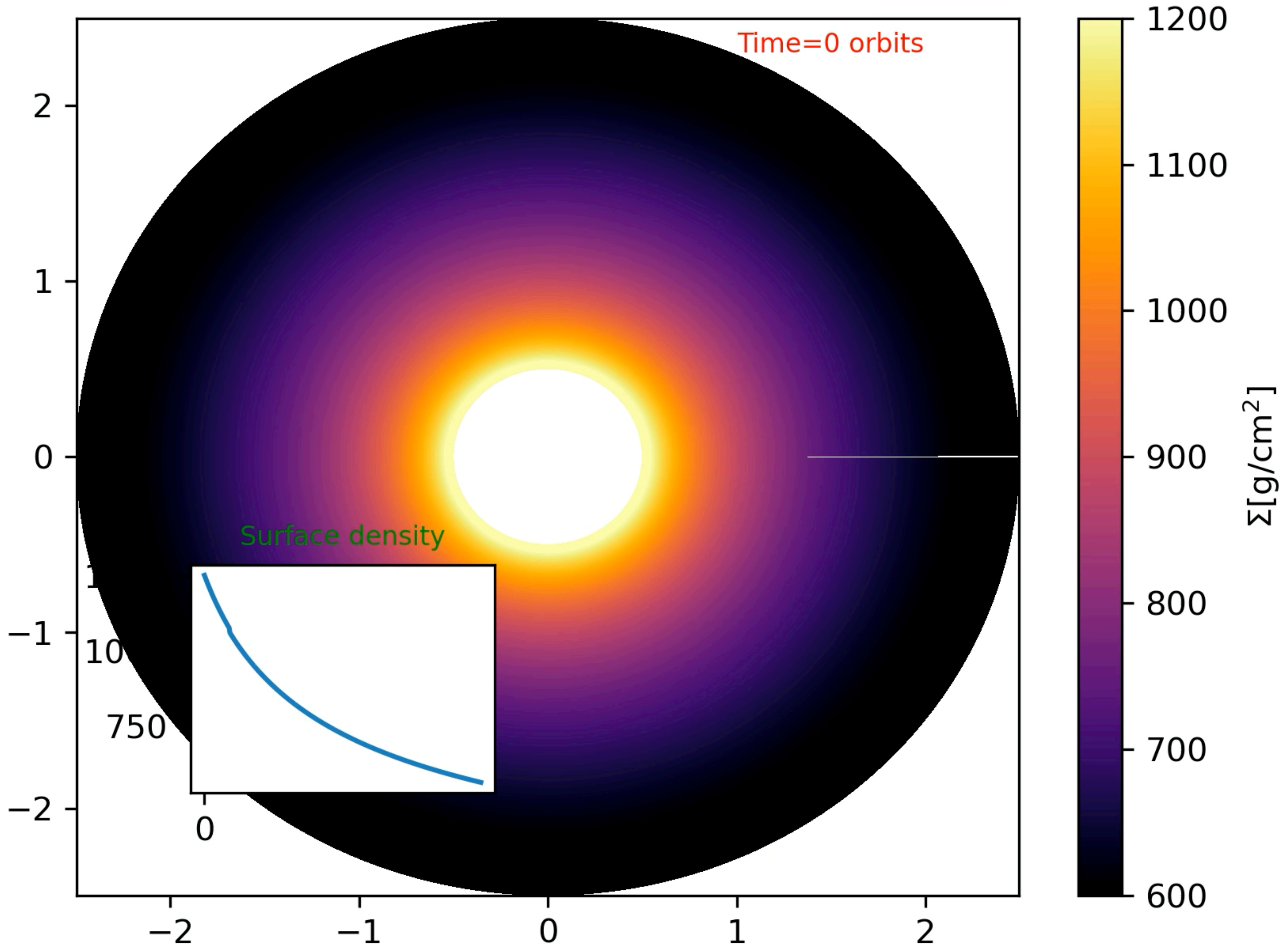

HL Tau @ ALMA 1.3 mm
~150 AU
The most detailed and highest quality image of a circumstellar disk ever obtained
ALMA Partnership, Brogan+2014

HL Tau @ ALMA 1.3 mm



7 pairs of BRIGHT and DARK rings

ALMA Partnership, Brogan+2014


Time=0 orbits
Surface density
Σ[g/cm²]
1200
1100
1000
900
800
700
600
750
0
−2
−1
1
2