

Astrophysical polarimetry

Dr. Frédéric Marin



Total radio continuum emission from the "Whirlpool" galaxy M51 (NRAO/AUI)



Overview

- **General introduction**
- **II Polarization : what is it and where can we observe it ?**
- **III** Theory
- **IV** Polarization mechanisms
- **V Observational techniques**
- **VI** Modeling polarization
- **VII** Project about radio-loud quasars and polarization

The basics: History

Vikings used the light-polarizing property of Iceland spar to tell the direction of the sun on cloudy days (for navigational purposes)

The polarization of sunlight in the Arctic can be detected, and the direction of the sun identified to within a few degrees in both cloudy and twilight conditions using a birefringent material and the naked eye

See:

T. Ramskou, "Solstenen," Skalk, No. 2, p.16, 1967

"Sky Compass," Review of Scientific Instruments, Vol. 20, p.460, June 1949

Varangian Guard

The basics: History

1669: Rasmus Bartholin re-discovers the phenomenon of double refraction of a light ray by Iceland spar (calcite)

Accurate description published, but since the physical nature of light was poorly understood at the time, he was unable to explain it



Rasmus Bartholin

1801: Thomas Young proposed the wave theory of light

1852: Description of polarization in terms of observables (experimental definition) by George Gabriel Stokes

The basics: birefringence

Birefringence is formally defined as the double refraction of light in a transparent, molecularly ordered material

 \rightarrow existence of orientation-dependent differences in refractive index

Many transparent solids are optically isotropic, meaning that the index of refraction is equal in all directions throughout the crystalline lattice

Examples: glass, table salt ...



The basics: birefringence

Sodium chloride → isotropic crystalline lattice structure (same optical properties along three mutually perpendicular axes)

Calcium carbonate \rightarrow complex but ordered anisotropic crystalline lattice structure (different optical properties according to the axis)

Polymer \rightarrow amorphous (no recognizable periodic crystalline structure)

Crystalline Structure of Isotropic and Anisotropic Materials



The basics: birefringence

Example of anisotropic material: the Viking's sunstone



The basics: birefringence

Anisotropic crystals, such as quartz, calcite, and tourmaline, have crystallographically distinct axes and interact with light by a mechanism that is dependent upon the orientation of the crystalline lattice with respect to the incident light angle

When light enters the <u>optical axis</u> of anisotropic crystals, it behaves in a manner similar to the interaction with isotropic crystals, and passes through at a single velocity

When light enters a <u>non-equivalent axis</u>, it is <u>refracted</u> into two rays, each polarized with the vibration directions oriented at right angles (mutually perpendicular) to one another and traveling at <u>different velocities</u>

→ Double refraction (or birefringence)

The basics: birefringence



The basics: birefringence

The index of refraction is actually a complex quantity:



of light depends on media • birefringence: phasevelocity of light also depends on P

• dichroism/diattenuation: also depends on P

The basics: retarders

Why is it important to introduce double refraction and a phase difference ?

- → we cannot measure the electric field vector amplitude in two directions in the same time
- → we can transform any degenerated state of polarization to another state



B. Linearly Polarized Light at 45 Degrees



C. Circularly Polarized Light



D. Elliptically Polarized Light Fig. 1: Various polarizations of light

The basics: retarders

Why is it important to introduce double refraction and a phase difference ?

- → we cannot measure the electric field vector amplitude in two directions in the same time
- → we can transform any degenerated state of polarization to another state

Knowing the dielectric properties of the crystal, we know the phase shift and the refraction angles



The basics: retarders

There are two kind of retarder used in astrophysics

- quarter-wave plate (retardation = $90^{\circ} \pm 2n\pi$)
- half-wave plate (retardation = $180^{\circ} \pm 2n\pi$)

Each is associated with a peculiar Mueller matrix (with ρ the position angle of the plate optical axis):

$$Quarter-wave \qquad \mathsf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 4\rho & \sin 4\rho & 0 \\ 0 & \sin 4\rho & -\cos 4\rho & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$
$$Half-wave \qquad \mathsf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho & \sin 2\rho \cos 2\rho & -\sin 2\rho \\ 0 & \sin 2\rho \cos 2\rho & \sin^2 2\rho & \cos 2\rho \\ 0 & \sin 2\rho & -\cos 2\rho & 0 \end{pmatrix}$$

