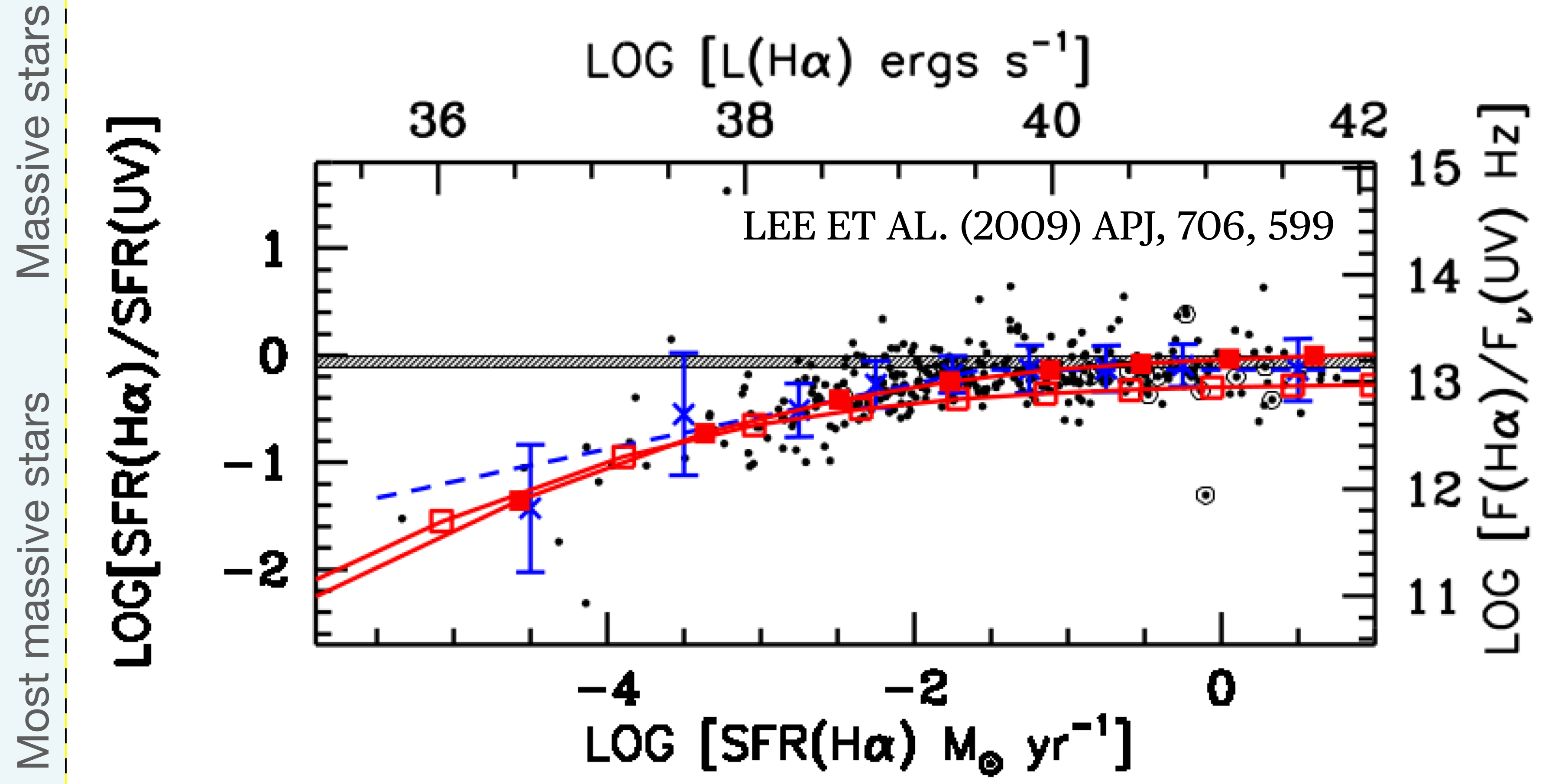


The IMF and chemical evolution in context

The *variability* of the IMF

THE IMF DEPENDS ON SFR



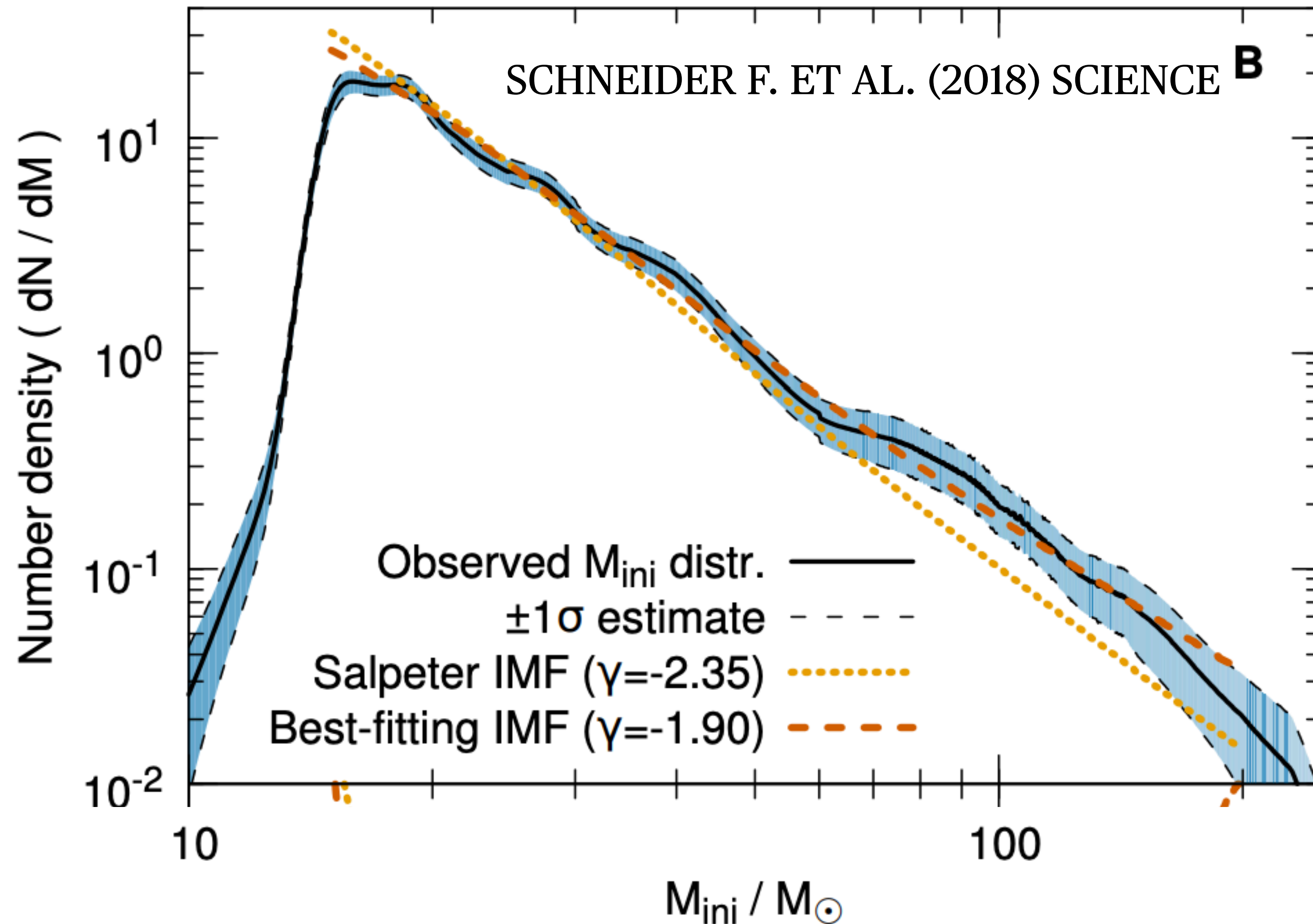
Black scatter points
dwarf galaxies in the local
volume

In the **blue data bins**,
dwarf galaxies have been
corrected for attenuation
by ISM dust

The open and filled **red squares** represent the
variable IMF models by
Weidner+04.

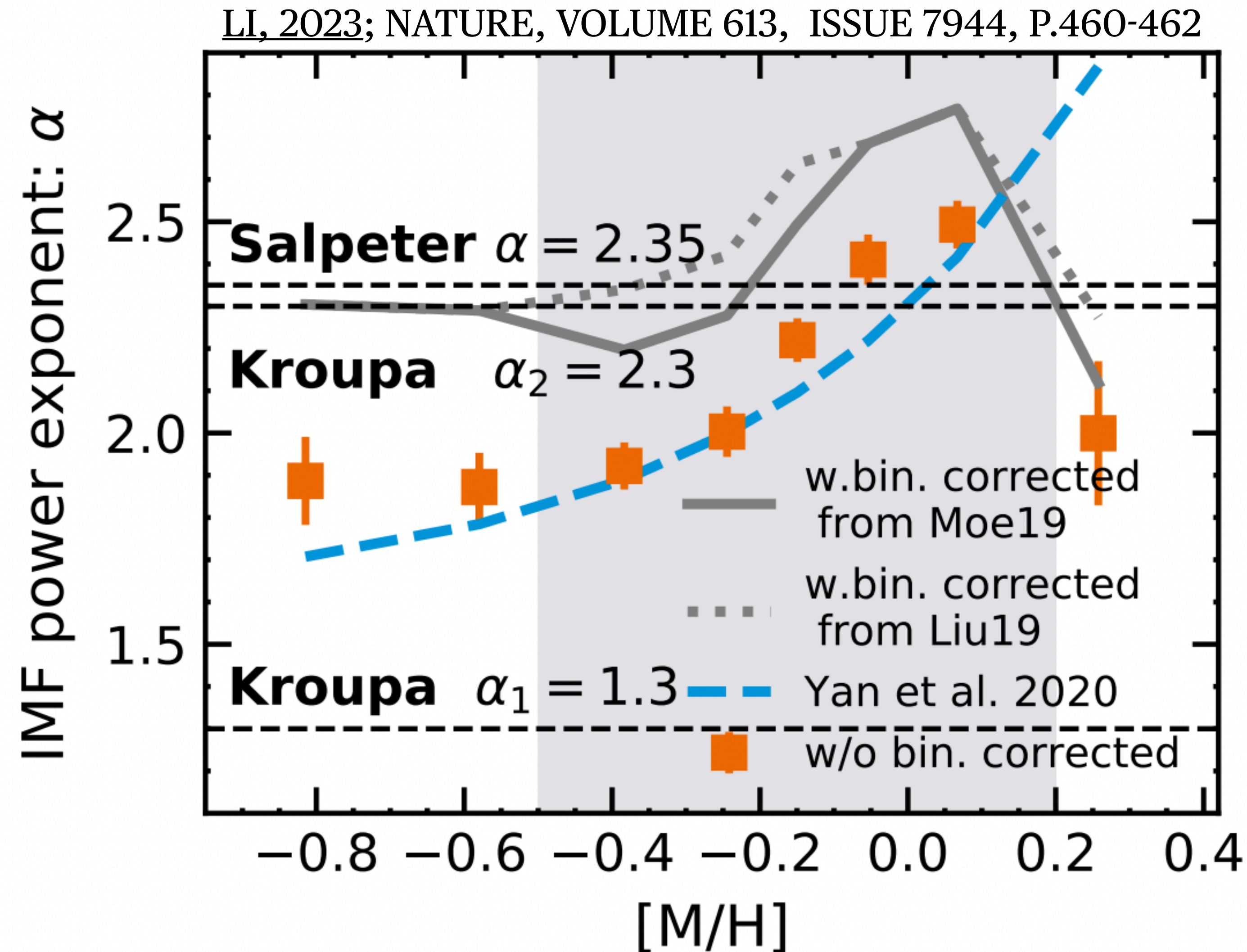
A case study: the starburst region 30 Dorados in the LMC

IMF



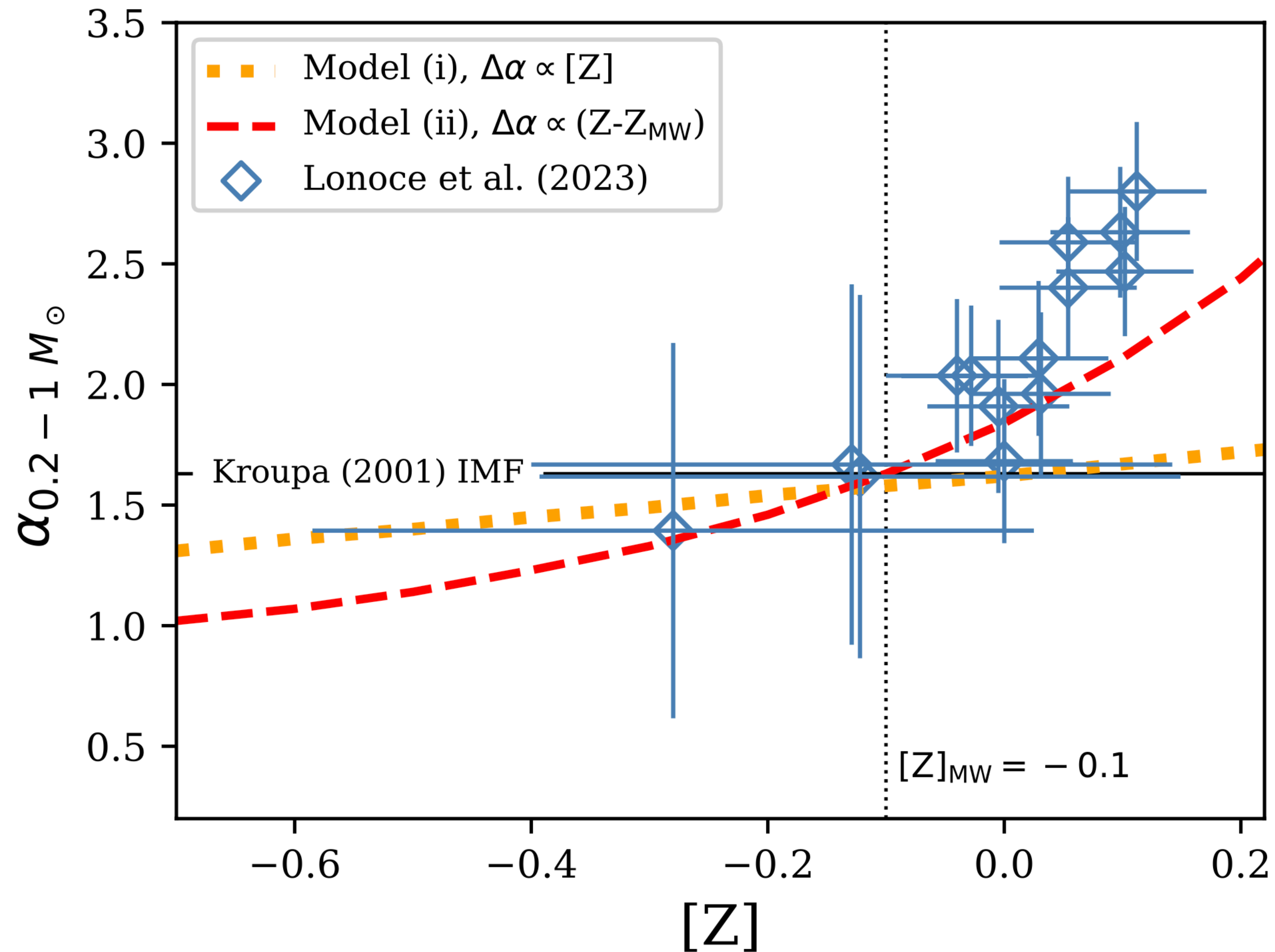
- a young (4–6 Myr) starburst region,
- Too many massive stars compared to the **Salpeter IMF slope of 2.35**.
- This suggests that 30 Dorados is inconsistent with a universal IMF

THE IMF DEPENDS ON METALLICITY



- 93,000 spectroscopically observed **M-dwarf stars** (low-mass) in the 100–300 parsec (pc) Solar neighborhood.
- They find unambiguous evidence of a **variable IMF** that depends on both metallicity and stellar age.

The stellar populations formed at early times contain fewer low-mass stars compared to the canonical IMF, independent of stellar metallicities.

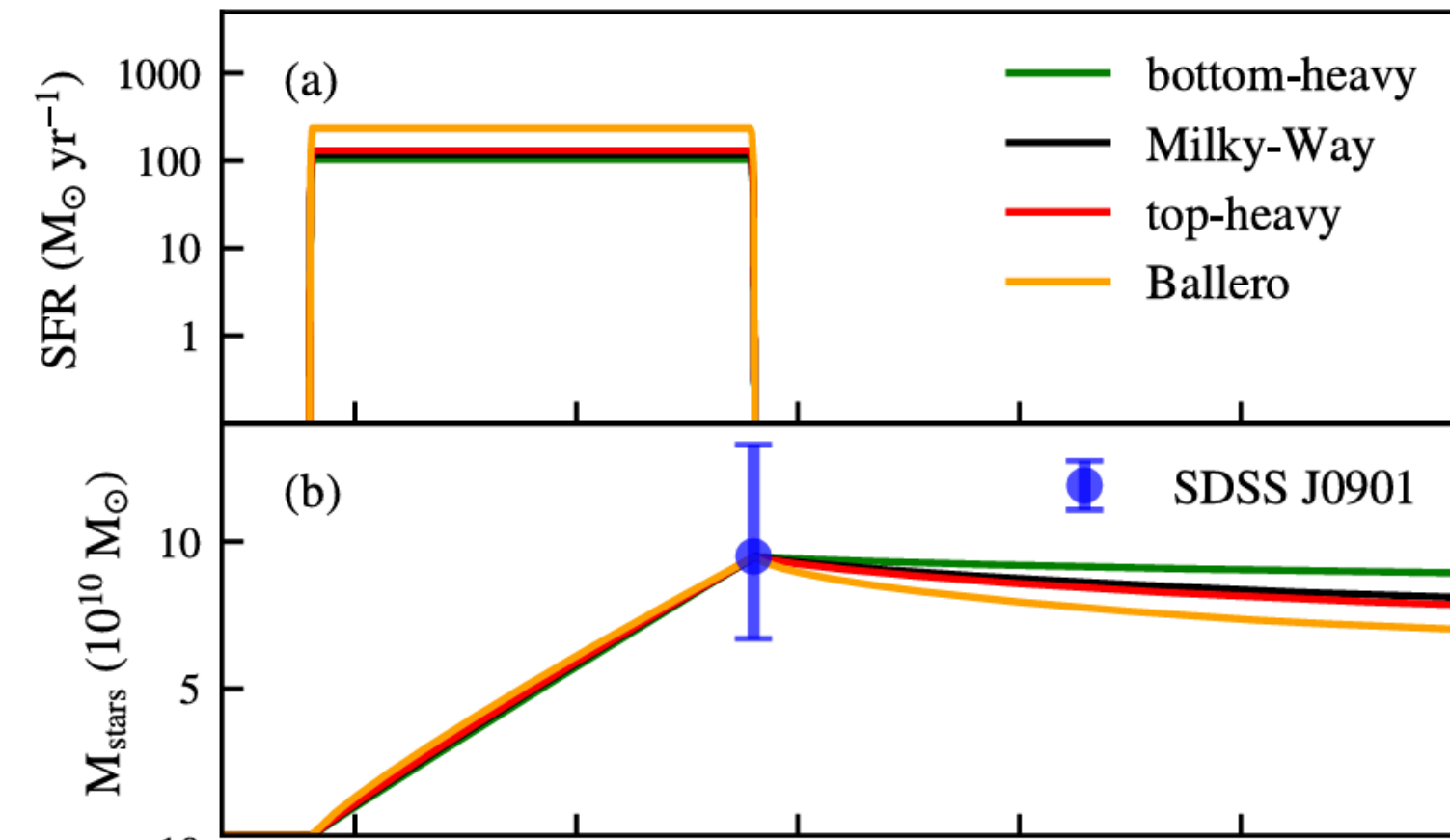


IGIMF FOR LOW MASS STARS
IN HIGH-Z ELLIPTICAL GALAXIES

NGC3309

YAN ET AL.,
INCL. **GJERGO**
(2024, APJ)