The basics: retarders

The quarter waves transforms circular polarization into linear one (and vice-versa) → it is thus possible to measure circular polarization with linear polarizers



B. Linearly Polarized Light at 45 Degrees



C. Circularly Polarized Light



D. Elliptically Polarized Light Fig. 1: Various polarizations of light

The basics: retarders

The quarter waves transforms circular polarization into linear one (and vice-versa) → it is thus possible to measure circular polarization with linear polarizers

The half-wave plate does not change the degree of polarization but rotates the plane of polarization of linearly polarized light to the symmetric direction with respect to its optical axis and reverses the direction of circular polarization



The basics: retarders

Instrumental troubles

- dependence on wavelength (chromatism)



The basics: retarders

Using a rotating birefringent material (retarder) and a fixed polarizer (analyzer), we can measure the orthogonal amplitudes of the electric field \rightarrow temporal modulation



Coupled to a lens, a spectropolarimeter and a photo-multiplier, one can easily create a simple polarimetric telescope



The basics: analyzer

Ideal linear polarizer (analyzer) at angle χ :

Linear
$$(\pm S_1)$$
 polarizer at 0°

Circular $(\pm S_3)$ polarizer at 0°

	(1	± 1	0	0)		(1	0	± 1	0)		1	0	0	± 1
0.5	±1	1	0	0	0.5	0	0	0	0		0	0	0	0
	0	0	0	0		±1	0	1	0	0.5	0	0	0	0
	0	0	0	0		0	0	0	0		<u>±1</u>	0	0	1)

Linear $(\pm S_{2})$ polarizer at 0°

The basics: polarization measurement

It is very easy to measure polarization in terms of intensities

The first three Stokes parameters $(S_0, S_1 \text{ and } S_2)$ are measured setting the retarder to its optical axis (no phase shift, $\rho = 0^\circ$) and rotating the transmission axis of the polarizer (θ) to the angles 0°, 45° and 90°, $S_{LHP} = I_0 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad S_{LVP} = I_0 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad S_{L+45P} = I_0 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

The parameter S₃ (circular polarization) is measured using a quarter-wave retarder ($\rho = 90^{\circ}$) and setting $\theta = 45^{\circ}$



The basics: polarization measurement

Thus, one can measure 4 intensities in terms of $I(\rho, \theta)$:

$$I(0^{\circ},0^{\circ}) = 0.5^{*}(S_{0} + S_{1})$$

$$I(45^{\circ},0^{\circ}) = 0.5^{*}(S_{0} + S_{2})$$

$$I(90^{\circ},0^{\circ}) = 0.5^{*}(S_{0} - S_{1})$$

$$I(45^{\circ},90^{\circ}) = 0.5^{*}(S_{0} + S_{3})$$

Or, expressed in terms of the Stokes vector:

$$S_{0} = I(0^{\circ}, 0^{\circ}) + I(90^{\circ}, 0^{\circ})$$

$$S_{1} = I(0^{\circ}, 0^{\circ}) - I(90^{\circ}, 0^{\circ})$$

$$S_{2} = 2*I(45^{\circ}, 0^{\circ}) - I(0^{\circ}, 0^{\circ}) - I(90^{\circ}, 0^{\circ})$$

$$S_{3} = 2*I(45^{\circ}, 90^{\circ}) - I(0^{\circ}, 0^{\circ}) - I(90^{\circ}, 0^{\circ})$$

The basics: polarization measurement

Another, simple technique consists of using the splitting capacity of polarizing birefringent crystals (such as Wollaston prisms) along their non-equivalent axes



The basics: polarization measurement

Modulation scheme	Advantages	Disadvantages
temporal	Negligible effects of flat field and optical aberrations	Influence of seeing if modulation is slow
	Potentially high polarimetric sensitivity	Read-out rate of regular array detectors limits modulation frequency
spatial	Off-the-shelf array detectors can be used	Requires up to four times larger array detector
	High photon collection efficiency	Influence of flat field
	Allows post-facto image reconstruction	Influence of differential aberrations

Astrophysical Spectropolarimetry: Proceedings of the XII Canary Islands Winter School of Astrophysics

The basics: polarization measurement

Modulation scheme	Advantages	Disadvantages				
temporal	Negligible effects of flat field and optical aberrations	Influence of seeing if modulation is slow				
	Potentially high polarimetric sensitivity	Read-out rate of regular array detectors limits modulation frequency				
spatial	Off-the-shelf array detectors can be used	Requires up to four times larger array detector				
	High photon collection efficiency	Influence of flat field				
	Allows post-facto image reconstruction	Influence of differential aberrations				
Very important !!	Astrophysical Spectropolarimetry: Proceedings of the XII Canary Islands Winter School of Astrophysics					

The biggest problem of polarimetry

Polarization measurements are photon hungry, as the Stokes parameters of light must be recored with sufficient statistics

In case of an X-ray polarimeter with an ideal response to polarization, to detect a 1% X-ray polarization signal against the statistical fluctuations in the instrumental response to the polarized signal, if we accept a 10σ standard deviation, 4×10^6 photons would be needed

In comparison, you only need 10 photons for source detection and 100 photons for a spectral slope measurement

→ polarization measurements take more time and are thus less used than spectroscopic instruments

Special cases: radio polarization



A radio telescope uses a large metal dish or wire mesh, usually parabolic-shaped, to reflect the radio waves to an antenna (feed) above the dish (for long wavelength radio waves, metal meshes are excellent reflector)

Special cases: radio polarization



Receiving antenna's convert electromagnetic radiation to an electrical current (or vice-versa for a transmitting antenna)

Feeds are designed to be <u>sensitive to one kind of</u> <u>polarization</u> (either native linear or native circular)

With an array of antenna with different feeds, <u>no needs for</u> <u>rotation</u>!

Special cases: high energy polarization

Above the UV band, photons are too energetic to interact with lenses and optical detectors. Measuring the polarization of X-rays in the 1 to 300 keV energy range is most easily done by Compton scattering and photoelectric effects.

The basic design of a Compton polarimeter consists of two detectors

→ the detectors are capable of determining the energies of the scattered photon and recoil electron along with the location of the Compton interaction site and direction of the scattered photon

The first, a low-Z (tracker) detector, has a high probability of Compton interactions, and the second, a high-Z detector (calorimeter) that has a high probability of absorbing the scattered photon

By analyzing the kinematics of each event we can determine the path of each photon that interacts within the detector

Special cases: high energy polarization

Remember the Compton scattering differential cross section is given by the Klein-Nishina formula:

$$d\sigma = \frac{r_o^2}{2} d\Omega \left(\frac{v'}{v_o}\right)^2 \left(\frac{v_o}{v'} + \frac{v'}{v_o} - 2\sin^2\theta\cos^2\eta\right) \quad \text{with} \quad \frac{v'}{v_o} = \frac{1}{1 + \left(\frac{hv_o}{mc^2}\right)(1 - \cos\theta)}$$

Fixing the scattering angle θ , the cross section for polarized photons is a maximum at azimuthal angles $\eta = 90^{\circ}$ and a minimum at $\eta = 0^{\circ}$

→ asymmetry in the number of photons scattered in the direction parallel and orthogonal to the incident photon electric vector; in other words, photons will tend to scatter 90° to their incident electric field vector

It is this phenomenon that is utilized in the Compton polarimeter

Special cases: high energy polarization

The polarization measurement is accomplished by recording the azimuthal modulation pattern of the scattered photons \rightarrow modulation pattern that can be fit with: $C(\eta) = A \cos\left(2\left(\eta - \varphi + \frac{\pi}{2}\right)\right) + B$

where φ is the polarization angle of the incident photons and A and B are constants used in the fit

With a proper modulation fit we can use the parameters A and B to find the polarization modulation factor μ_{n} :

$$\mu_p = \frac{C_{p,\max} - C_{p,\min}}{C_{p,\max} + C_{p,\min}} = \frac{A}{B}$$

The quality of the polarization signature is quantified by the polarization modulation factor

Special cases: high energy polarization

$$\mu_p = \frac{C_{p,\max} - C_{p,\min}}{C_{p,\max} + C_{p,\min}} = \frac{A}{B}$$

 $C_{p,max}$ and $C_{p,min}$ represent the maximum and minimum number of counts measured in the polarimeter

In order to determine the polarization of the incident beam we need to know the response of the detector to a beam of 100% polarized photons.