POSTDOC AT
NANJING UNIVERSITY



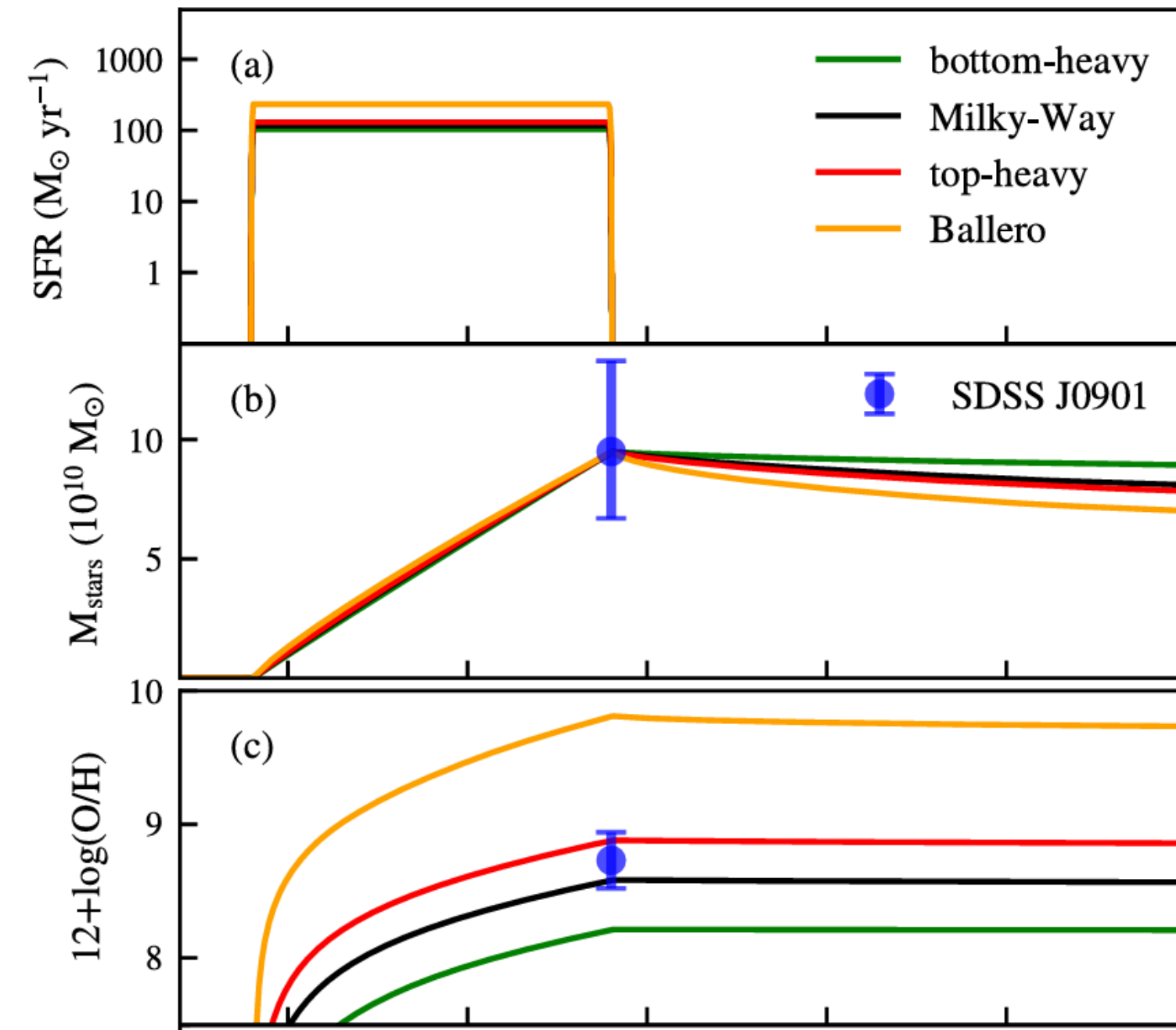
Hubble time [Gyr]

CO ISOTOPOLOGUE STUDY

GUO ET AL.,
INCL. **GJERGO**
(2024, MNRAS)

PHD STUDENT AT
NANJING UNIVERSITY

THE IMF SLOPE FOR
MASSIVE STARS IN
HIGH REDSHIFT
STARBURSTS IS TOP-HEAVY



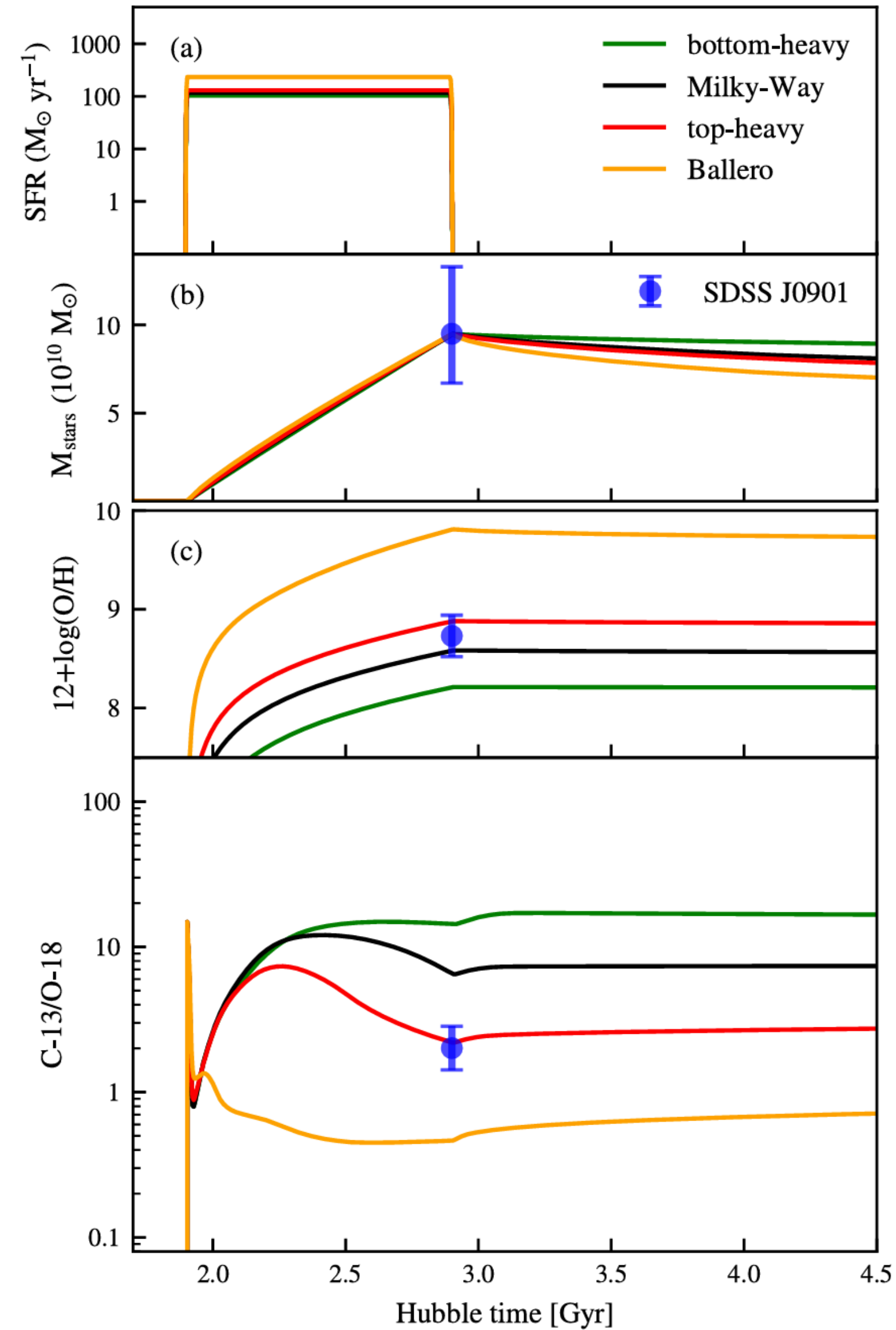
Hubble time [Gyr]

CO ISOTOPOLOGUE STUDY

GUO ET AL.,
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THE IMF SLOPE FOR
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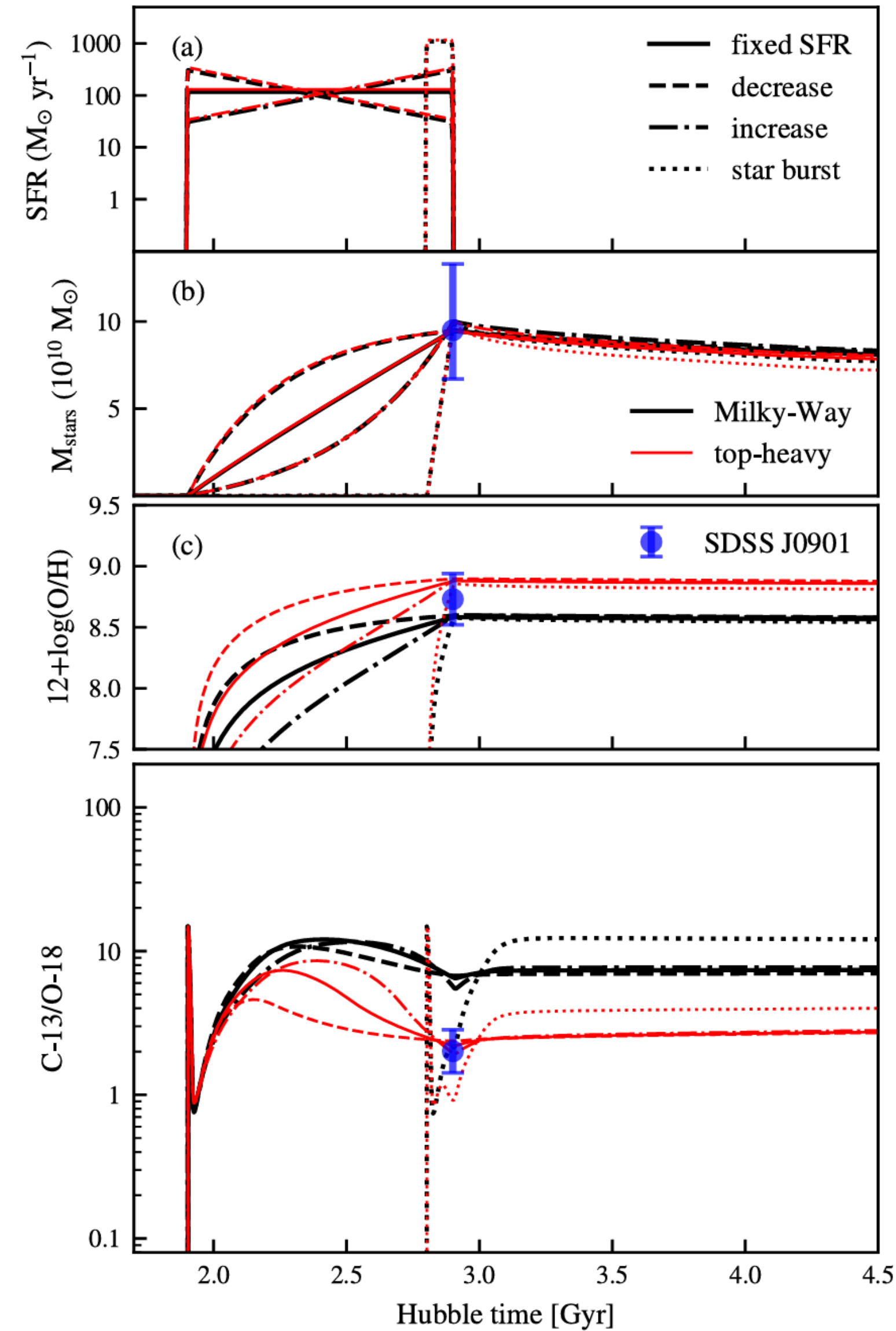
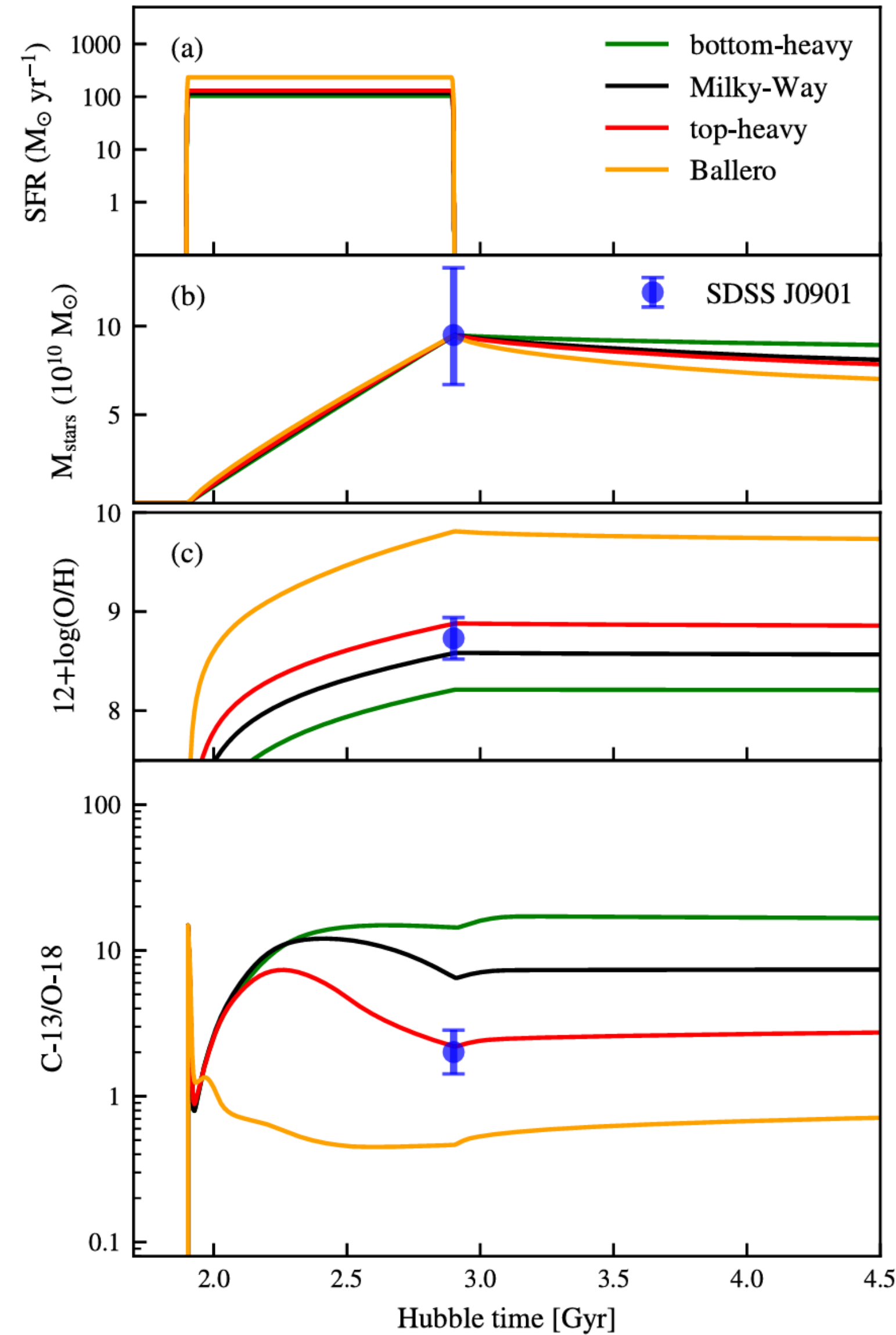


CO ISOTOPOLOGUE STUDY

GUO ET AL.,
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THE IMF SLOPE FOR
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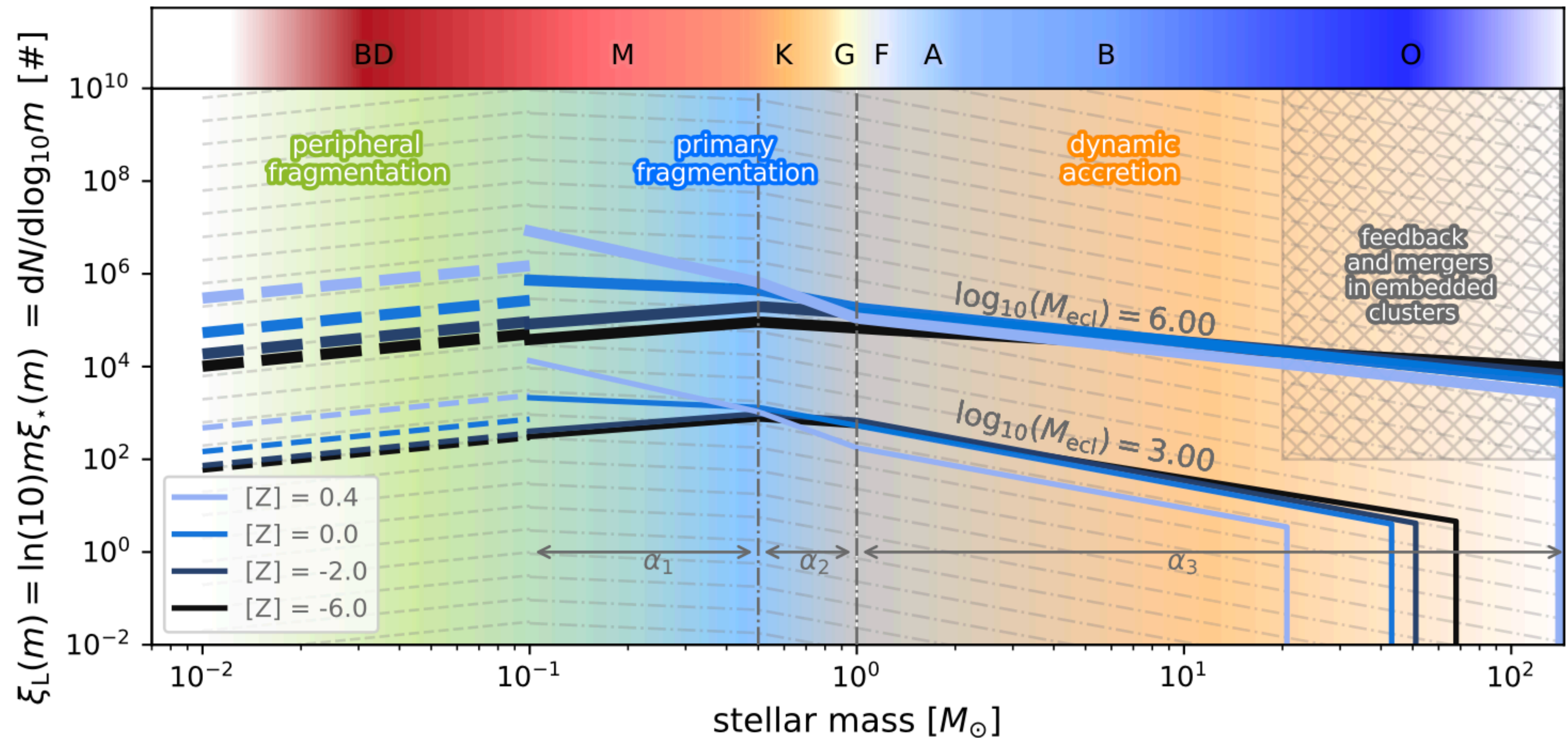
CO ISOTOPOLOGUE STUDY