This can be measured experimentally or by performing Monte Carlo simulations for the particular detector design and structure





Special cases: high energy polarization

The modulation factor for a completely polarized beam (μ_{100}) can be used to find the unknown polarization P is: μ_p

$$P = \frac{\mu_p}{\mu_{100}}$$

To determine the minimum detectable polarization (MDP) of a specific detector design we use:

$$MDP(\%) = \frac{n_{\sigma}}{\mu_{100}R_{src}} \sqrt{\frac{2\left(R_{src} + R_{bgd}\right)}{T}}$$

where n_{σ} is the significance level (number of sigma), R_{src} is the total source counting rate, R_{bgd} is the total background counting rate and T is the observation time

Special cases: high energy polarization

Example of Compton polarimeter: GRAPE, X-Calibur, XPE, ...

Classical techniques (Bragg diffraction and Thompson/Compton scattering) can provide "good" performances but with very low efficiency

Special cases: high energy polarization

Example of Compton polarimeter: GRAPE, X-Calibur, XPE, ...

Classical techniques (Bragg diffraction and Thompson/Compton scattering) can provide "good" performances but with very low efficiency

Another, more efficient way to measure X-ray polarization

→ photoelectric effect



Special cases: high energy polarization

Performances / real tracks



Muleri et al. (2008)

5.41 keV, He-DME mixture at 1 atm

Special cases: high energy polarization

Performances / real tracks

Overview

- **General introduction**
- **II Polarization : what is it and where can we observe it ?**
- **III** Theory
- **IV** Polarization mechanisms
- **V** Observational techniques
- **VI** Modeling polarization
- **VII** Project about radio-loud quasars and polarization

Radiative transfer

<u>Radiative transfer</u> is the branch of physics aimed to study the <u>change in the</u> properties of a light beam while it passes through a medium

The study of radiative transfer allows us to establish the link between the microscopic interactions occurring between photons and atoms and the macroscopic properties of the objects studied in astrophysics

By comparing observed and simulated spectra it is possible to put constraints on the physical conditions (density, temperature, geometry, ...) prevailing in stars, galaxies or quasars

Radiative transfer

The energy flux radiated by an extended source in a given direction, divided by the apparent area of the source in the same direction is called luminance

$$L_{\nu}(\vec{r}, \vec{s}, t) = \frac{d^3 \phi_{\nu}(\vec{r}, t)}{(\vec{s}.\vec{n}) dt d\Omega dS}$$

Let's now consider that the incident photon beam passes through a dense medium composed of spherical particles

→ the photons probably interact with the matter and three phenomena can occur (absorption, emission and scattering)

$$dL_{\nu}(\vec{r}, \vec{s}) = dL_{\nu,a}(\vec{r}, \vec{s}) + dL_{\nu,e}(\vec{r}, \vec{s}) + dL_{\nu,d}(\vec{r}, \vec{s}) = -\sigma_{\nu,a}L_{\nu}(\vec{r}, \vec{s})ds + \sigma_{\nu,e}L_{b,\nu}(\vec{r}, \vec{s})ds - \sigma_{\nu,d}L_{\nu}(\vec{r}, \vec{s})ds + \sigma_{\nu,d} \left[\frac{1}{4\pi} \int_{0}^{\infty} \int_{0}^{4\pi} \Phi(\vec{s'}, \vec{s}, \nu, \nu')L_{\nu'}(\vec{s'})d\Omega'd\nu'\right] ds$$