GUO ET AL.,
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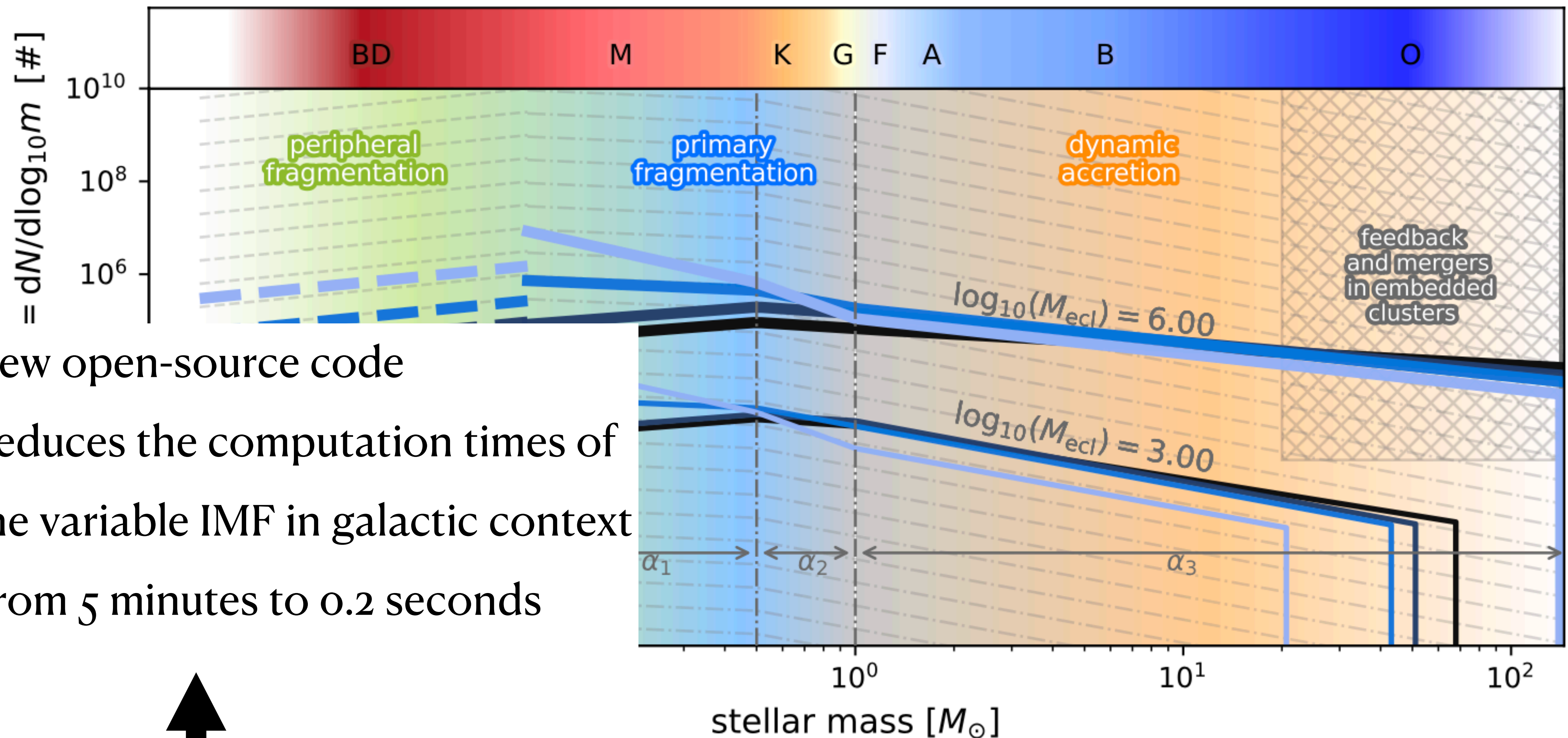
THE IMF SLOPE FOR
MASSIVE STARS IN
HIGH REDSHIFT
STARBURSTS IS TOP-HEAVY

Overview of the variable stellar IMF



Using pyGIMF (Gjergo et al., 2025, RAA) Kroupa, **Gjergo**, Jerabkova, Yan (2024)

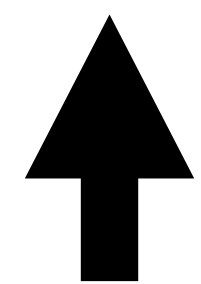
Overview of the variable stellar IMF



New open-source code

Reduces the computation times of
the variable IMF in galactic context

From 5 minutes to 0.2 seconds



Using `pylGIMF` (Gjergo et al., 2025, RAA) Kroupa, **Gjergo**, Jerabkova, Yan (2024)

Embedded cluster mass function

Parameter to compute the ECMF

$$\beta = -0.106 \log_{10} \bar{\psi}_{\delta t} + 2. \quad (10)$$

Embedded cluster mass function

$$\xi_{\text{ecl}} = dN_{\text{ecl}}/dM_{\text{ecl}} = k_{\text{ecl}} M_{\text{ecl}}^{-\beta}, \quad 5M_{\odot} \leq M_{\text{ecl}} < M_{\text{ecl,max}}. \quad (11)$$

extract k_{ecl} and $M_{\text{ecl,max}}$

$$1 = \int_{M_{\text{ecl,max}}}^{M_{\text{ecl,max}*}} k_{\text{ecl}} M_{\text{ecl}}^{-\beta} dM_{\text{ecl}} \quad (12)$$

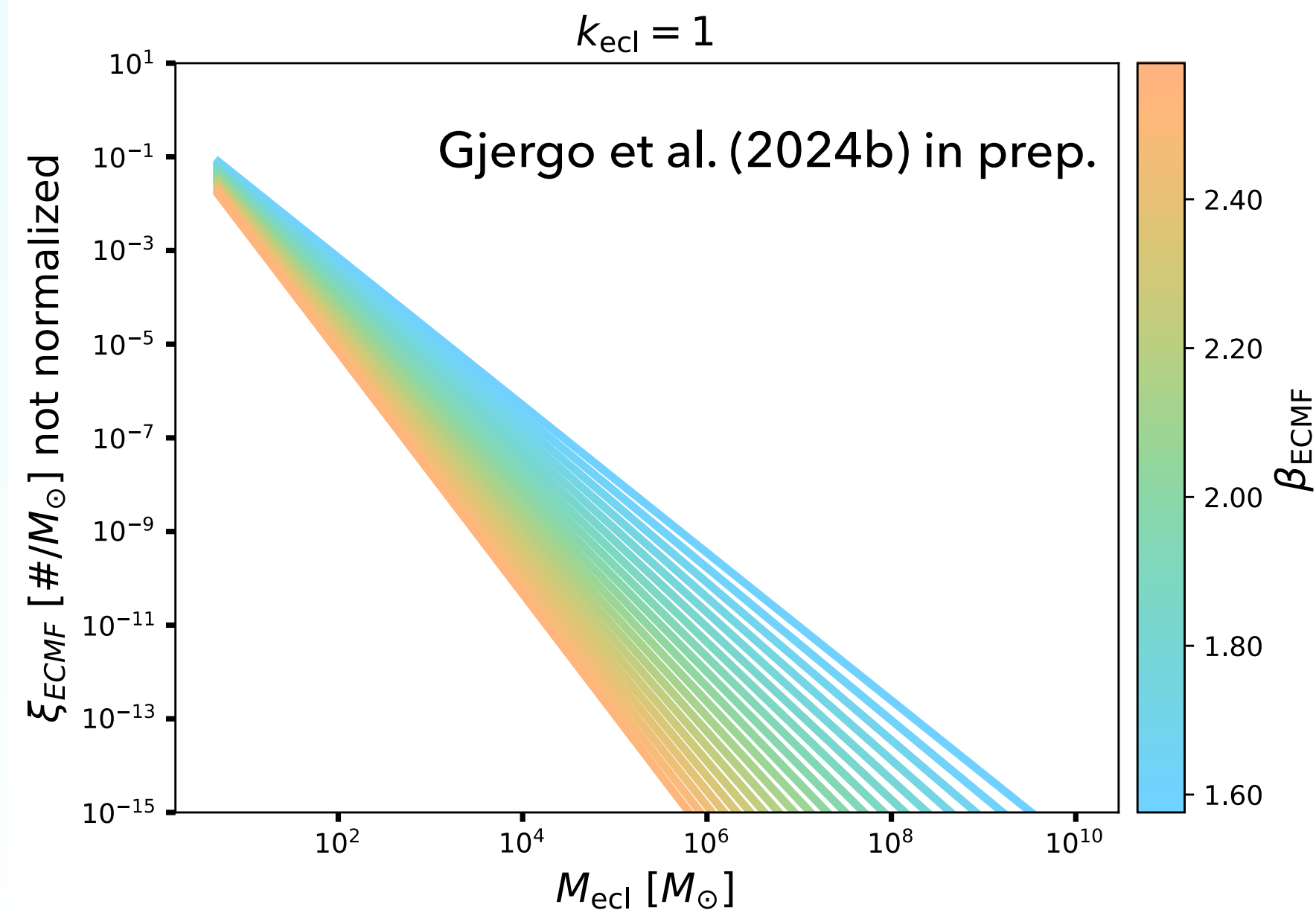
$$M_{\text{tot}} = \bar{\psi}_{\delta t} \delta t = \int_{M_{\text{ecl,min}}}^{M_{\text{ecl,max}}} M_{\text{ecl}} \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}} \quad (13)$$

IGIMF parameters

δt	$= 10^7 \text{ yr}$
$Z_{\odot}$	$= 0.0142$
$\Delta\alpha$	$= 63$
$m_{\text{max}*}$	$= 150M_{\odot}$
m_{min}	$= 0.08M_{\odot}$
$M_{\text{ecl,max}*}$	$= 10^{10}M_{\odot}$
$M_{\text{ecl,min}}$	$= 5M_{\odot}$

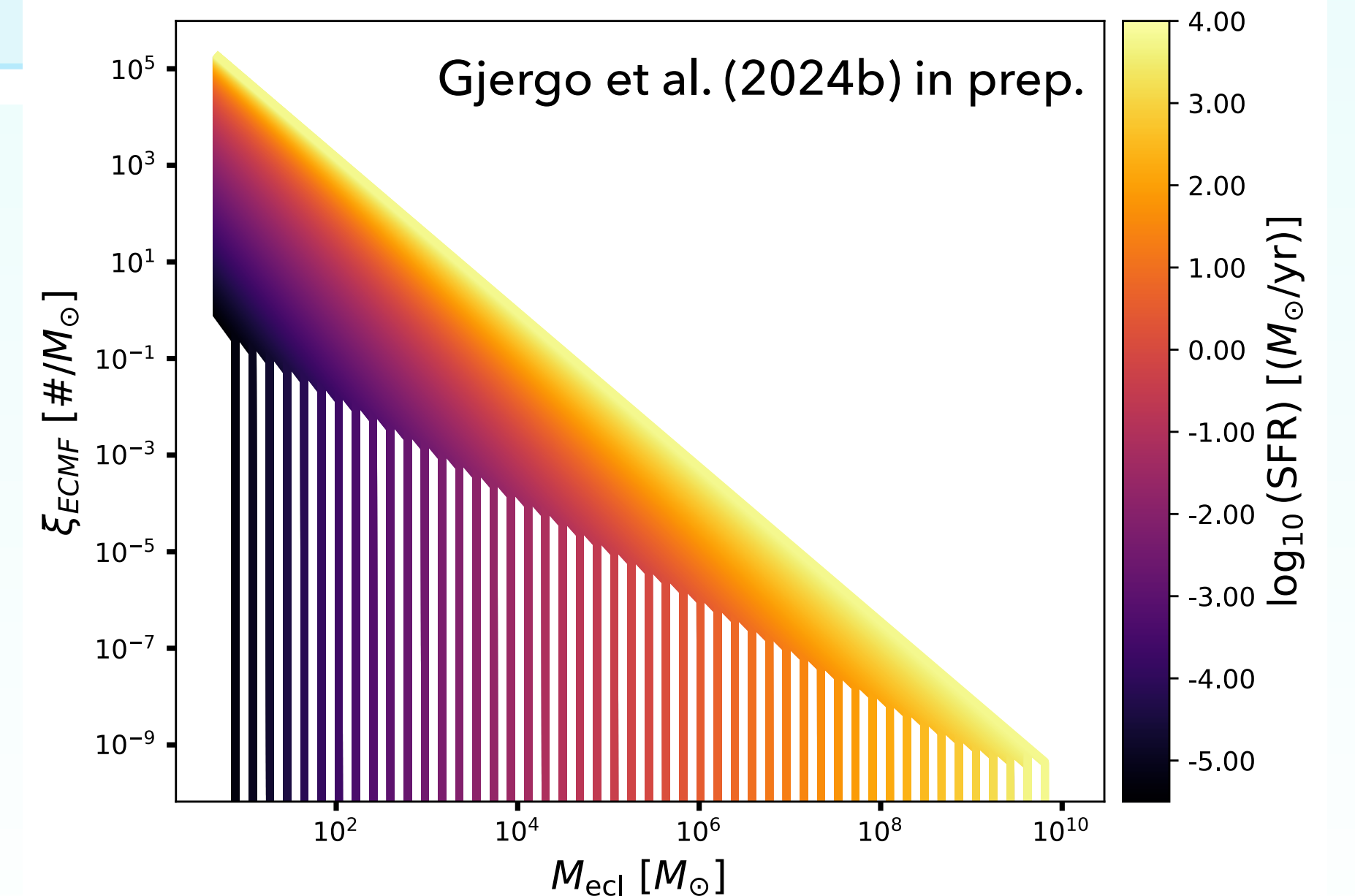
Table 1: Parameters

Dependence on the SFR



Formulation from

- Kroupa & Weidner (2003)
- Kroupa & Weidner (2005)
- Marks & Kroupa (2012)
- Kroupa (2013)
- [...]
- Jerabkova et al. (2018)
- Yan et al. (2019,2020, 2021)



Stellar Initial Mass Function

Dependence on
the embedded cluster
stellar mass
and metallicity

Gjergo et al. (2024b) in prep.

Formulation from
Kroupa & Weidner (2003)
Kroupa & Weidner (2005)
Marks & Kroupa (2012)
Kroupa (2013)
[...]
Jerabkova et al. (2018)
Yan et al. (2019,2020, 2021)

Parameters to compute the stellar IMF

$$\begin{aligned}\alpha_1 &= 1.3 + \Delta\alpha \cdot (Z - Z_\odot), \\ \alpha_2 &= 2.3 + \Delta\alpha \cdot (Z - Z_\odot),\end{aligned}\tag{3}$$

M_{ecl} -dependent exponent

$$\alpha_3 = \begin{cases} 2.3, & x < -0.87, \\ -0.41x + 1.94, & x > -0.87. \end{cases}\tag{4}$$

$$x = -0.14[Z] + 0.99 \log_{10}(\rho_{\text{cl}}/10^6)\tag{5}$$

$$\log_{10} \rho_{\text{cl}} = 0.61 \log_{10} M_{\text{ecl}} + 2.85,\tag{6}$$

Stellar IMF for individual embedded clusters

$$\xi_\star(m, M_{\text{ecl}}, Z) = dN_\star/dm = \begin{cases} 2k_\star m^{-\alpha_1(Z)}, & 0.08 \leq m/M_\odot < 0.50, \\ k_\star m^{-\alpha_2(Z)}, & 0.50 \leq m/M_\odot < 1.00, \\ k_\star m^{-\alpha_3(Z, M_{\text{ecl}})}, & 1.00 \leq m/M_\odot < m_{\text{max}}, \end{cases}\tag{7}$$

extract $k_\star$ and m_{max}

$$M_{\text{ecl}} = \int_{0.08 M_\odot}^{m_{\text{max}}} m \xi_\star(m) dm,\tag{8}$$

$$1 = \int_{m_{\text{max}}}^{m_{\text{max}^*}} \xi_\star(m) dm,\tag{9}$$

IGIMF parameters

δt	$= 10^7 \text{yr}$
$Z_\odot$	$= 0.0142$
$\Delta\alpha$	$= 63$
m_{max^*}	$= 150 M_\odot$
m_{min}	$= 0.08 M_\odot$
$M_{\text{ecl}, \text{max}^*}$	$= 10^{10} M_\odot$
$M_{\text{ecl}, \text{min}}$	$= 5 M_\odot$

Table 1: Parameters

Stellar Initial
Mass Function

Dependence on
the embedded cluster
stellar mass
and metallicity

Gjergo et al. (2024b) in prep.

Parameters to compute the stellar IMF

$$\begin{aligned}\alpha_1 &= 1.3 + \Delta\alpha \cdot (Z - Z_\odot), \\ \alpha_2 &= 2.3 + \Delta\alpha \cdot (Z - Z_\odot),\end{aligned}\tag{3}$$

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IGIMF parameters

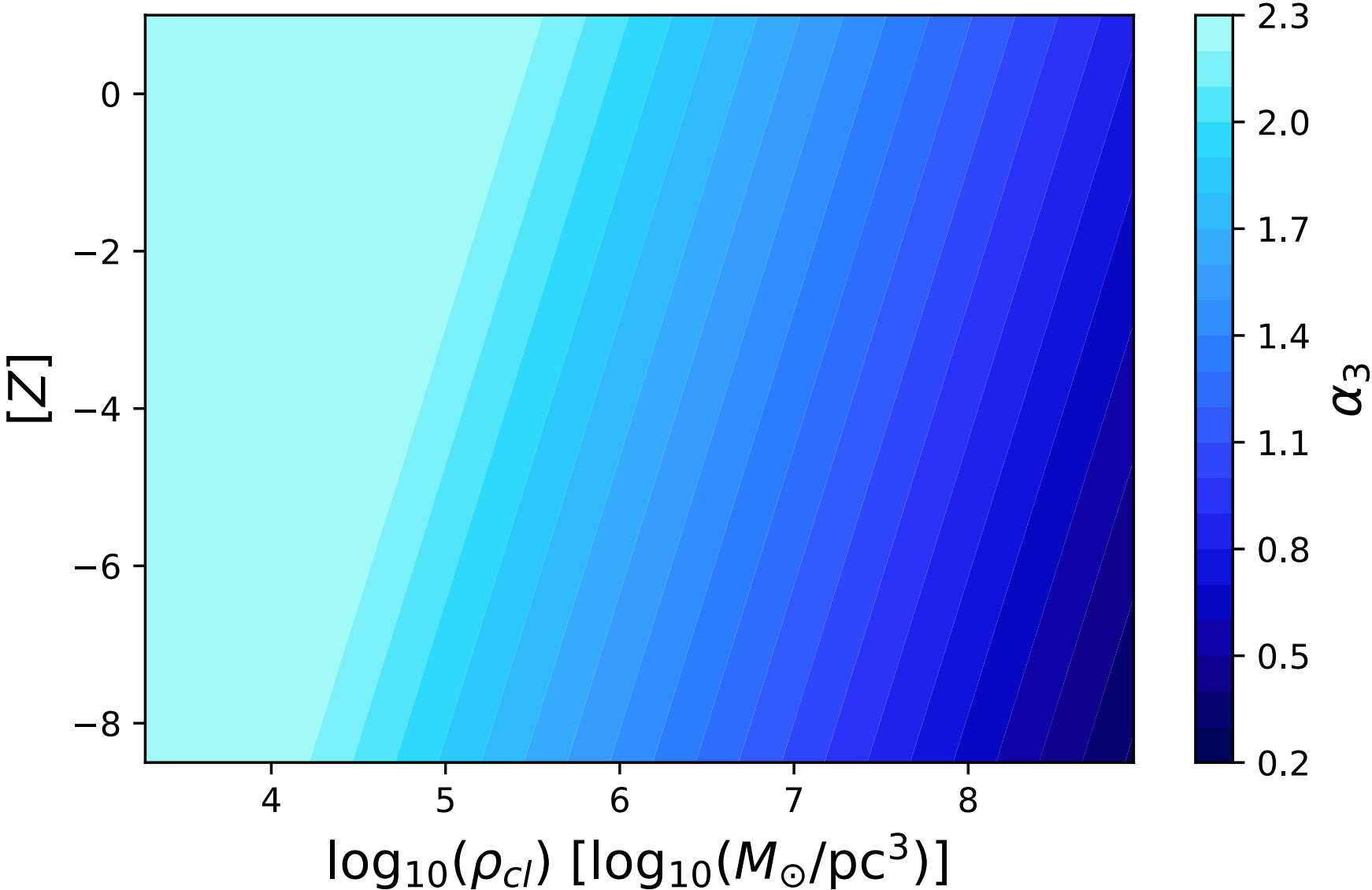
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$M_{\text{ecl,max}*}$	$= 10^{10} M_\odot$
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Table 1: Parameters

Stellar IMF for ind

$\xi_\star(n)$

extract $k_\star$ and m_{m}



(7)

(8)

(9)

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- Jerabkova et al. (2018)
- Yan et al. (2019,2020, 2021)

Actual galaxy-wide IMF

Dependence on SFR and metallicity (both functions of time)