Radiative transfer

Defining an extinction coefficient βv representing the beam attenuation by out-coming scattering and absorbing particles as well as the system albedo κv , both wavelength-dependent, one can write:

$$\frac{\mathrm{d}L_{\nu}(\vec{r},\vec{s})}{\mathrm{d}s} = \beta_{\nu}(S_{\nu}(\vec{r},\vec{s}) - L_{\nu}(\vec{r},\vec{s}))$$

$$L_{\nu}(\vec{r},\vec{s}) = L_{\nu}(\vec{r_0},\vec{s})\Gamma_{\nu}(\vec{r_0}\rightarrow\vec{r}) + \int_{\vec{r_0}}^{\vec{r}} S_{\nu}(\vec{r},\vec{s}) \frac{\partial\Gamma_{\nu}(\vec{r'}\rightarrow\vec{r})}{\partial\vec{r'}} \mathrm{d}\vec{r'}$$

Which are the differential and the integral formulation of the radiative transfer equation, respectively

Monte Carlo

Solving the radiative transfer equation is not straightforward; at least <u>four</u> <u>methods</u> can be found through the literature: the differential, integral, statistic and hybrid methods

The differential method resolves the differential equation of the radiative transfer formulation, while the integral method solves the integral equation

The hybrid methods are combinations of differential, integral and statistic methods, adapted to specific needs

In modern research, it is convenient to simulate the radiative transfer equation using the Monte Carlo statistical method

Monte Carlo

The essence of the Monte Carlo Method is sampling from probability distribution functions (PDFs), and this is referred to as the 'Fundamental Principle'

Lets denote a random number ξ associated with a uniform distribution function over the interval [0, 1] such as:

$$P(\xi)\mathrm{d}\xi = \begin{cases} \mathrm{d}\xi & \text{if } 0 < \xi < 1\\ 0 & \text{outside of the interval} \end{cases}$$

Monte Carlo

Once such a ξ number chosen, it is easy to obtain a random number from a distribution function f (x) using a transformation method

→ if $y \in \{xmin, xmax\}$ is a variable whose distribution function is f(y), it is possible to determine a random value x_0 from such a distribution function by generating a random number ξ and solving the following equation:

$$\xi = P(x_0) = \frac{\int_{x_{min}}^{x_0} f(y) dy}{\int_{x_{min}}^{x_{max}} f(y) dy}$$

where the denominator is used to normalize the cumulative distribution function such as:

$$\int_{x_{min}}^{x_{max}} P(x_0) dx_0 = \int_{x_{min}}^{x_{max}} \frac{\int_{x_{min}}^{x_0} f(y) dy}{\int_{x_{min}}^{x_{max}} f(y) dy} dx_0 = 1$$

Monte Carlo

Example 1

isotropic distribution

 $P(\mu,\phi)d\mu d\phi = d\mu/2d\phi/(2\pi)$

Sampling a scattering angle from an

As $x_{_0}$ ranges from $x_{_{min}}$ to $x_{_{max}}$, ξ ranges from 0 to 1 uniformly

 $\mu_0 = 2\xi_1 - 1$

 $\phi_0 = 2\pi\xi_2$

Thus, to sample a random variable x_0 , it is just needed to call a random number generator (RNG) that uniformly samples numbers between 0 and 1, and invert the first equation to get x_0 .

$$\xi = P(x_0) = \frac{\int_{x_{min}}^{x_0} f(y) dy}{\int_{x_{min}}^{x_{max}} f(y) dy}$$

where $\mu = \cos \theta$, $d\mu = \sin \theta d\theta$

 ξ_1 and ξ_2 = random numbers

Monte Carlo

Example 2 Sampling a scattering angle from a non-isotropic differential scattering cross section; the case of electrons bounds to atom and submitted to ionizing radiation Z=1

Monte Carlo

In complicated cases, one can generate random number with arbitrary distribution P(x) using the Rejection Method (or other methods)

Monte Carlo

Construct your own code !

Flow chart of the Monte Carlo code STOKES (Goosmann & Gaskell 2007)

Overview

- **General introduction**
- **II Polarization : what is it and where can we observe it ?**
- **III** Theory
- **IV** Polarization mechanisms
- **V** Observational techniques
- **VI** Modeling polarization
- **VII Project about radio-loud quasars and polarization**

Student project

Radio-loud quasars and polarimetry

See with the teacher (not part of the final exam)