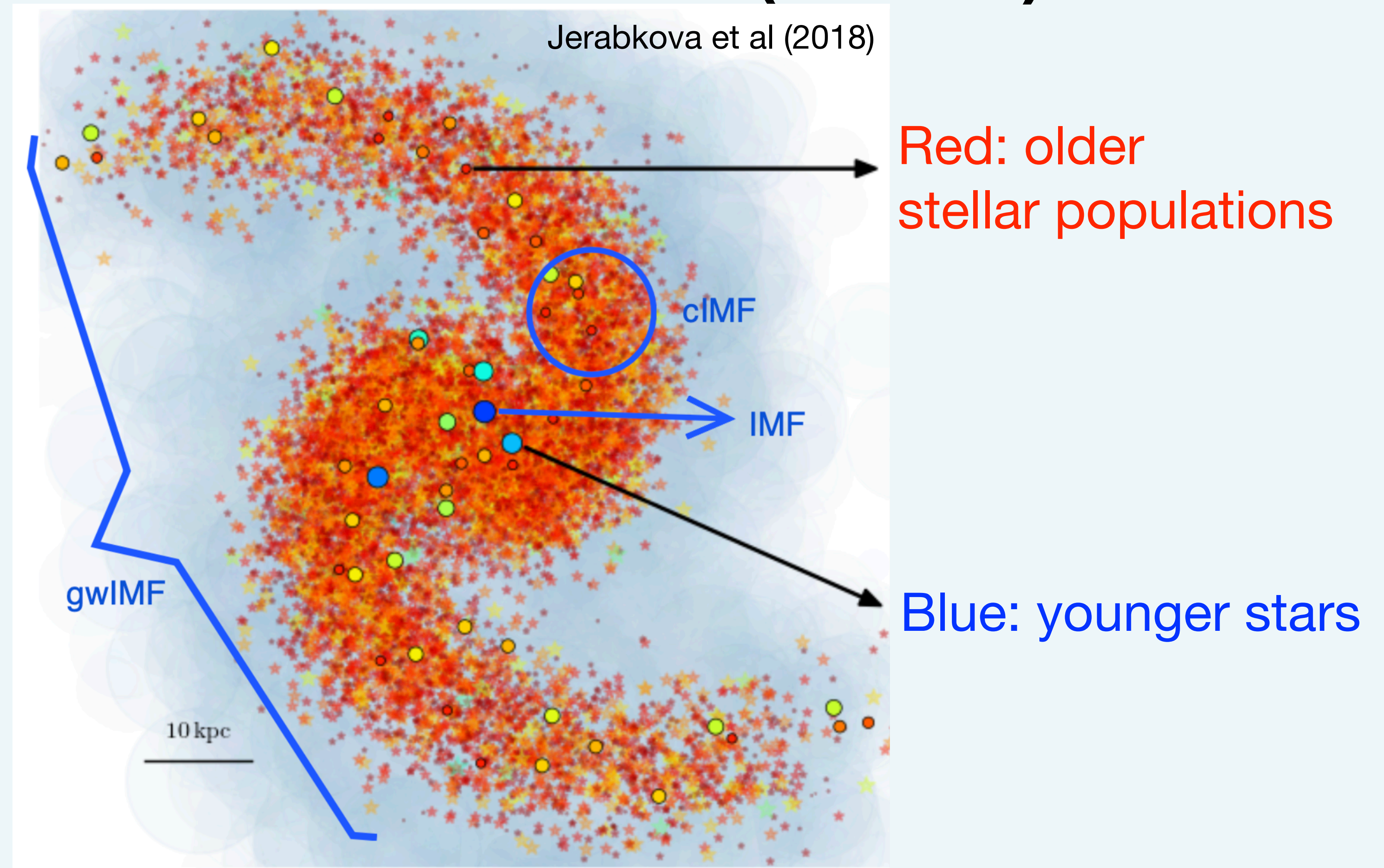
IGIMF (or equivalently, gwIMF)

$$\xi_{\text{IGIMF}}(m; t) = (dN_{\star}/dm)(t) = \int_{5M_{\odot}}^{M_{\text{ecl}, \text{max}}} \xi_{\text{ecl}}(M_{\text{ecl}}, \bar{\psi}_{\delta t}(t)) \xi_{\star}(m, M_{\text{ecl}}, [Z](t)) dM_{\text{ecl}}. \quad (22)$$

Effectively,

$$IN(Z, SFR) = OUT(m) \quad (23)$$

Integrated Galaxy-wide Initial Mass Function (IGIMF)



Formulation from

Kroupa & Weidner (2003)

Weidner & Kroupa (2005, 06, 10, 13)

Marks & Kroupa (2012)

Kroupa et al. (2013)

[...]

Jerabkova et al. (2018)

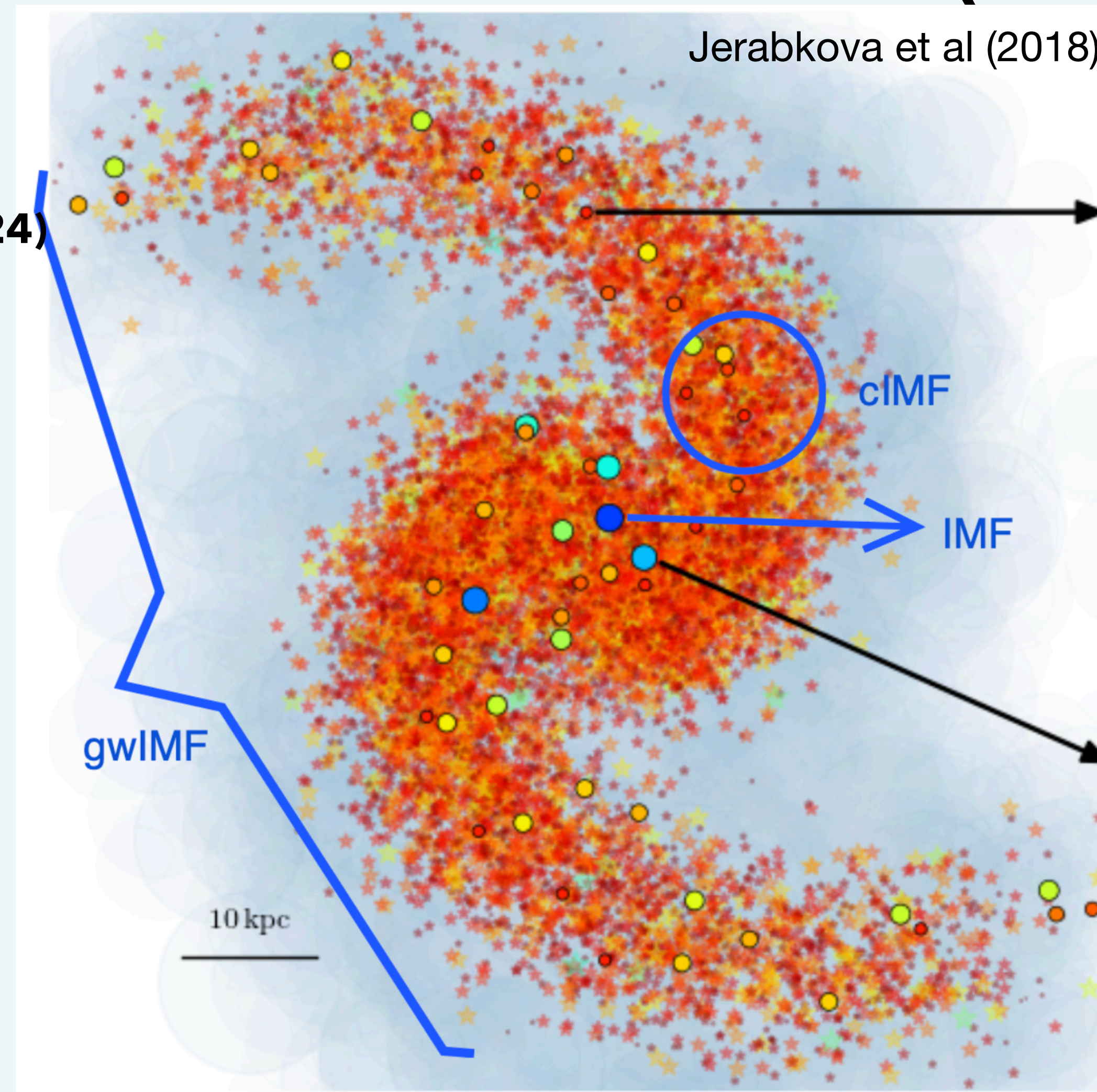
Yan et al. (2017, 2021, 2023)

Kroupa, Gjergo, Jerabkova, Yan (2024)

Gjergo et al. (2025)

Gjergo, Zhang, Kroupa (subm.)

Integrated Galaxy-wide Initial Mass Function (IGIMF)



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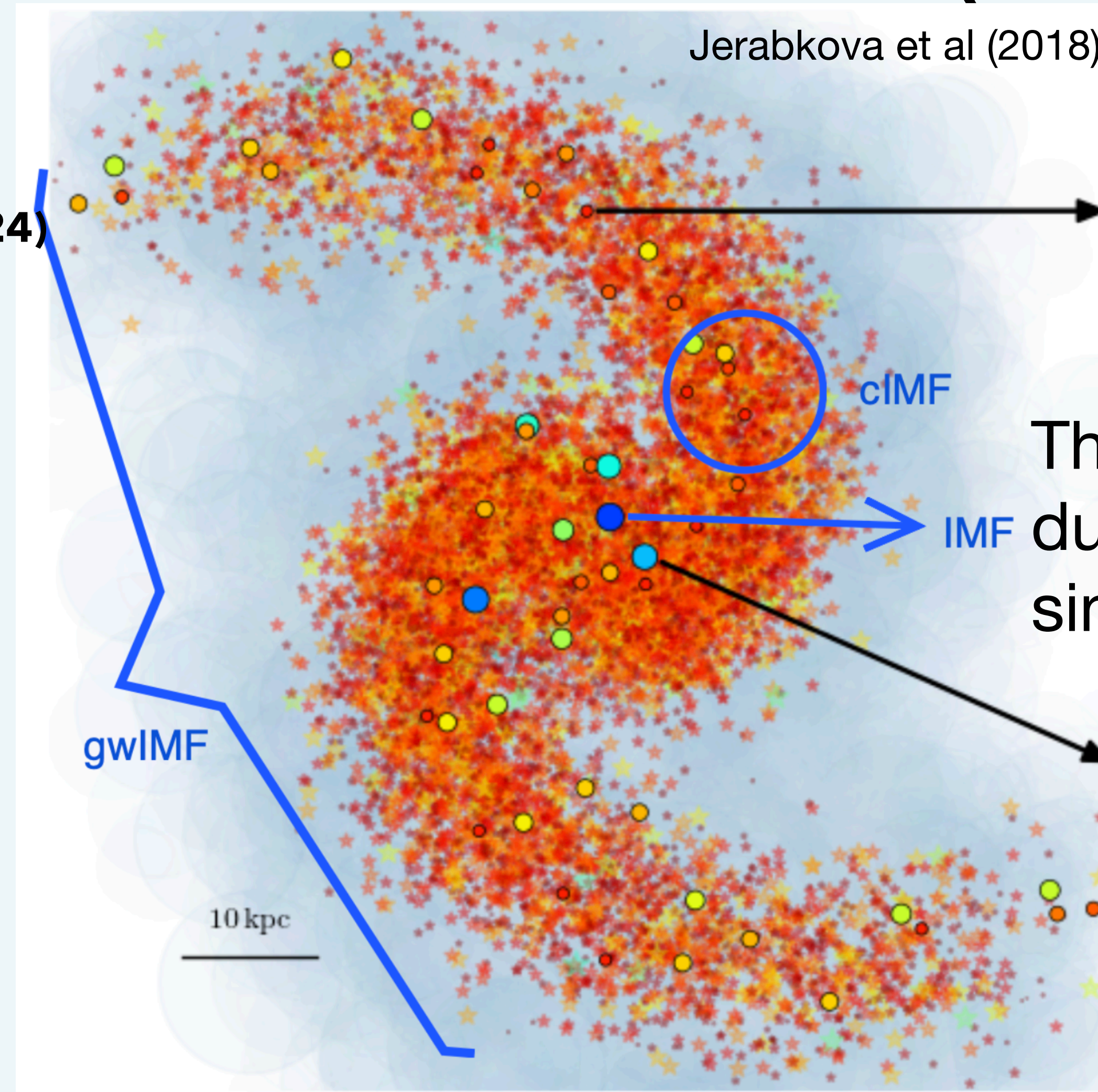
Jerabkova et al. (2018)

Yan et al. (2017, 2021, 2023)

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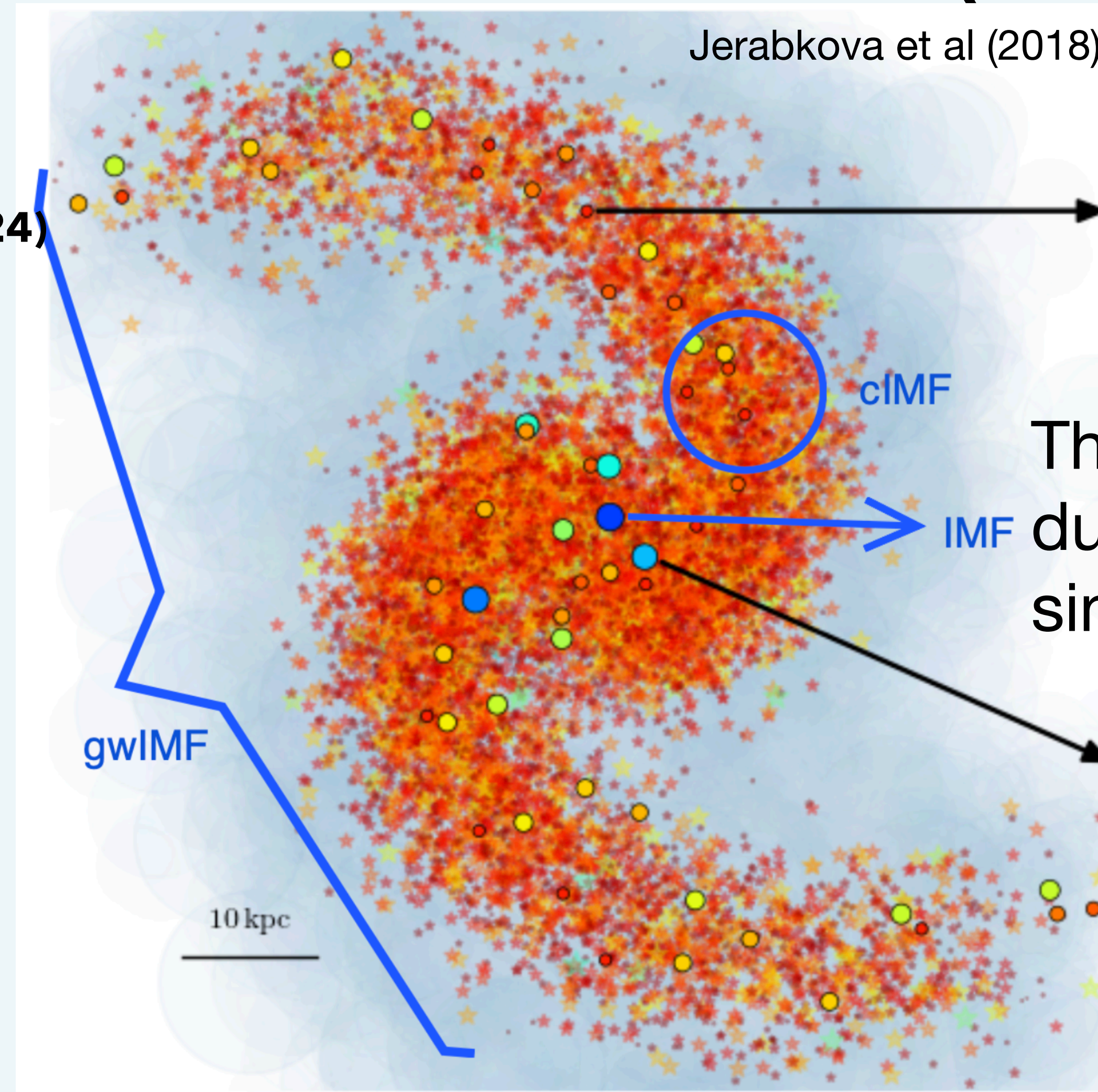
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Red: older
stellar populations

The stellar IMF emerges only
during the formation of
single stellar populations

Blue: younger stars

$$\text{IMF} = \frac{\Delta N \text{ of stars}}{\Delta \text{ mass of stars}}$$

Integrated Galaxy-wide Initial Mass Function (IGIMF)

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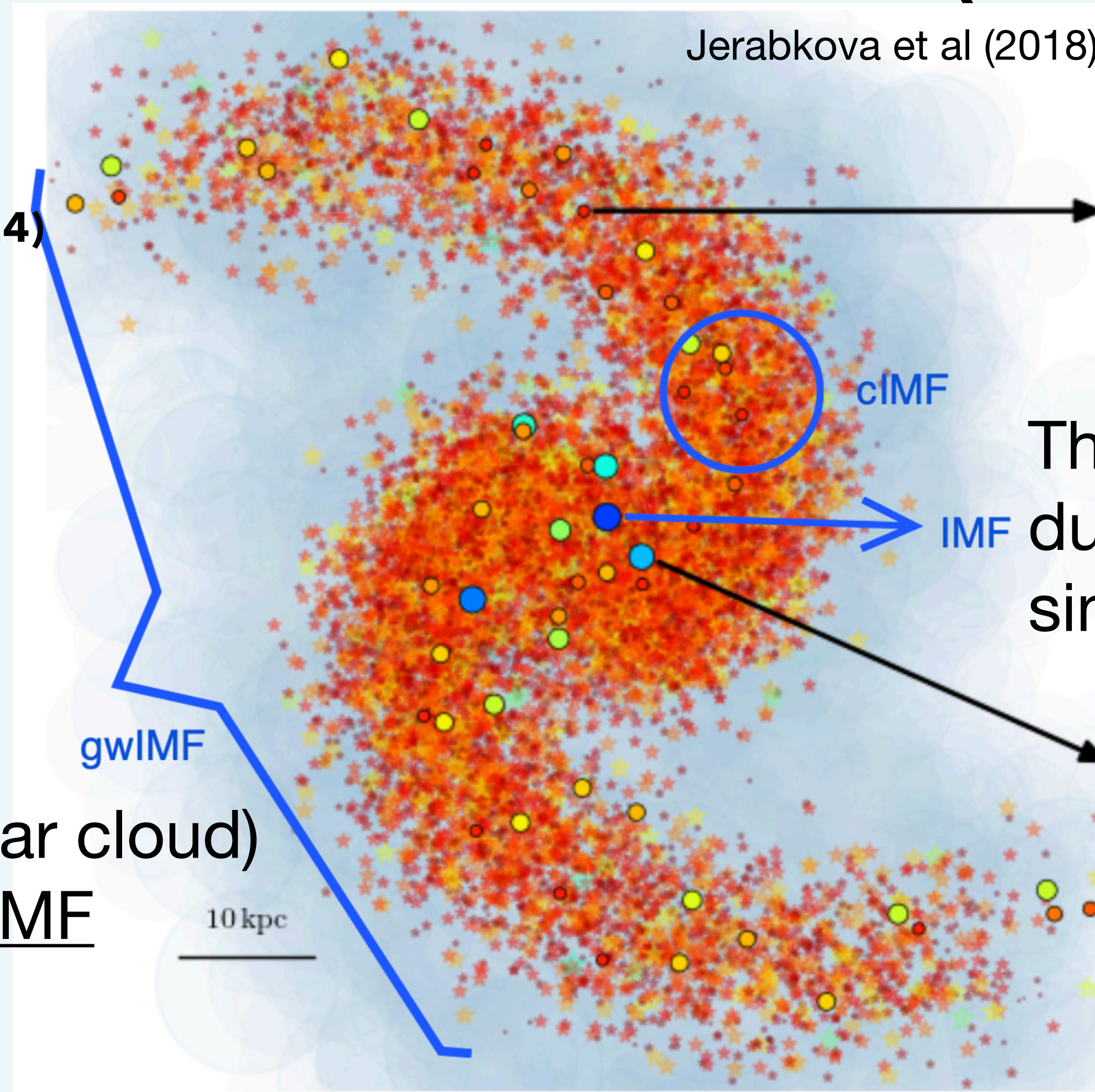
Yan et al. (2017, 2021, 2023)

Kroupa, Gjergo, Jerabkova, Yan (2024)

Gjergo et al. (2025)

Gjergo, Zhang, Kroupa (subm.)

The **cumulative sum**
of all stellar IMFs
formed in past 10 Myr
(The lifetime of a molecular cloud)
defines the galaxy-wide IMF
(gwIMF)



Jerabkova et al (2018)

**Red: older
stellar populations**

The stellar IMF emerges only
during the formation of
single stellar populations

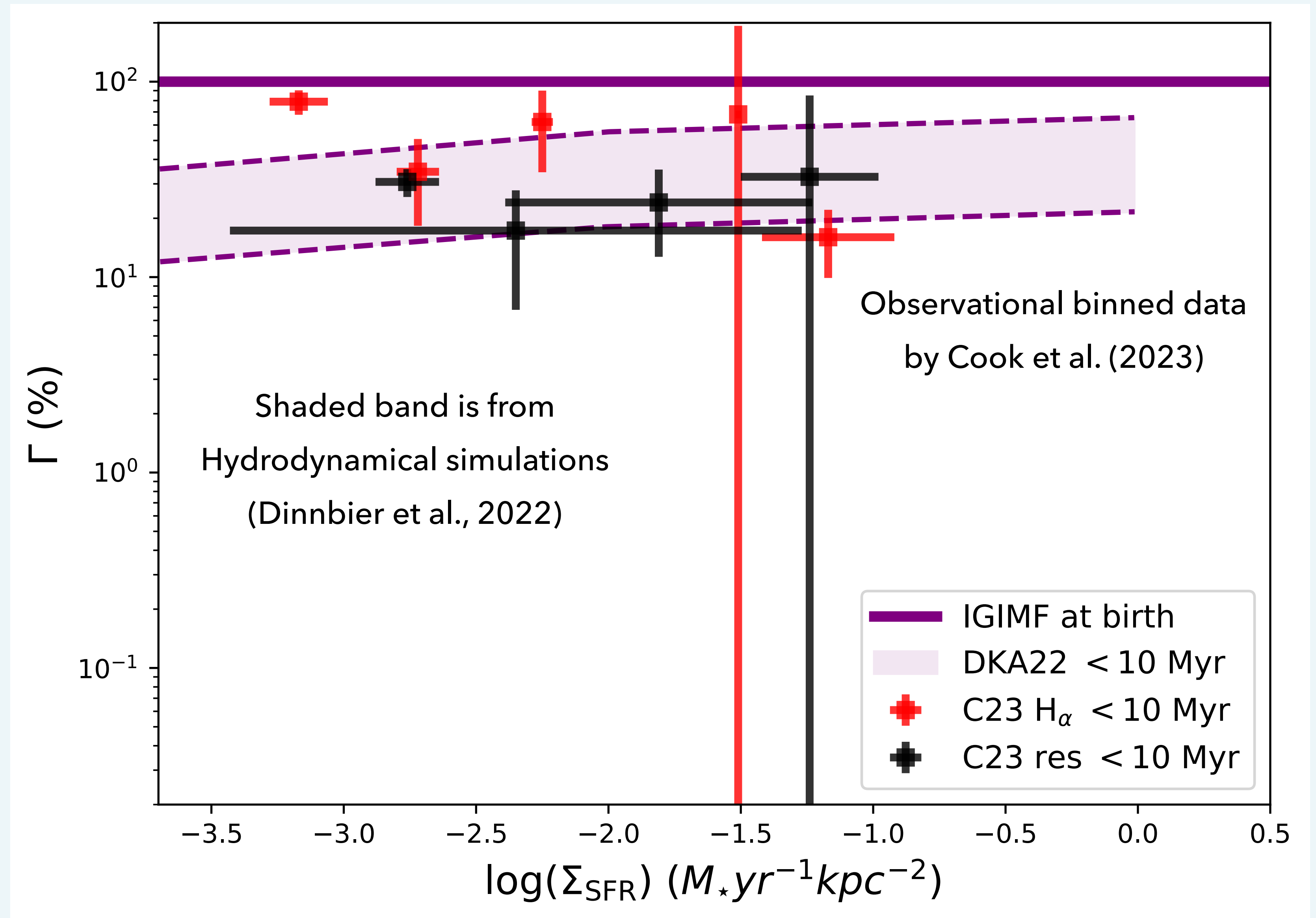
Blue: younger stars

$$\text{IMF} = \frac{\Delta N \text{ of stars}}{\Delta \text{ mass of stars}}$$

Where do stars form?

Γ is the percentage of stars born in embedded clusters

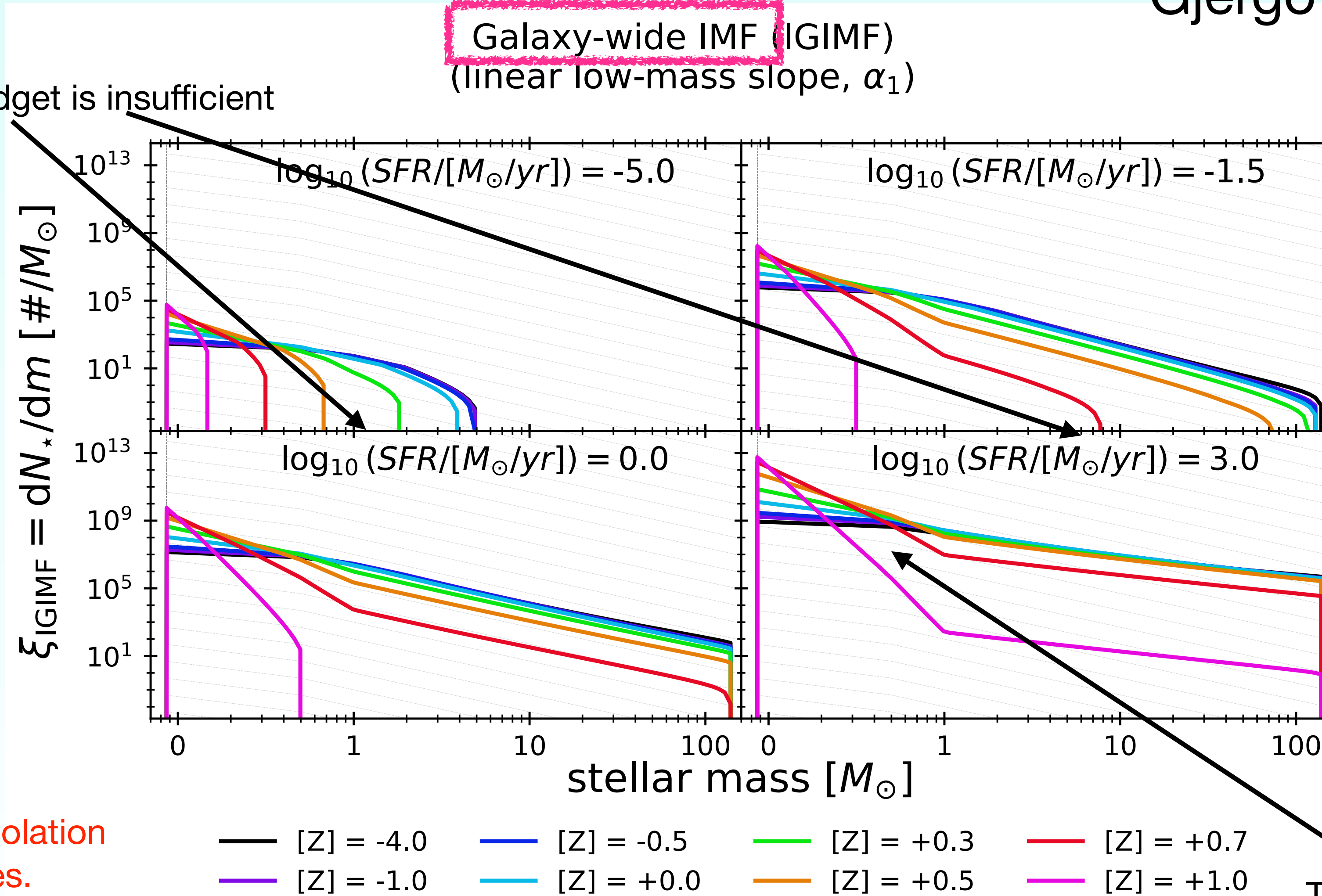
Gjergo et al. (2025, RAA)



IGIMF in practice

Gjergo et al. (2025, RAA)

Optimal sampling:
If the star-forming mass budget is insufficient
Massive stars cannot form



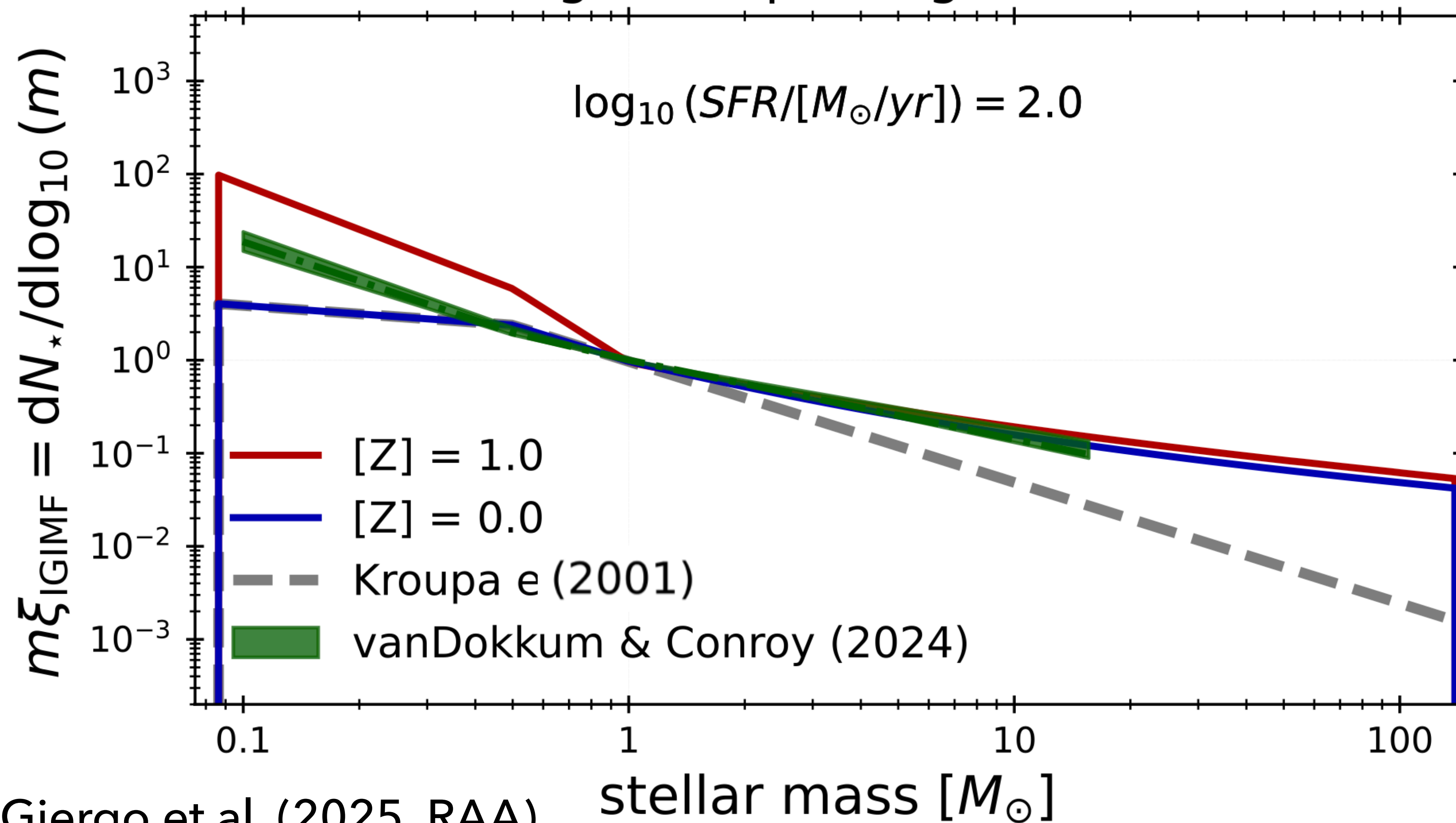
Higher SFR:
1) Raises the entire gwIMF
2) Makes it top-heavy
(if the mass budget is sufficient)

The higher the metallicity, [Z]
The steeper the low-mass slope

Disclaimer: untested extrapolation
for high $[Z] > 0.3$ metallicities.
Alternative proposed in
Gjergo et al. (2025)

A special case of the IGIMF: high- z elliptical galaxies

$$\log_{10} (SFR/[M_{\odot}/\text{yr}]) = 2.0$$



Gjergo et al. (2025, RAA)

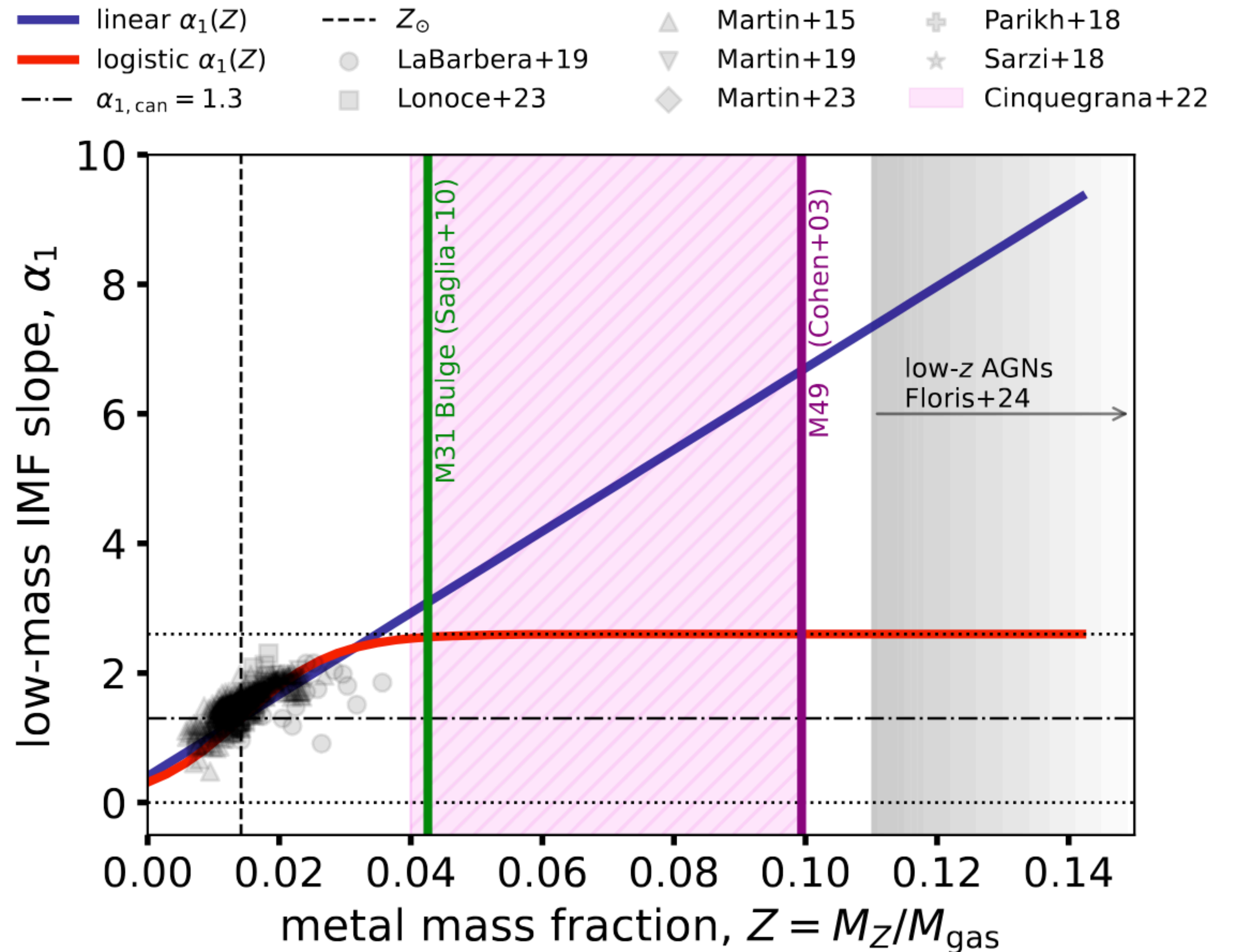
While the average IMF of most spiral galaxies looks like Kroupa (2001) (**Dashed-gray line**)

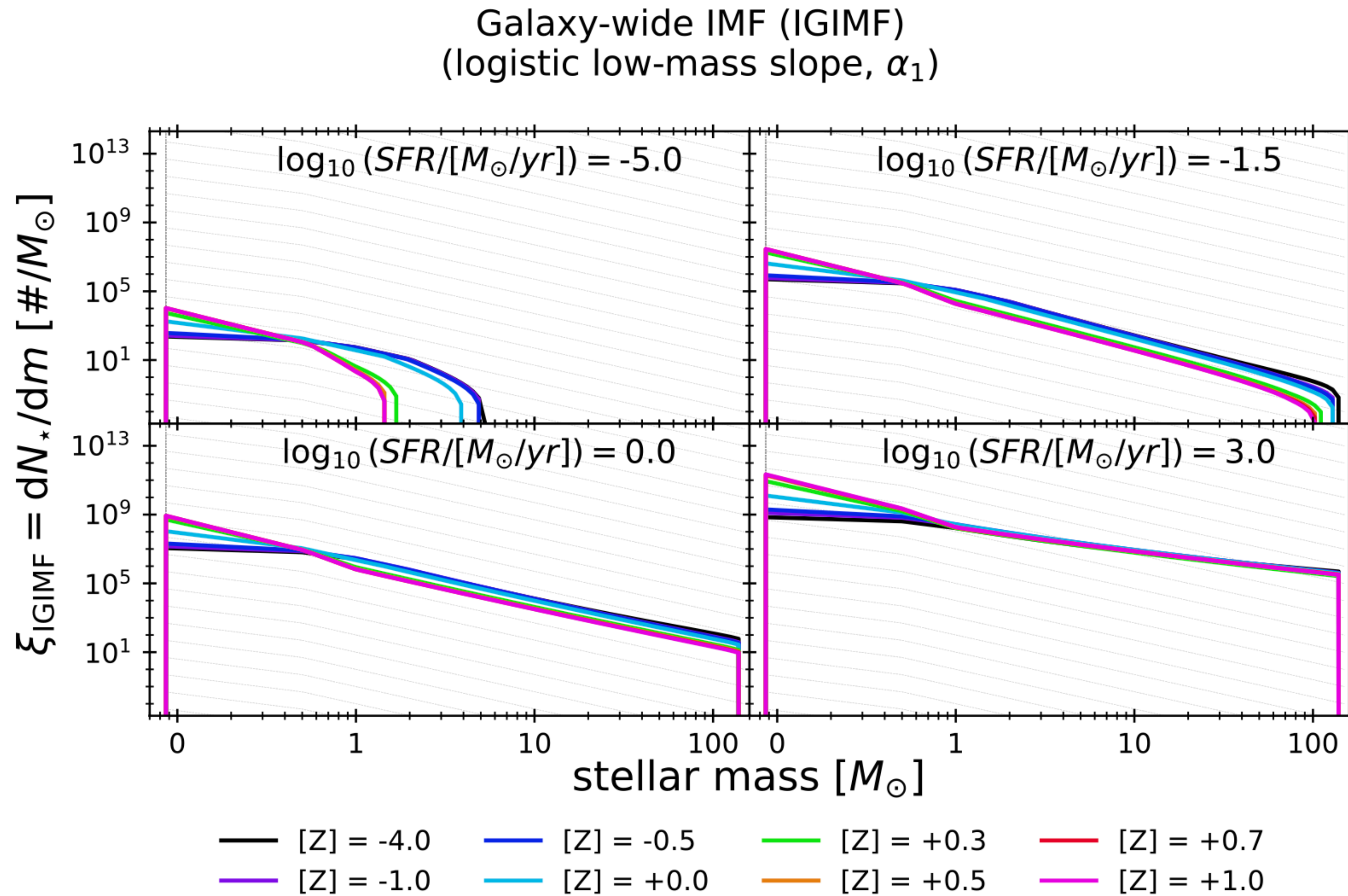
Distant elliptical galaxies have an average IMF like the **shaded green curve**.

It happens to be a special case of our galaxy-wide variable IMF model (IGIMF theory) for metal-rich starburst galaxies (**red** and **blue** curves)

Dependence of α_1 (the low-mass slope) on metal content

Gjergo et al. (2025, RAA)

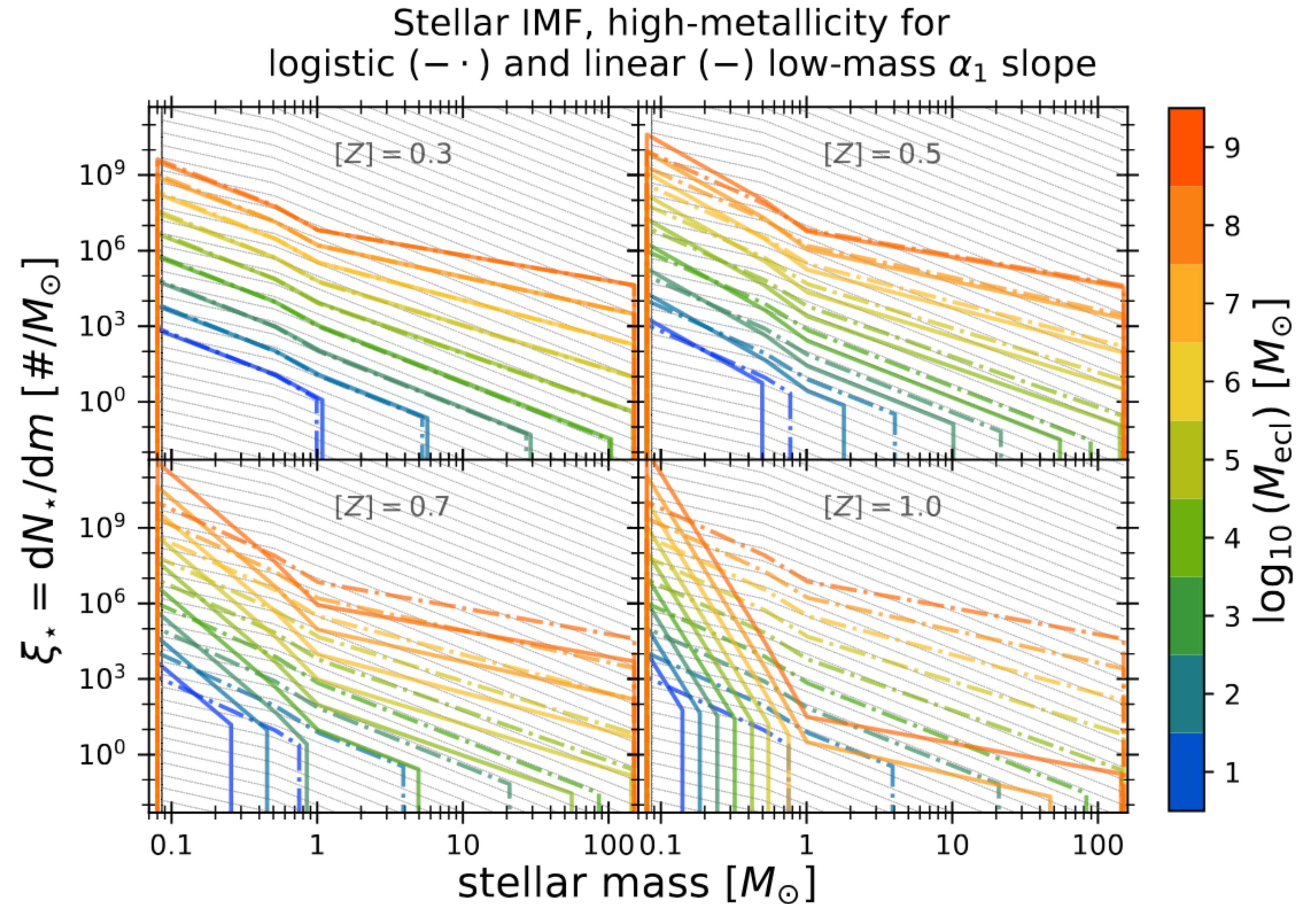




Gjergo et al. (2025, RAA)

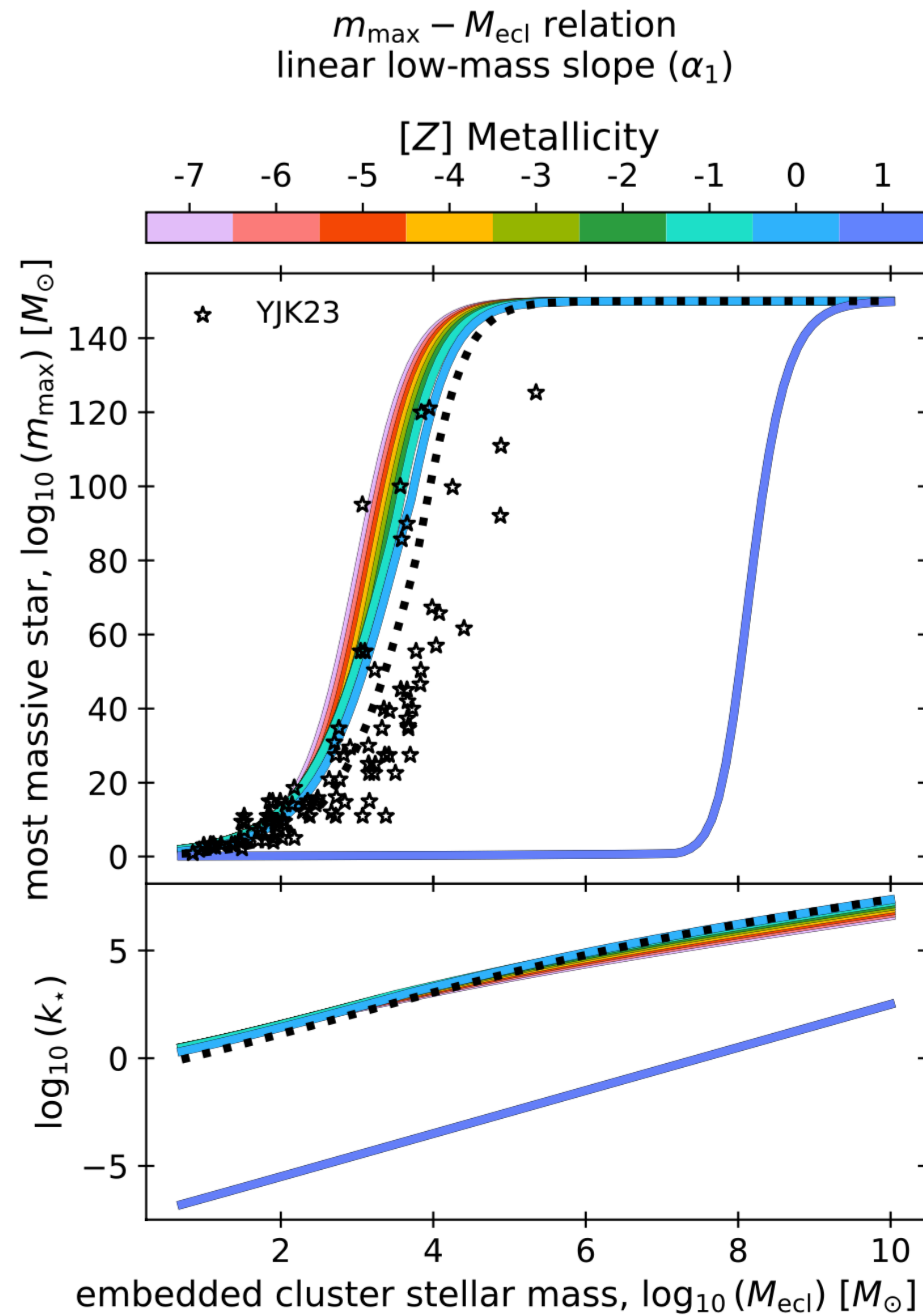
Variation of the stellar IMF at extreme supersolar metallicities

Gjergo et al. (2025, RAA)



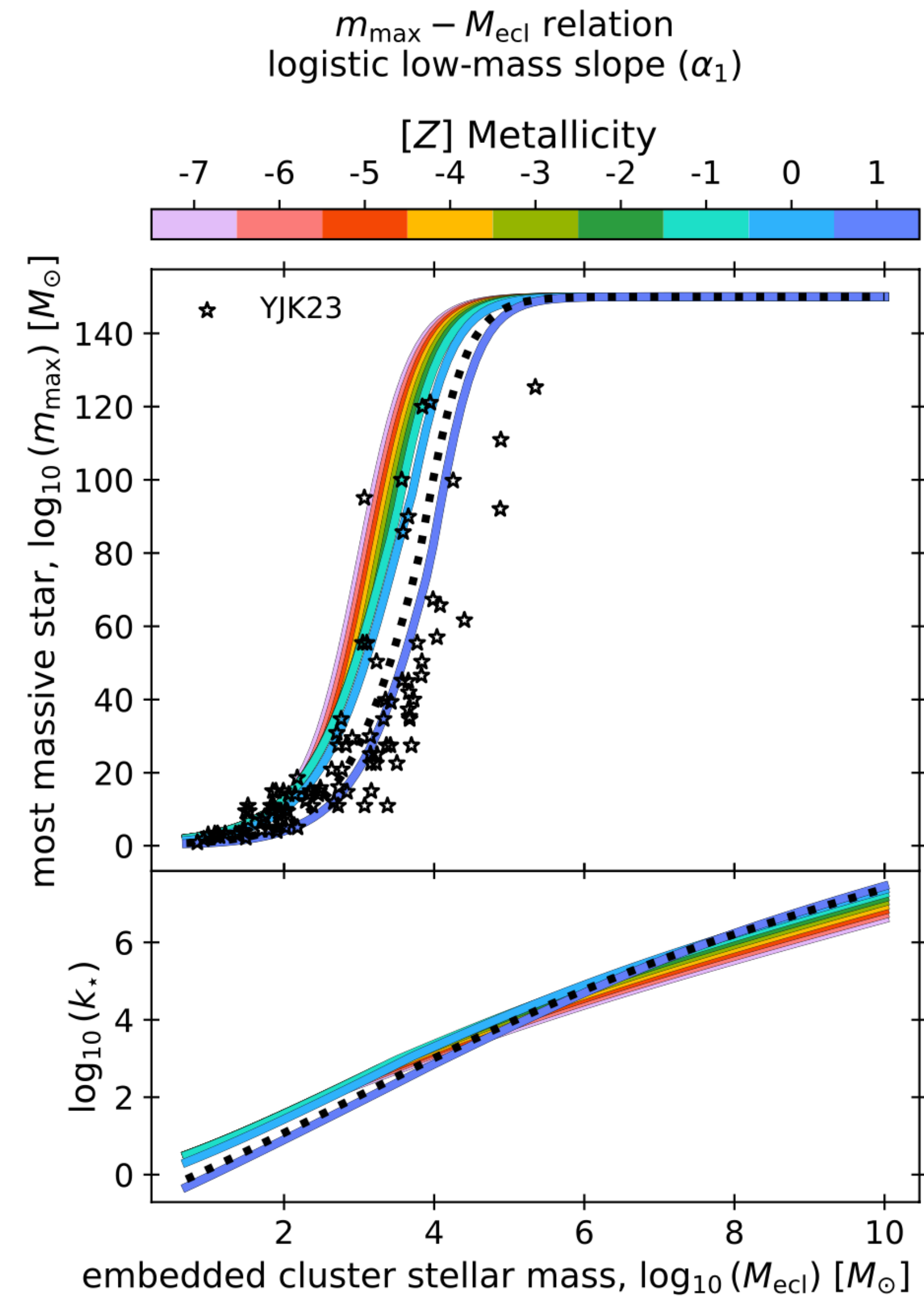
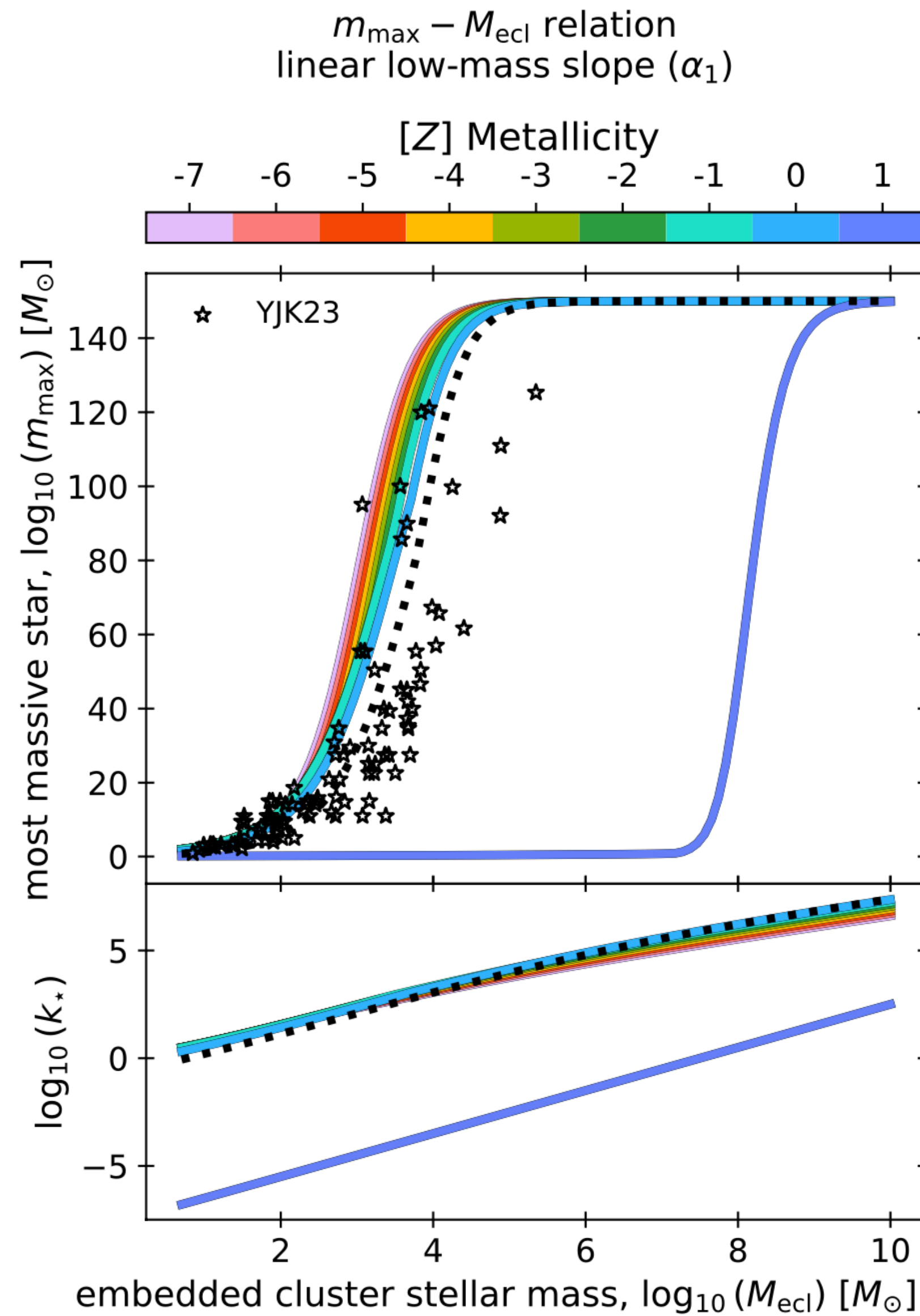
Extrapolation of the
m-max-Mecl relation
At high metallicities

Gjergo et al. (2025,
RAA)

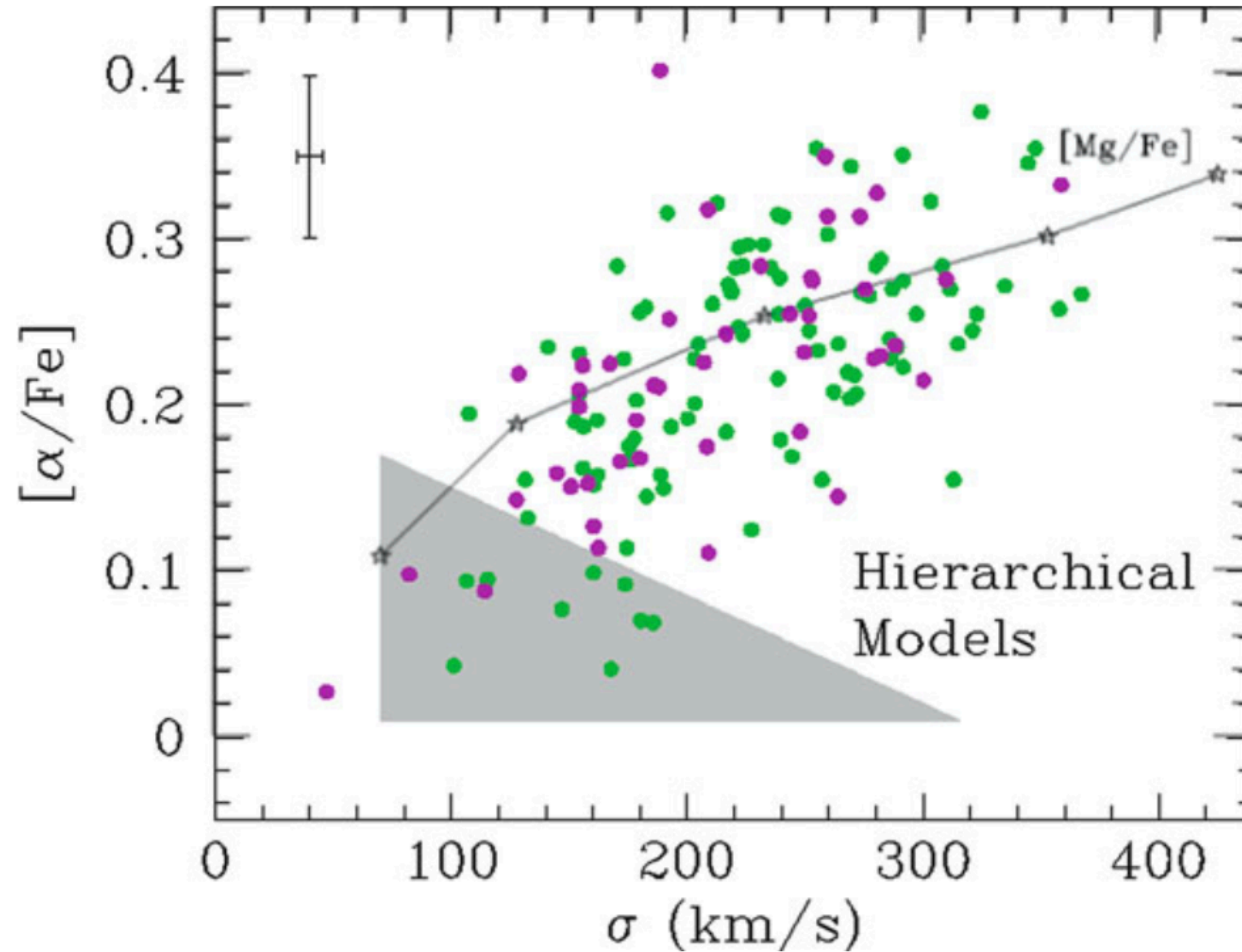


Extrapolation of the
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At high metallicities

Gjergo et al. (2025,
RAA)



Chemical Evolution

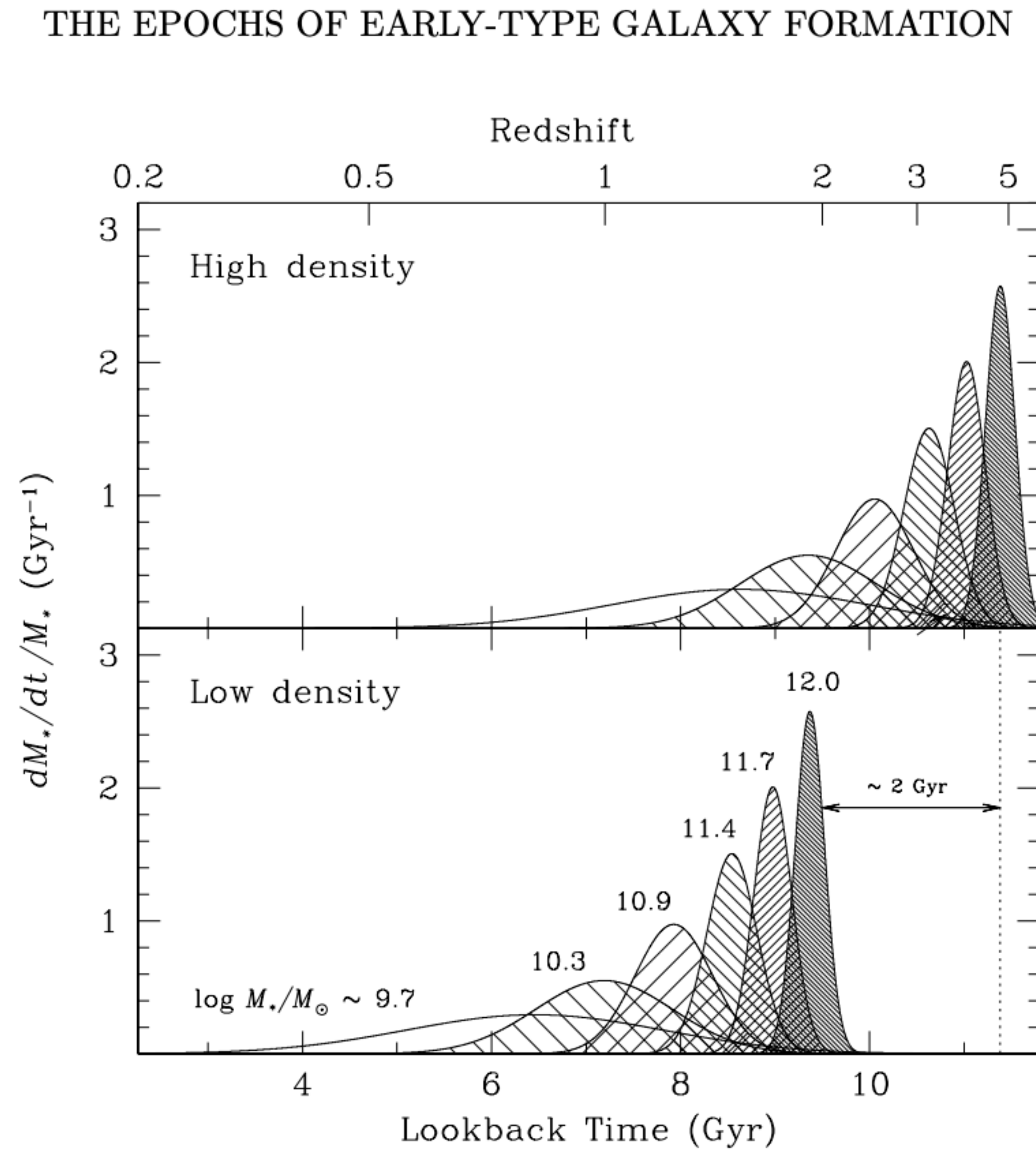


Matteucci (2012)

Figure adapted from
Thomas et al. (2002)

Curve from Matteucci & Pipino (2004)

Thomas et al. (2005)



**The CMB's meaning for
cosmological models
and the
role elliptical galaxies have
on this matter**



Pavel Kroupa

Helmholtz-Institut für Strahlungs- und Kernphysik (HISKP)

University of Bonn

c/o Argelander-Institut für Astronomie

University of Bonn

Astronomical Institut,

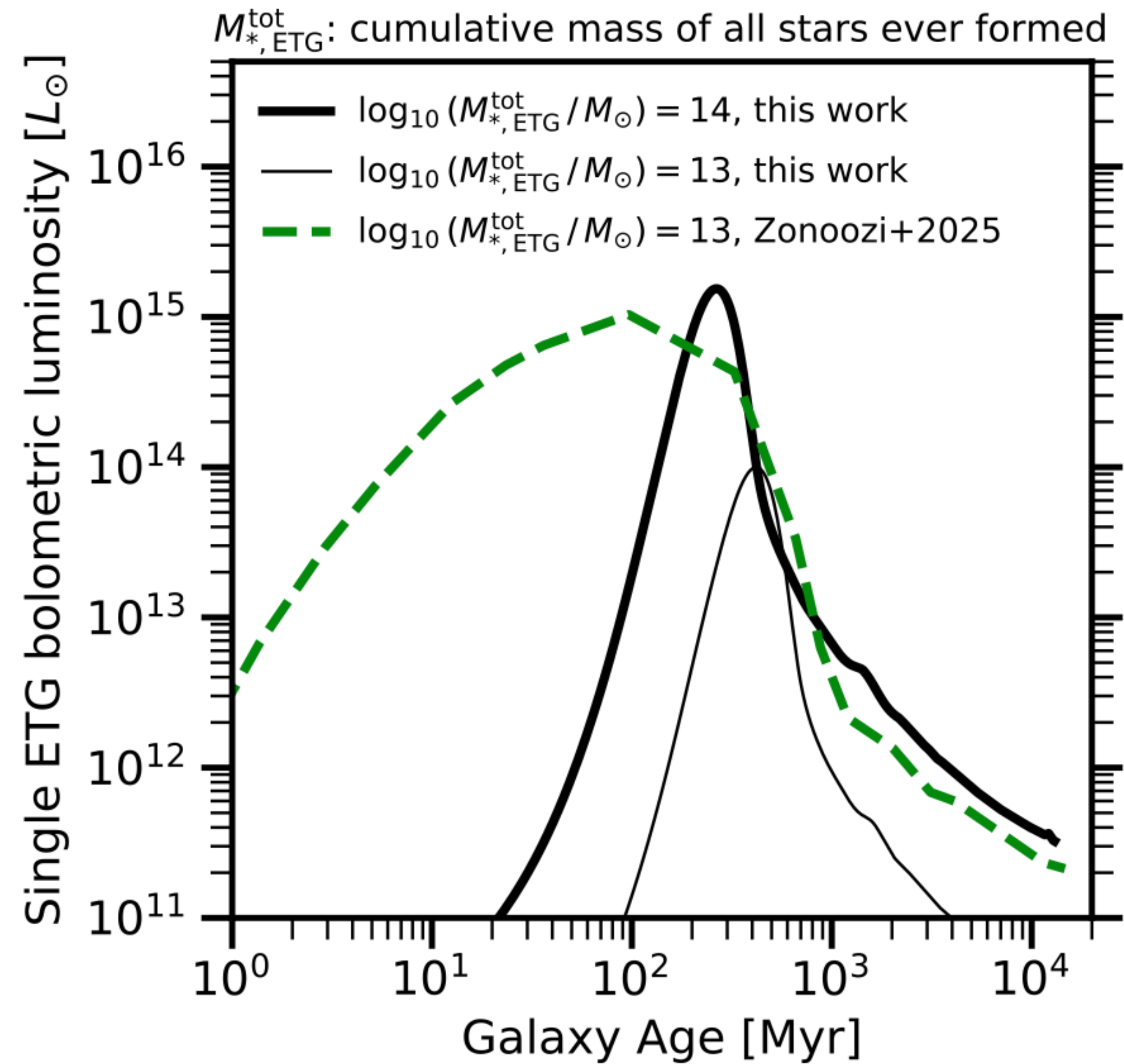
Charles University in Praha

AUUK, Praha

December 10th, 2025

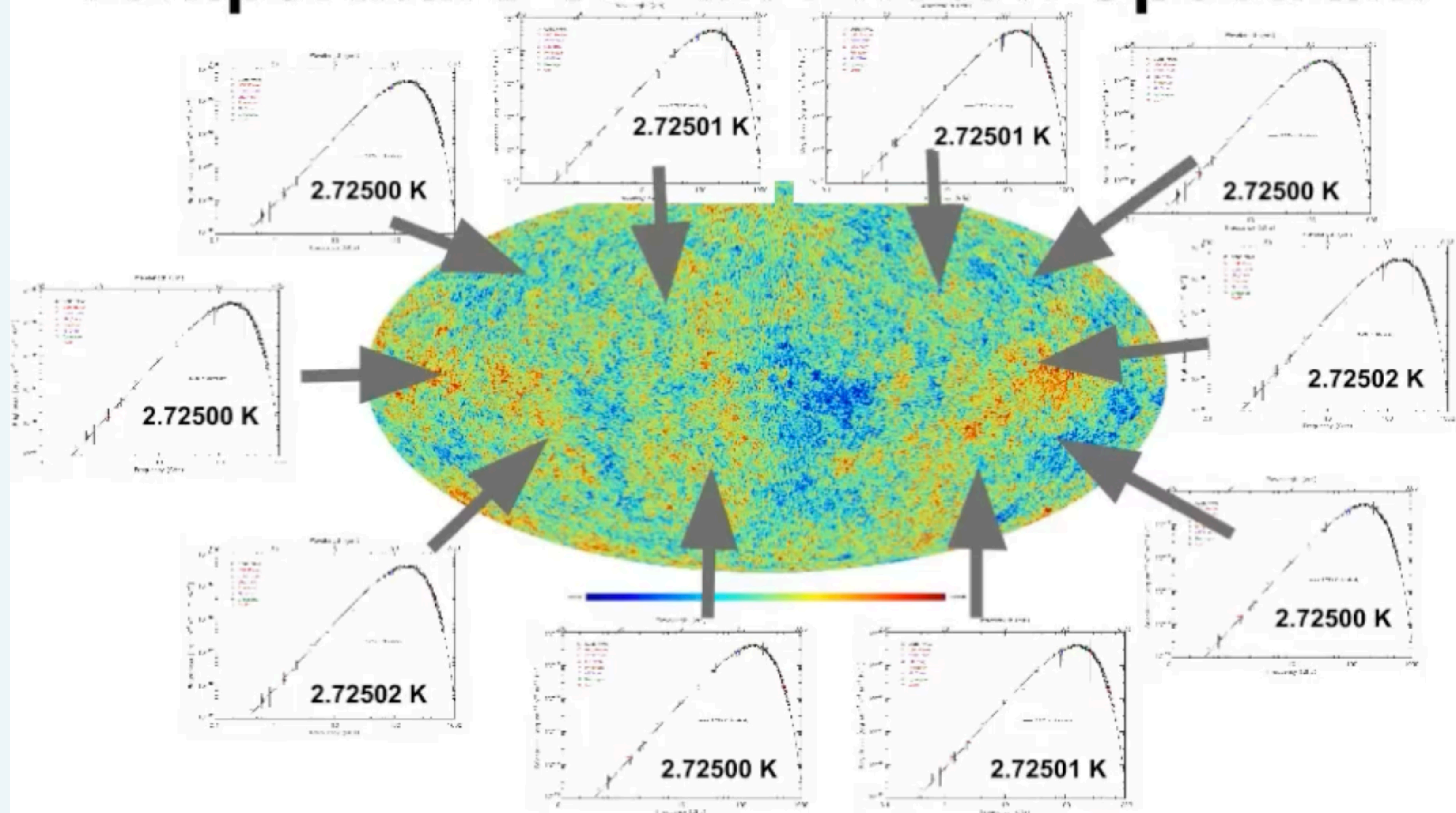
**With
Eda Gjergo
and
Nikolaos Samaras**

Pavel Kroupa: University of Bonn / Charles University

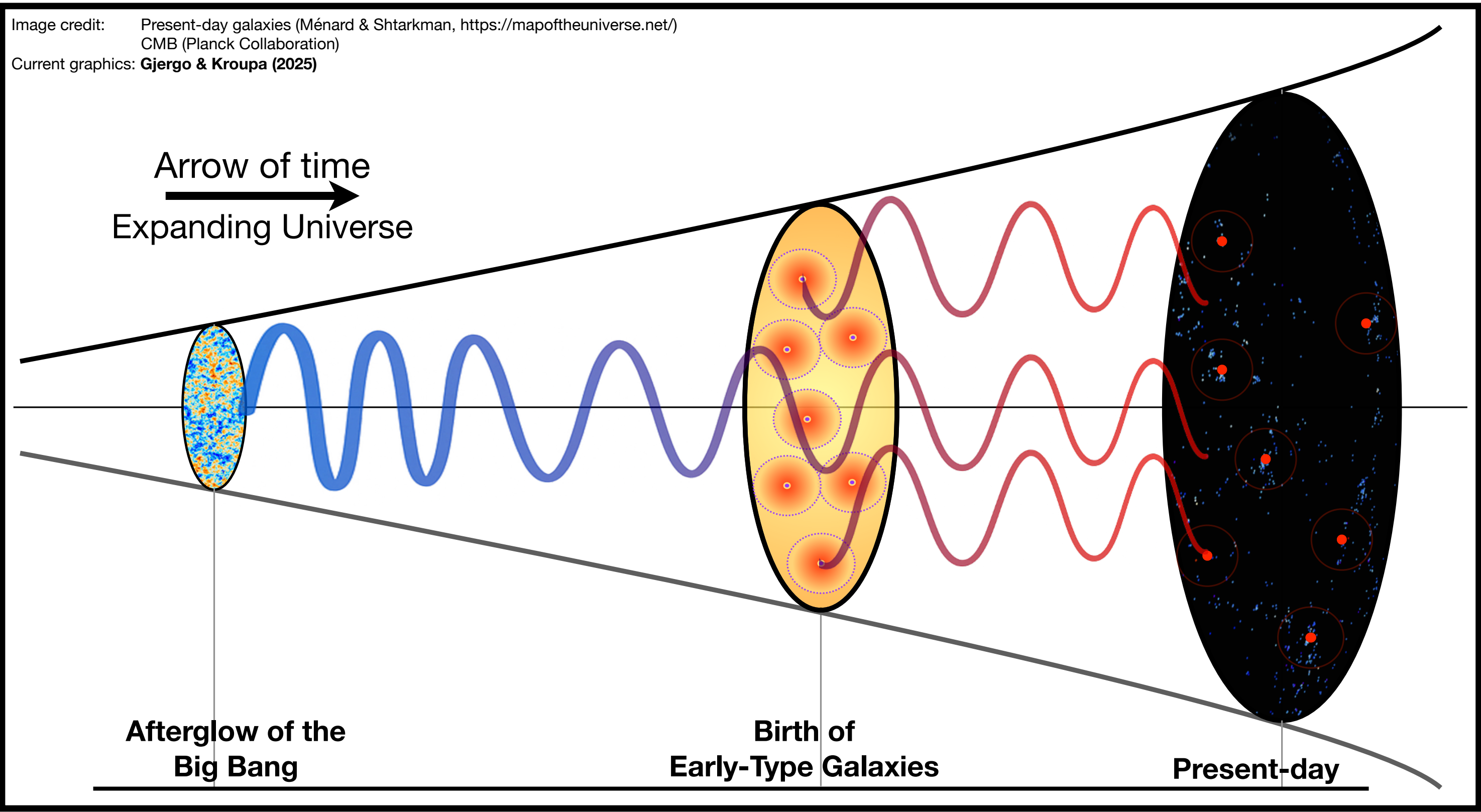


Gjergo & Kroupa (2025)

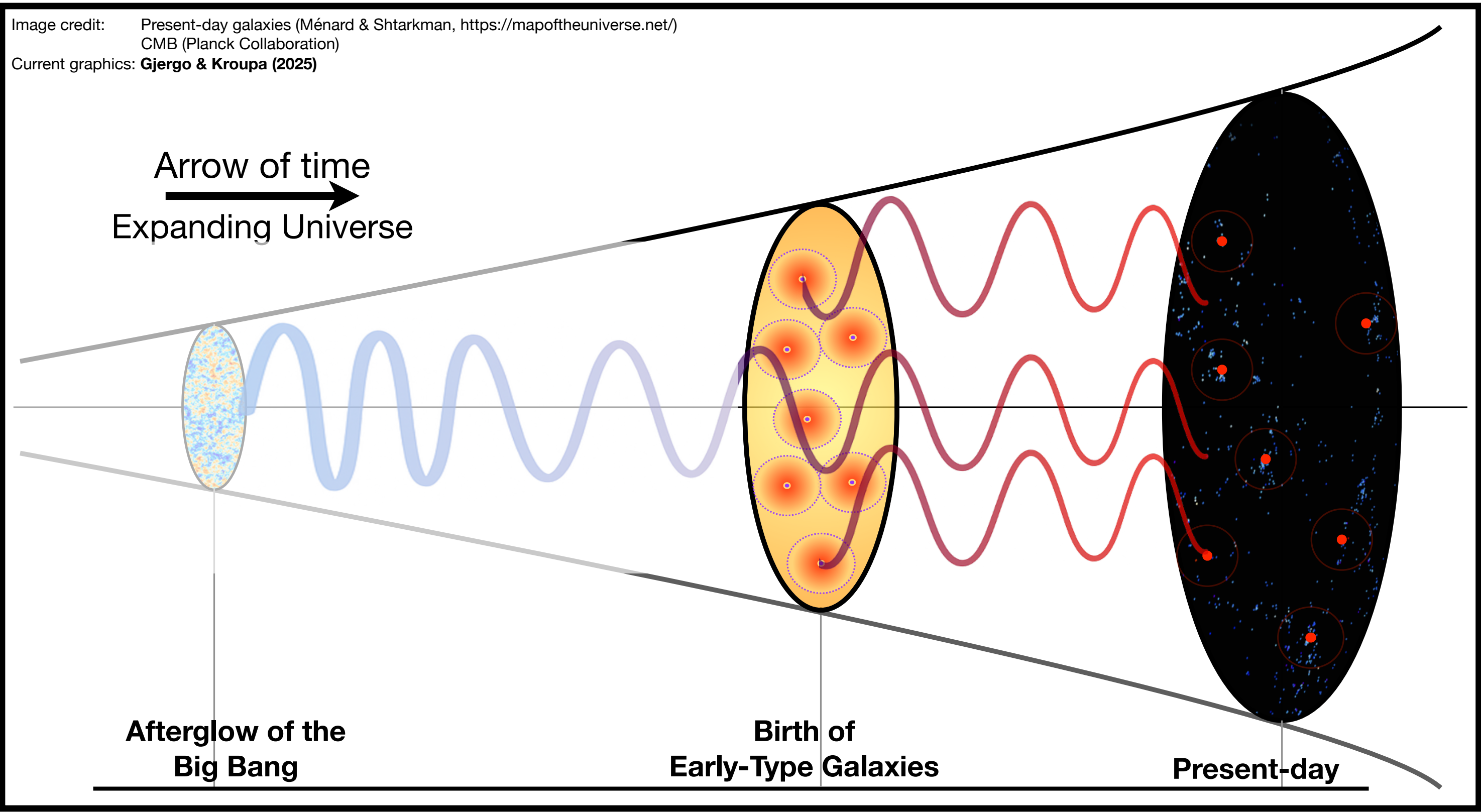
Every Tiny Color Spot on this Map is Temperature of Full Photon Spectrum



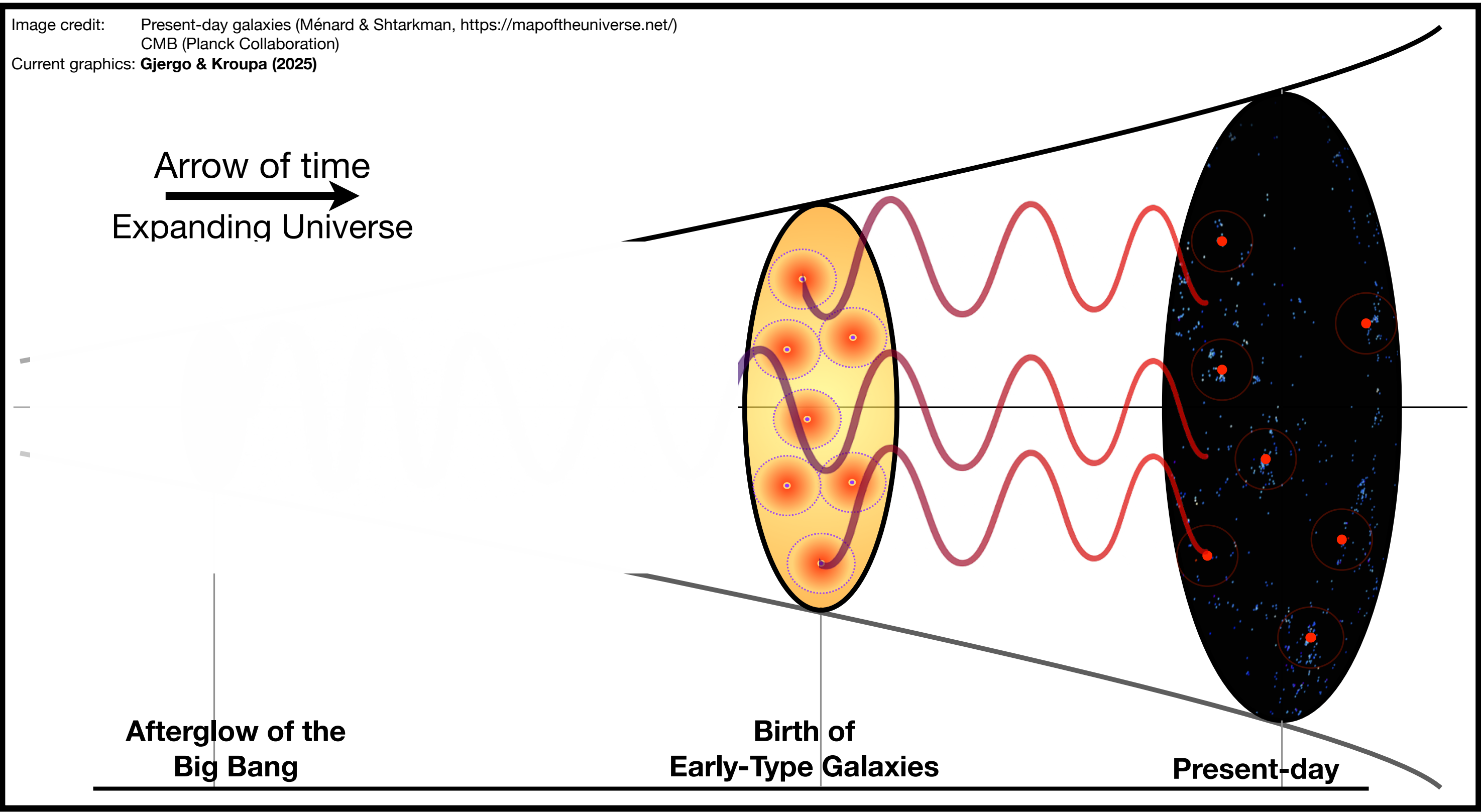
20 microKelvin variations



Gjergo & Kroupa (2025)

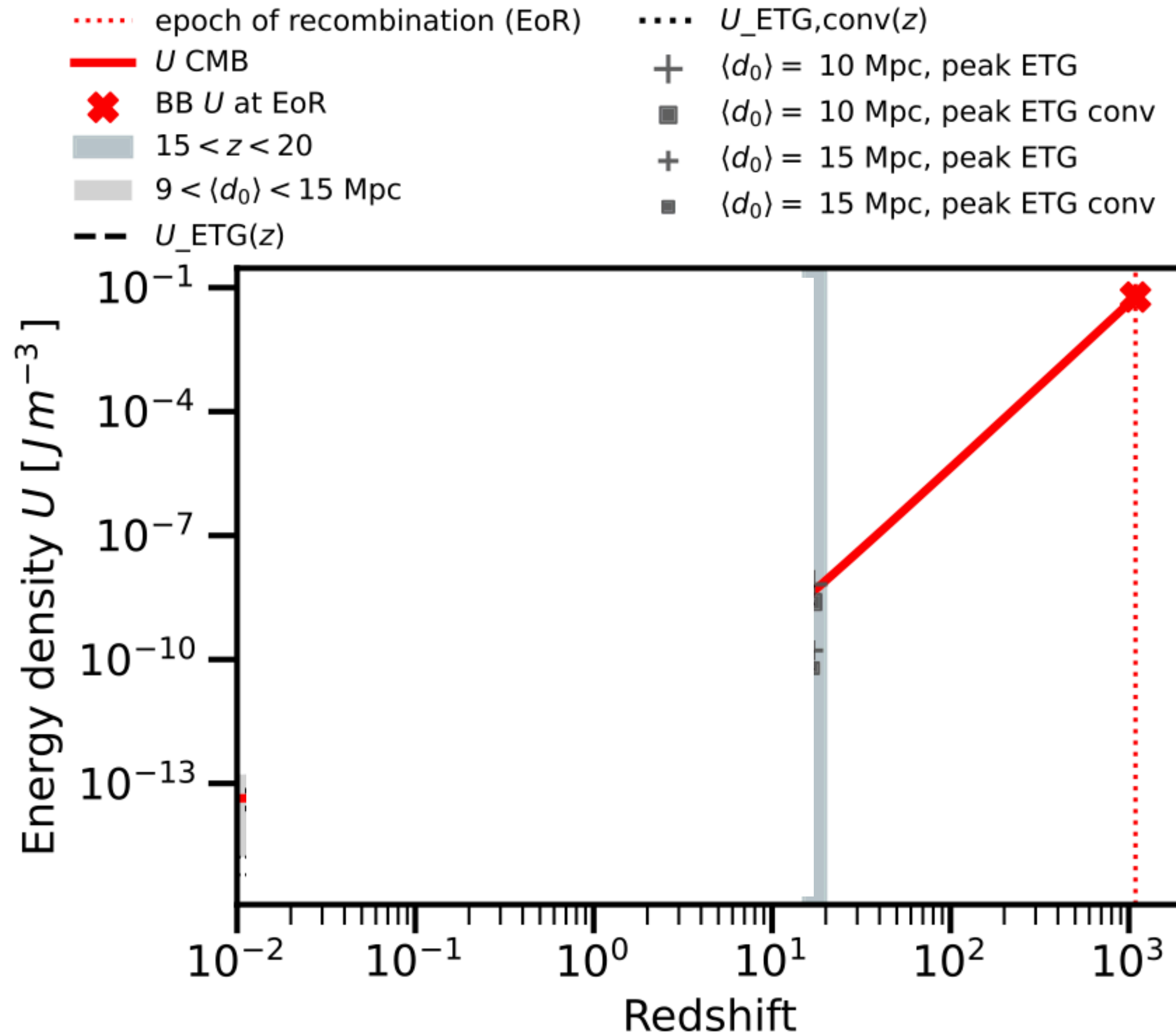


Gjergo & Kroupa (2025)



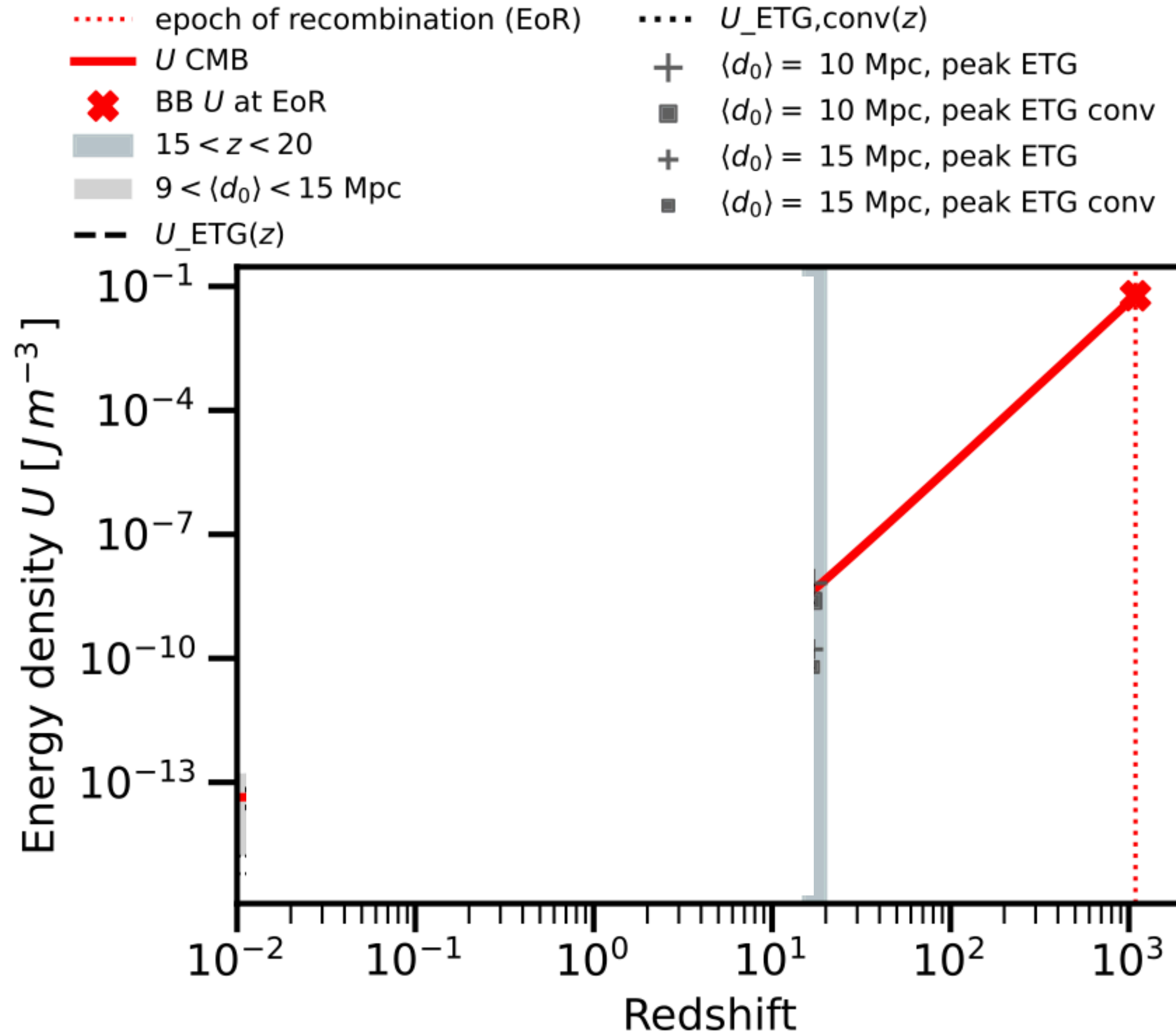
Gjergo & Kroupa (2025)

Gjergo & Kroupa (2025)



x Photosphere
of
Hot Big Bang

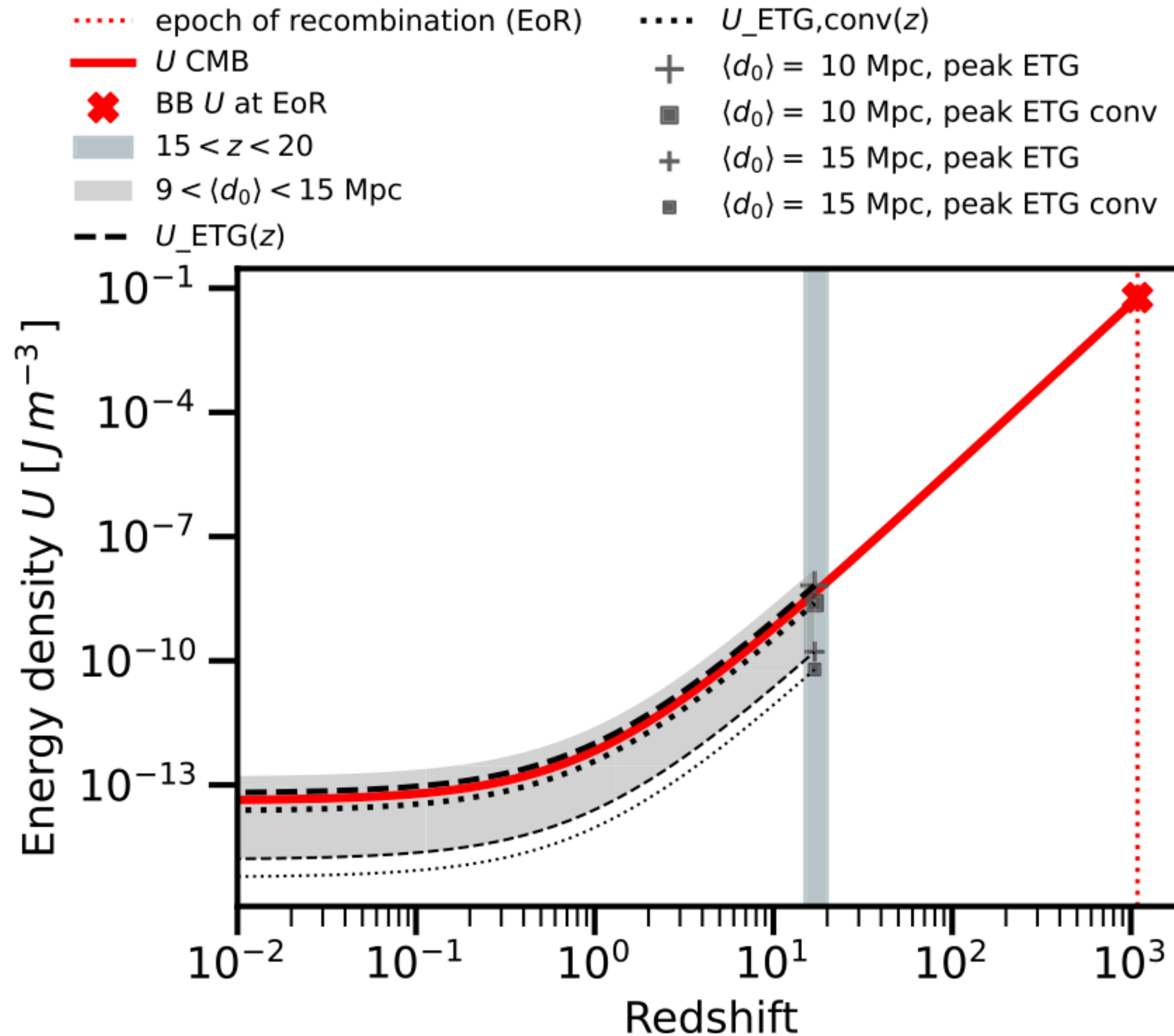
Gjergo & Kroupa (2025)



x Photosphere
of
Hot Big Bang

+ Elliptical
galaxies
form

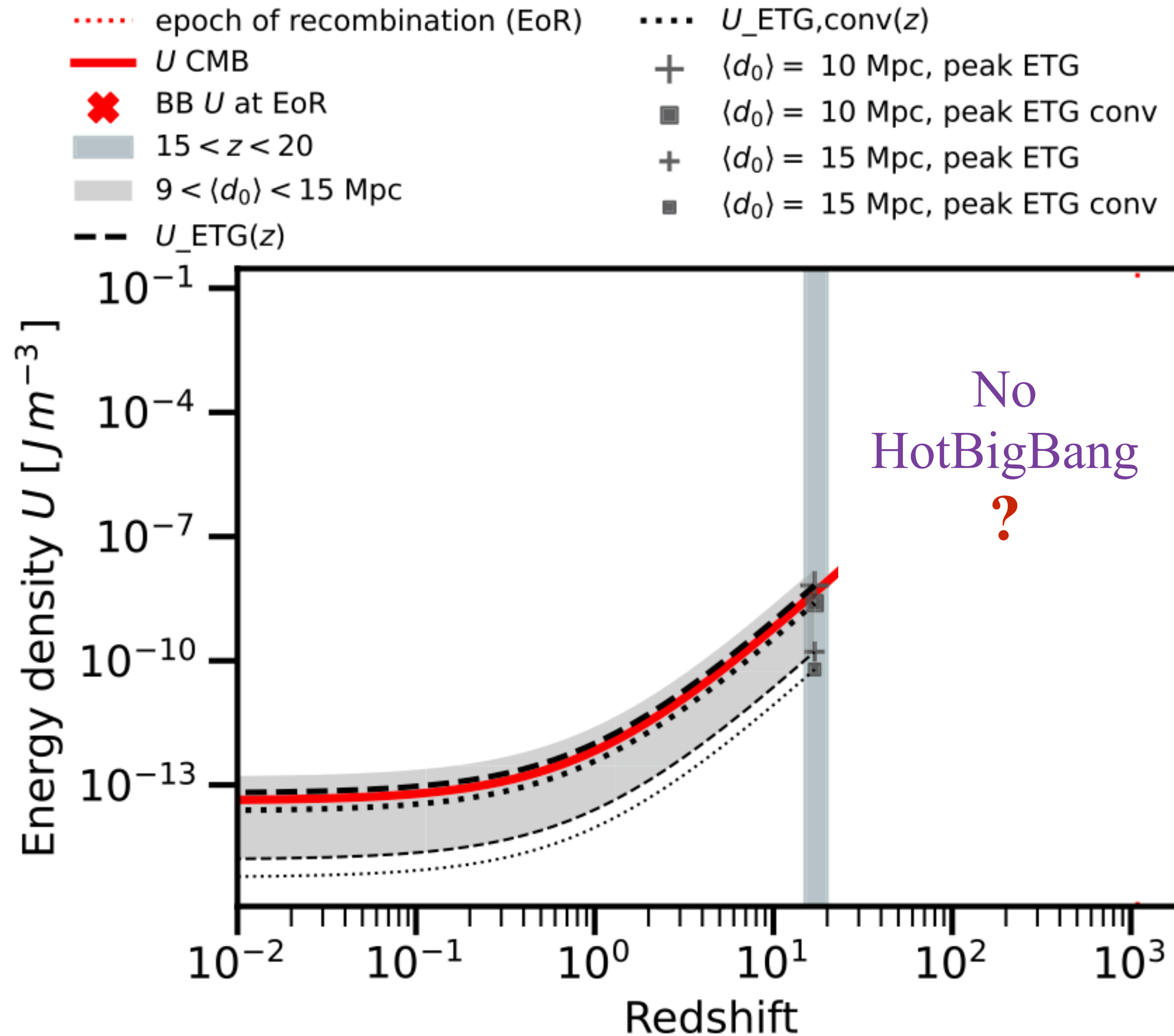
Gjergo & Kroupa (2025)



✕ Photosphere
of
Hot Big Bang

+ Elliptical
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Gjergo & Kroupa (2025)



x Photosphere
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Take-home messages

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- **Stars make the elements:** Each star synthesizes and ejects different isotopic yields, set by its initial mass and composition, and by its evolutionary pathway.

Take-home messages

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- **What GCE does:** Galactic chemical evolution models combine a galaxy's IMF (initial mass function) and SFH (star formation history) with stellar yields and stellar lifetimes to predict the time evolution of elemental and isotopic abundances.

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- **What GCE does:** Galactic chemical evolution models combine a galaxy's IMF (initial mass function) and SFH (star formation history) with stellar yields and stellar lifetimes to predict the time evolution of elemental and isotopic abundances.
- **Why it matters:** Abundance patterns act as tight constraints on galaxy evolution (Lecture 3)

Take-home messages

- **Same baryons everywhere:** The same baryonic matter found on Earth is present throughout the Universe
 - but elemental and isotopic abundances vary across environments and cosmic time.
- **Stars make the elements:** Each star synthesizes and ejects different isotopic yields, set by its initial mass and composition, and by its evolutionary pathway.
- **What GCE does:** Galactic chemical evolution models combine a galaxy's IMF (initial mass function) and SFH (star formation history) with stellar yields and stellar lifetimes to predict the time evolution of elemental and isotopic abundances.
- **Why it matters:** Abundance patterns act as tight constraints on galaxy evolution (Lecture 3)
- **Where it is incomplete:** Key uncertainties and open questions remain (Lecture 3)